A Labelled Deduction System for Kanger's Deontic-Praxeological Logic

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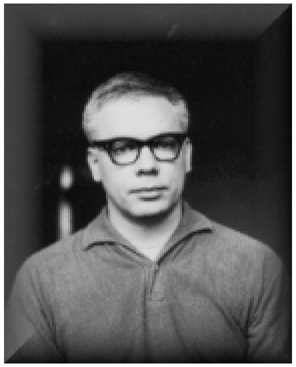
Plan of paper "A LDS for KTR"

- What kind of theory is Kanger's theory of rights?
  - Set of implications.
- What kind of logic is adequate for it?
  - Segerberg's criteria.
- How to asses adequacy of a logic for KTR?
  - Proof strategy.
- Can a labelled deduction system be adequate for KTR?
  - Yes.



Stig Kanger (1924-1988)

References

Kanger's theory of rights

- Kanger, Stig (1972) Law and Logic. *Theoria* **38** : 105-129.
- Kanger, Stig (1966) Rights and Parliamentarianism. *Theoria* **32** : 85-115.

Reprinted in:

- G. Holmström-Hintikka, S. Lindström and R. Sliwinski (eds.) *Collected Papers of Stig Kanger with Essays on his Life and* Work, Vols. I-II. Kluwer Academic Publishers, 2000.

Criteria of adequacy

- Segerberg, Krister (1992) Getting Started: Beginnings in the Logic of Action. *Studia Logica*, **51**: 347-378.

Action semantics

- Hilpinen, Risto. (1997) On Action and Agency. In: E. Ejerhed and S. Lindström (eds.), *Logic, Action and Cognition – Essays in Philosophical Logic*, pp. 3-27. Kluwer Academic Publishers

Labelled deduction systems

- Basin, David, Matthews, Seán and Viganò, Luca (1998) Natural Deduction for Non-Classical Logics. *Studia Logica* **60**:119-160

Kanger’s theory of rights

“Strength diagram”

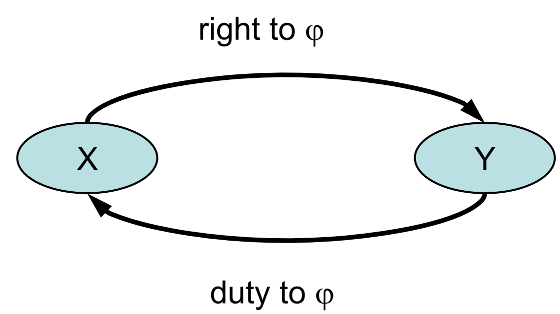
Some history: Wesley Newcomb Hohfeld

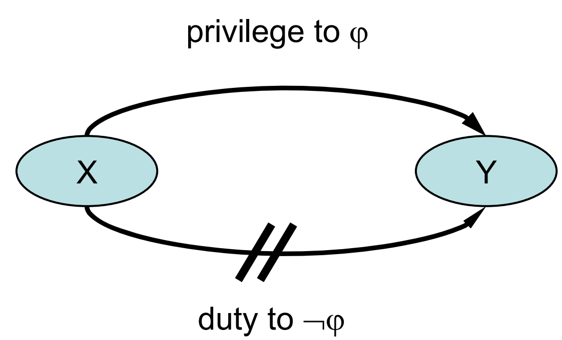
- Seminal paper on logic of rights: (1913) Some Fundamental Legal Conceptions as Applied in Judicial Reasoning. I. *Yale Law Journal* **23**: 16-59.

Jural Opposites	right	privilege	power	immunity
	no-right	duty	disability	liability
Jural Correlatives	right	privilege	power	immunity
	duty	no-right	liability	disability

Hohfeldian classification

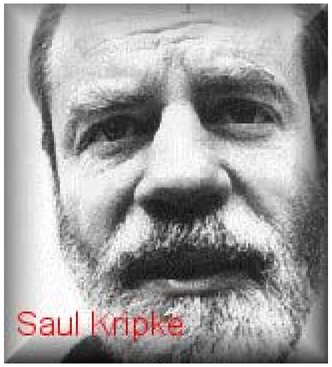
- Interdefinability:
  - Correlatives as a kind of converse relations: RightTo[φ](X,Y) iff DutyTo[φ](Y,X).
  - Opposites as similar to “complement” relation: PrivilegeTo[φ](X,Y) iff ¬DutyTo [¬φ](X,Y).

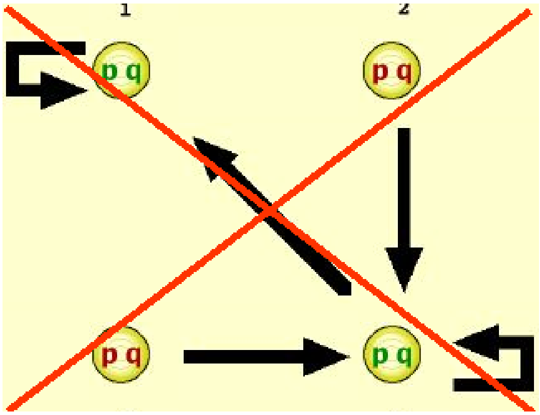




Modal logic and theory of rights

- Stig Kanger.
  - Regarded his theory of rights as one of his substantial contributions to philosophy.
  - Worked on it intermittently for thirty years.
  - Methods of modal logic.
- Interesting fact:
  - Kanger and Kripke independently invented semantics of “possible worlds with accessibility relations”.
  - But in KTR Kanger used only axiomatic (not formal semantic) methods.
    - Explanation: the semantics of rights involves the semantics of action, which he was trying to develop.





Active and passive rights

- Hohfeld-Kanger notion of right.
  - There are eight types of “simple” rights.
  - Right is a relation between: Two agents,
    - Bearer and Counterparty
  - w.r.t. Action,
    - s. t. one of two agents has duty or permission to execute or to omit the action (which is the “object-matter” of the right).
  - In accordance with traditional distinction between:
    - “Passive rights”, right to have something done (Counterparty obligatives) and “Active rights”, right to do something (Bearer’s permissives).

COUNTERPARTY OBLIGATIVES

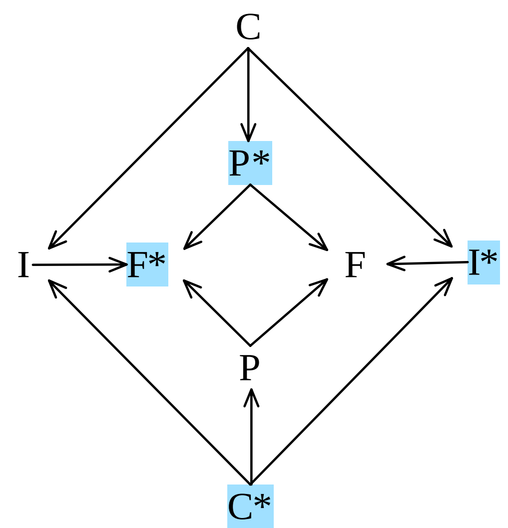
$$Shall\ \pm Do\left(y,\pm F\right)$$

BEARER PERMISSIVES

$$\neg Shall\ \pm Do\left(x,\pm F\right)$$

Looking inside the rights

- Two modal operators:
  - Deontic [Shall],
  - Praxeological [Do].
    - Historical remark:
      - Fitch, Frederic (1963) A Logical Analysis of Some Value Concepts. *The Journal of Symbolic Logic* **26**: 135-142
- Types of simple rights: P: [(Counter) Claim, (Counter) Immunity], A: [(Counter) Power, (Counter) Freedom].

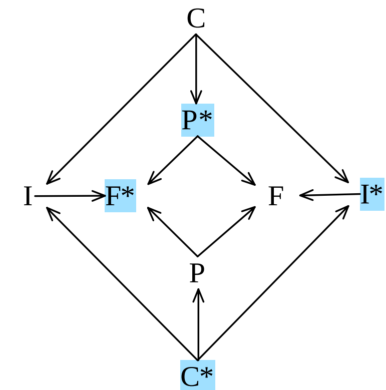


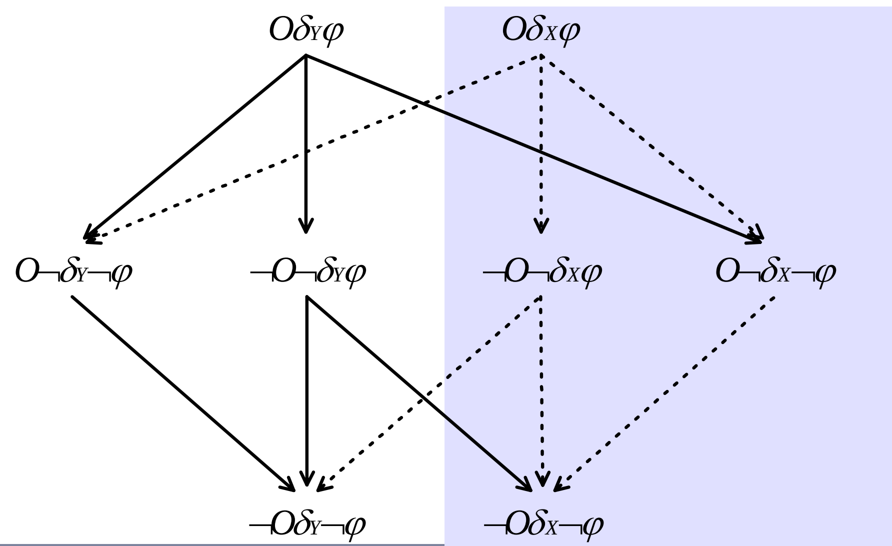
(X has a claim) $O\delta_Y\varphi$

(X has a caounterclaim) $O\delta_{Y\neg}\varphi$

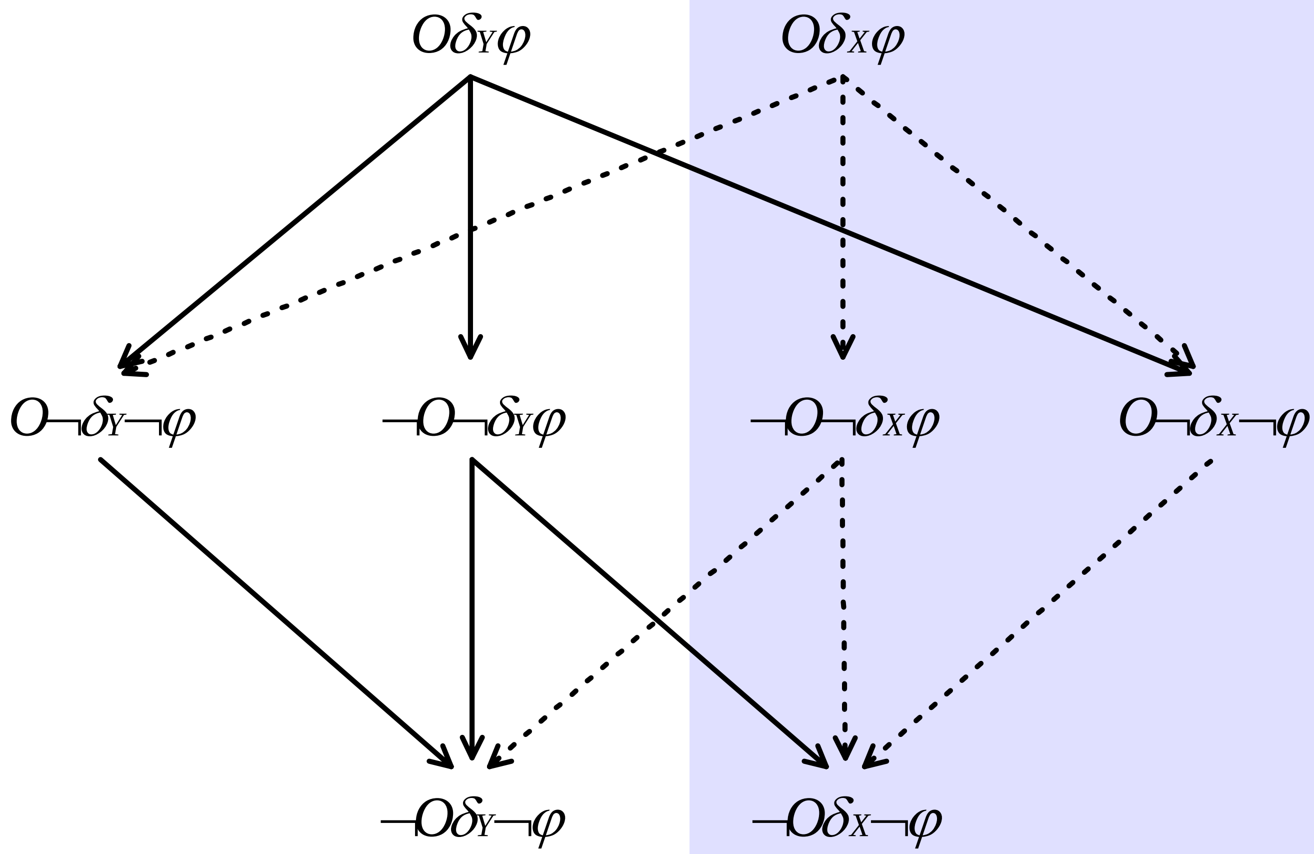
Strength diagram

- How to read it?
  - C implies P*:= ‘X has a claim-right versus Y to the effect that φ’ implies ‘Y has a power-right versus X to the effect that φ’.
- How to translate it to deontic-praxeological logic?
  - C implies P*:=‘It shall be that Y sees to it that φ’ implies ‘Not: it shall be that not: Y sees to that φ’ .
    - More informal reading: If an action is obligatory, it is permitted.

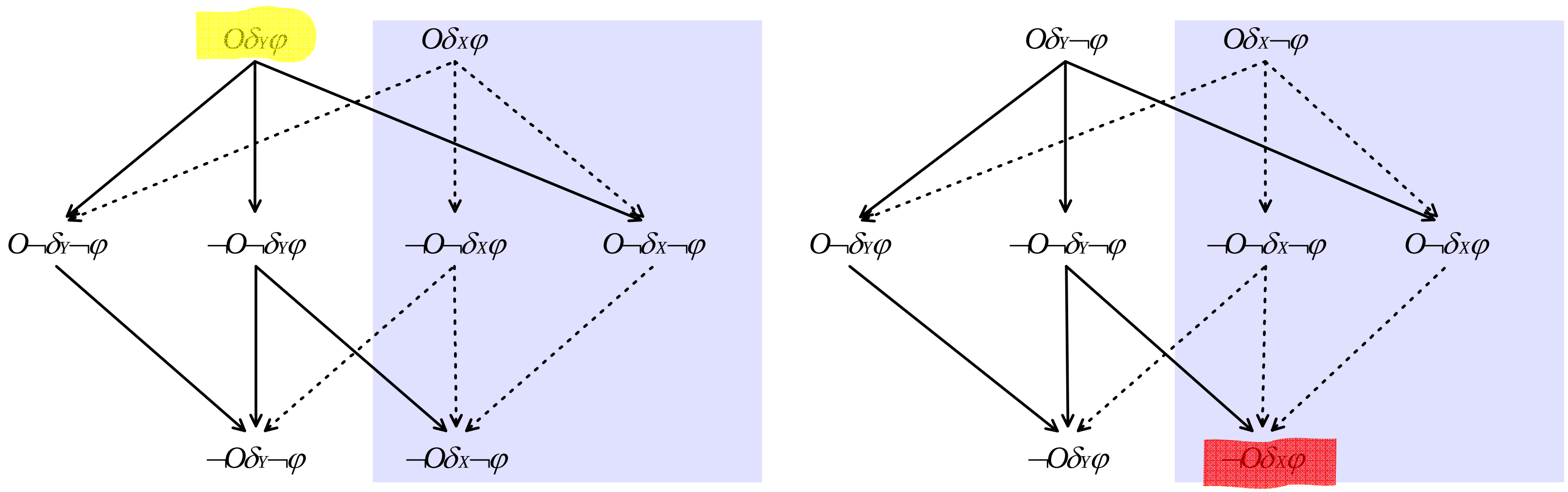




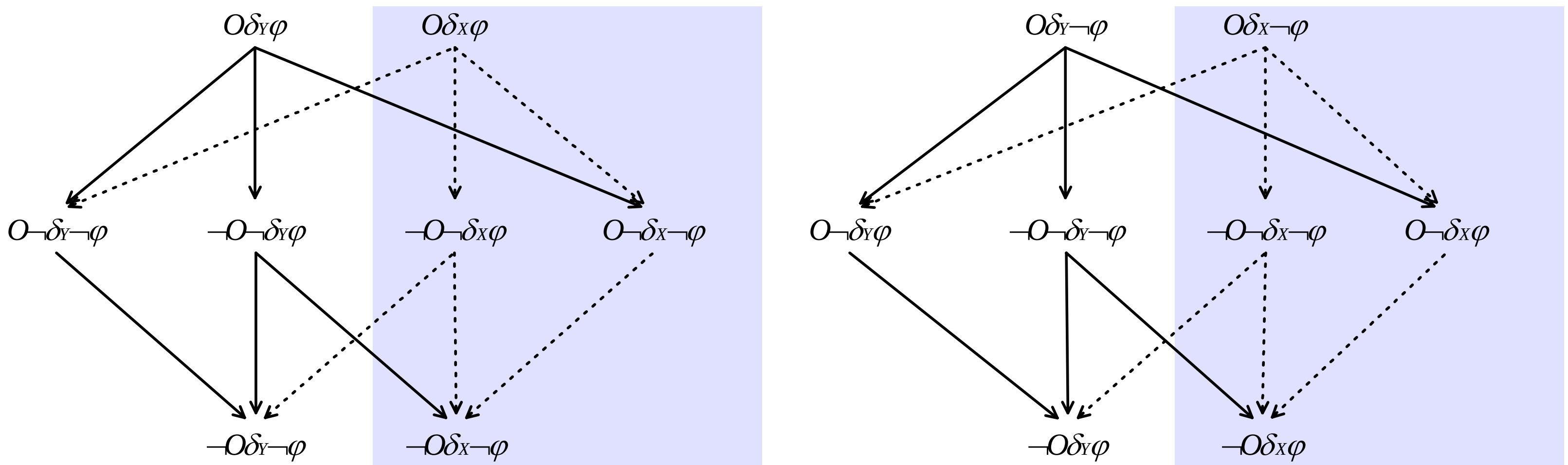
Strength diagram: DP explication



Atomic rights: combination of affirmations and negations of simple rights [ar3]

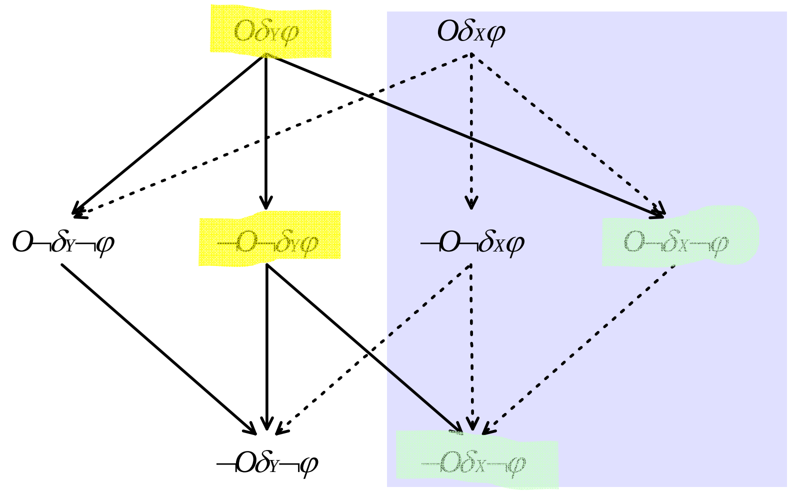


Atomic rights: combination of affirmations and negations of simple rights



Two-agents deontic system

- Relations between deontic statuses of actions of two agents.
- Left side: agent Y. Right side: agent X.
- "Transversal" implications show: that if an action of one agent is obligatory, then the "opposite" action of another agent is forbidden (and she is not obliged to do it).



What kind of theory is KTR?

- Strength diagram shows in a maximal way necessary configurations of any logically viable normative system.
- What kind of logic would be adequate for KTR (i.e. “strength diagram”)?
  - The provability of SD-implications is not sufficient for adequacy.
  - No other deontic-praxeological implication should be provable.

$$RP^{imp} = \left\{ \begin{array}{l} C \Rightarrow I, C \Rightarrow P*, C \Rightarrow I*, P* \Rightarrow F*, P* \Rightarrow F, \\ C* \Rightarrow I*, C* \Rightarrow P, C* \Rightarrow I, P \Rightarrow F, P \Rightarrow F* \end{array} \right\}$$

Necessary condition of adequacy of logic $\vdash_l$ for KTR:

If $\varphi \in RP^{imp}$, then $\vdash_l \varphi$.

Segerberg's criteria

- "[...] Kanger himself does not tie his presentation to a unique underlying logic. Rather, he notes the conditions a logic must satisfy in order to fit his theory. [...] the logic must support the logical relationships portrayed in the diagrams [...] "That is to say,
- **the minimum condition** is that the logic must be strong enough to allow the derivation of logical implications in the diagram;
- and **the maximum condition** is that the logic must not be so strong that any new implications are added in those diagrams."

Construction of logic for KTR and applying Segerberg's criteria

Tasks:
Constructing LDS and
[minimum condition] proving provability, and
[maximum condition] proving unprovability

First step: defining KTR

- We set that the relation of logical implication is reflexive and transitive.
- KTR is reflexive and transitive closure of RP^{imp} (i.e. implications depicted in the "strength diagram").

For simple right types X, Y, Z, KTR is the smallest set such that:

(i) $RP^{imp} \subseteq KTR$,

(ii) $X \Rightarrow X \in KTR$,

(iii) If $X \Rightarrow Y \in KTR$ and $Y \Rightarrow Z \in KTR$, then $X \Rightarrow Z$.

Second step: choosing a logic

- Labelled deduction for normal propositional modal logics.
  - Approach developed by: Basin, Matthews, and Viganò.
- Modification of BMV approach:
  - Application to multimodal case.
  - Adjustment to “Fitch style” proofs.

The basic ideas of BMV approach

- The elimination and introduction rules for universal and existential modalities remain the same for different logics.
- The difference is established *via* “relational theory” which encodes the properties of the frames characterized by the logic in question.
  - Example: instead of *D* axiom we use assumption free rule *R(w,f(w))*, where *f* is Skolem function.
- The non-relational language is enriched with set of indices.
  - Syntax for non-relational sentences: [index] [colon] [unlabelled sentence].

L-deduction rules for connectives

- The sentences must have the same index (label) for a rule to be applicable.
- Example: conditional introduction [proof-to-proof rule] and conditional elimination [sentence(s)-to-sentence rule].
- Important exception: negation rule.

$\rightarrow Intro$

$$\frac{\Gamma, w_i : \varphi \vdash w_i : \psi}{\Gamma \vdash w_i : \varphi \rightarrow \psi}$$

$\rightarrow Elim$

$$w_i : \varphi \rightarrow \psi, w_i : \varphi \vdash w_i : \psi$$

Global negation rule

- Local negation is not strong enough.
- Intuitively: impossible world in a frame shows that the frame itself is not possible.

$$\neg Intro$$

$$\frac{\Gamma, w_i : A \vdash w_j : \bot}{\Gamma \vdash w_i : \neg A}$$

Rules for universal and existential modalities

$\Box$ Intro

$$\frac{\Gamma, R(w_i, w_j) \vdash w_j : \psi}{\Gamma \vdash w_i : \Box\psi}$$

$[w_j \text{ is new index}]$

$\Diamond$ Intro

$$R(w_i, w_j), w_j : \varphi \vdash w_i : \Diamond\varphi$$

$\Box$ Elim

$$R(w_i, w_j), w_i : \Box\varphi \vdash w_j : \varphi$$

$\Diamond$ Elim

$$\frac{\Gamma, R(w_i, w_j), w_j : \varphi \vdash w_k : \psi}{\Gamma, w_i : \Diamond\varphi \vdash w_k : \psi}$$

$[w_j \text{ is new index}]$

Geometry of meaning

- Hypothesis: LDS open new modelling possibilities.
  - Example: irreflexive frames.
    - Uncharacterizable using “box/diamond” language.
    - Could be easily represented within relational theory.

Choosing modalities

- For O the choice seems obvious.
  - Universal modality.
  - Relational theory: seriality (D).
- For δ?
  - Universal modality.
  - Relational theory: reflexivity (T).
    - On the right side: Kanger's axiomatization from Rights and Parliamentarianism.

$$\frac{\vdash \varphi \rightarrow \psi}{\vdash O\varphi \rightarrow O\psi}$$

$$(O\varphi \wedge O\psi) \rightarrow O(\varphi \wedge \psi)$$

$$O\varphi \rightarrow \neg O \neg \varphi$$

$$\frac{\vdash \varphi \leftrightarrow \psi}{\vdash \delta\varphi \leftrightarrow \delta\psi}$$

$$\delta\varphi \rightarrow \varphi$$

Rules for LDS for KTR

$\odot$ *Intro*

k.	$R_{\odot}(w_i, w_j)$		
	$\vdots$		
l.	$w_j : \varphi$		
m.	$w_i : \odot\varphi$	$\odot$ Intro: k-l	

[w_j is new index, may not appear outside its subproof]

$\odot$ *Elim*

k.	$R_{\odot}(w_i, w_j)$	
	$\vdots$	
l.	$w_i : \odot\varphi$	
	$\vdots$	
m.	$w_j : \varphi$	$\odot$ Elim: k, l

O seriality

	$\vdots$	
k.	$R_O(w_i, f(w_i))$	O seriality
	$\vdots$	

δ reflexivity

	$\vdots$	
k.	$R_{\delta X}(w_i, w_i)$	δ reflexivity
	$\vdots$	

	$\vdots$	
k.	$R_{\delta Y}(w_i, w_i)$	δ reflexivity
	$\vdots$	

Proving minimum condition

- Easy part in proof of adequacy og logic for KTR.
  - Construct a proof for each implication from RP[imp].
    - Other implications (gained due to reflexivity and transitivity) are trivial (reiteration and hypothetical sylogism are provable).
- Example: transversal implication.

Example <4.10>. $\vdash^{ld}_{O\delta} w_0 : O\delta_Y\varphi \rightarrow O\neg\delta_X\neg\varphi$

1.		
2.	$w_0 : O\delta_Y\varphi$	
3.	$R_O(w_0, w_1)$	
4.	$w_1 : \delta_Y\varphi$	O Elim: 2, 3
5.	$R_{\delta Y}(w_1, w_1)$	δ reflexivity
6.	$w_1 : \varphi$	δ_Y Elim: 4, 5
7.	$w_1 : \delta_X\neg\varphi$	
8.	$R_{\delta X}(w_1, w_1)$	δ reflexivity
9.	$w_1 : \neg\varphi$	δ_X Elim: 7, 8
10.	$w_1 : \bot$	$\bot$ Intro: 6, 9
11.	$w_1 : \neg\delta_X\neg\varphi$	$\neg$ Intro: 7-10
12.	$w_0 : O\neg\delta_Y\neg\varphi$	O Intro: 3-11
13.	$w_0 : O\delta_Y\varphi \rightarrow O\neg\delta_X\neg\varphi$	$\rightarrow$ Intro: 2-12

Example <4.10>. $\vdash^{Id}_{O\delta} w_0 : O\delta_Y\varphi \rightarrow O\neg\delta_X\neg\varphi$

1.		
2.	$w_0 : O\delta_Y\varphi$	
3.	$R_O(w_0, w_1)$	
4.	$w_1 : \delta_Y\varphi$	O Elim: 2, 3
5.	$R_{\delta Y}(w_1, w_1)$	δ reflexivity
6.	$w_1 : \varphi$	δ_Y Elim: 4, 5
7.	$w_1 : \delta_X\neg\varphi$	
8.	$R_{\delta X}(w_1, w_1)$	δ reflexivity
9.	$w_1 : \neg\varphi$	δ_X Elim: 7, 8
10.	$w_1 : \bot$	$\bot$ Intro: 6, 9
11.	$w_1 : \neg\delta_X\neg\varphi$	$\neg$ Intro: 7-10
12.	$w_0 : O\neg\delta_Y\neg\varphi$	O Intro: 3-11
13.	$w_0 : O\delta_Y\varphi \rightarrow O\neg\delta_X\neg\varphi$	$\rightarrow$ Intro: 2-12

Harder : proving unprovability

- All rights-implications not belonging to KTR should be unprovable in LD system.

U is the set of right types.

Logic $\vdash_l$ satisfies maximum condition iff

for any $R \in (U \times U - KTR)$ it holds that $\nvdash_l R$.

Strategy of the proof

1. Introducing a simple semantics for deontic and praxeological modalities.
2. Proving soundness of LD system.
3. Contraposition: unsatisfiable sentences are unprovable.
4. Construction of counterexamples for each “undesirable” implication.

The simplest semantics of action

- Hilpinen attributes the simplest semantics of action to Chellas (1969).
  - “[...] ‘*a* sees to it that *p*’ is true in a world *u* if and only if *p* is true in every alternative to *u*. The alternatives to a given world *u* are its ‘practical’ alternatives: they are worlds in which agent behaves in the same way as in *u*.”

Simple model

- Three accessibility relations:
  - one reflexive deontic relation, and two serial praxelogical relations (one for each agent).
- Valuation of propositional letters at worlds.
- Additionally: interpretation function for predicates and terms of relational theory.

$$\mathrm{R}_O \text{ is serial}, \mathrm{R}_{\delta X}, \mathrm{R}_{\delta Y} \text{ are reflexive.}$$

$$[\![R_\odot]\!] = \mathrm{R}_\odot,\ \odot \in \{O, \delta X, \delta Y\}$$

$$[\![w]\!] \in W,\ [\![f(w)]\!] \in \{v : R_O(w, v)\}$$

$$\mathfrak{M} = \langle \langle W, \mathrm{R}_O, \mathrm{R}_{\delta X}, \mathrm{R}_{\delta Y} \rangle, V, [\![\cdot]\!] \rangle$$

Standard semantic definitions

- Separate definitions for sentences of relational theory and for labelled sentences.
  - Examples show generalized modalities.

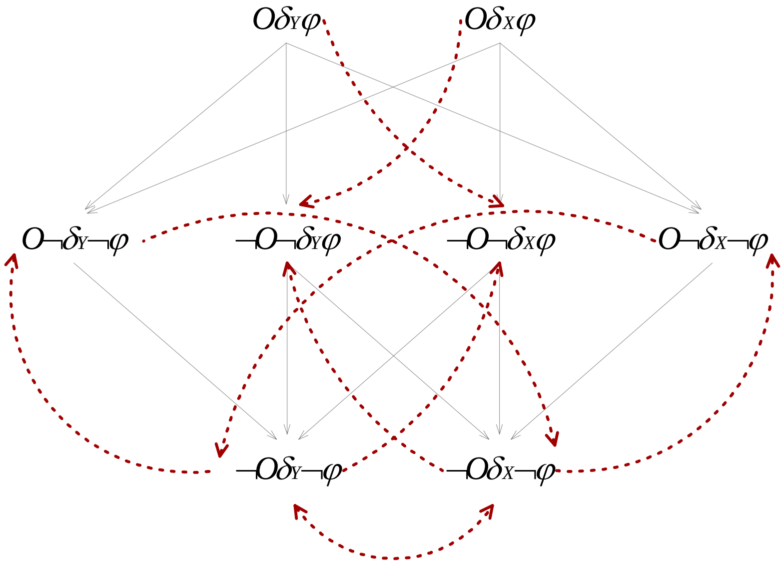
$$\mathfrak{M} \models R_{\odot}(t_1, t_2) \text{ iff } \langle [\![t_1]\!], [\![t_2]\!] \rangle \in [\![R_{\odot}]\!]$$

...

$$\mathfrak{M} \models w : \odot\varphi \text{ iff for all } v : \mathrm{R}_{\odot}([\![w]\!], v) \rightarrow \mathfrak{M} \models v : \varphi$$

Soundness proof and counterexample construction

- 1. Straightforward.
- 2. Laborious [40 counterexamples; reducible to 5 cases].



Conclusion

- LD system for KTR meets Segerberg criteria of adequacy.
- [Not discussed] LD system is complete with respect to "the simplest semantics" of action.
  - Modification of standard proof.

$$\Gamma_{i+1} \cup \Delta_{i+1} = \begin{cases} \Gamma_i \cup \Delta_i \text{ if } \Gamma_i \cup \{S_{i+1}\} \cup \Delta_i \vdash u : \bot \quad \text{for some } u \in I^*, \\ \text{otherwise } \begin{cases} \Gamma_i \cup \{S_{i+1}\} \cup \Delta_i \text{ if } S_{i+1} \neq w : \neg \odot \varphi, \\ \Gamma_i \cup \{w : \neg \odot \varphi, w_{\neg \odot \varphi} : \neg \varphi\} \cup \Delta_i \cup \{R_{\odot}(w, w_{\neg \odot \varphi})\} \text{ if } S_{i+1} = w : \neg \odot \varphi. \end{cases} \end{cases}$$

Thank you!