A Logical Typology of Normative Systems

Berislav Žarnić

University of Split, Croatia

Abstract

In this paper, the set-theoretic approach in the logical theory of normative systems is extended using Broome's definition of the normative code function. The syntax and semantics for first order metanormative language is defined, and metanormative language is applied in the formalization of the basic principles in Broome's approach and in the construction of a logical typology of normative systems. Special attention is given to the types of normative systems which are not definable in terms of the properties of singular sets of requirements (e.g. the realization equivalence of codes, the social compatibility of codes, and the compatibility of codes issued by different normative sources). Examples are given of the application of the typology in the interpretation of philosophical texts. Von Wright's hypothesis on the connection of logical properties of normative systems, conceived set-theoretically, with standard deontic logic is proved by introducing the translation function between the metanormative language and the restricted language of standard deontic logic. The translation reveals that von Wright's hypothesis must be appended. The problems of narrow and wide scope readings of the deontic conditionals and of the meaning of iterated deontic operators are addressed using the distinction between relative and absolute normative codes. The theorem on the existence of a realization equivalent absolute code for any relative code is proved.

Key words: deontic logic, metaethics

1 The Set-theoretic Approach to Normative Systems

What use can one make of the logic of intentionality (i.e. the logic of propositional attitude reports) in predicting and explaining human behaviour if in reality this logic can fail? For example, the logic of belief requires any agent not to have contradictory beliefs, and yet in reality agents' inconsistent belief systems abound. The status of the logic of intentionality has been a puzzling issue, since two intuitions on the nature of the laws of logic seem to collide. On the one hand, the laws of logic are construed as unavoidable in reality. On the other hand, it is well known that the laws of the logic of intentionality may fail in human theoretical and practical reasoning. The standard solution assigns a normative role to the logic of intentionality.

John Broome has developed a general metanormative perspective which provides a fruitful framework for the logical analysis of intentionality. In general, according to Broome, a normative source (e.g. rationality) may accord with reality and then the corresponding property (e.g. the property of being rational) is realized. Broome's distinction between normative sources and normative properties fits in well with the thesis of "normative essentialism" proposed by Zangwill (2005). Zangwill has put forward the thesis that the essence of the mental is to be subject to norms, not to conform to them. Using Broome's conceptual distinction, one might rephrase Zangwill's thesis as follows: the mental is essentially subjected to the requirements of normative sources, and it accidentally might conform to them, in which case some normative property becomes instantiated.

There has been a long debate on the logical character of normativity and on the normative character of logic. I will not argue for the logicality of the normative, or for the normativity of the logical. Rather, I will focus on the typology of normative systems in order to provide a formal explication of the different senses that the statements 'a normative system is logical' and 'a logical system is normative' may have. For the purpose of explication, I will rely on a set-theoretical approach in the logical theory of normative systems. The approach was introduced by Alchourrón and Bulygin (1998) who represented the force of the norm by the membership of its norm-content in a set (normative system); later von Wright (1999) discussed the approach as a possible interpretation of deontic logic; and, more recently, Broome (2007b) generalized the approach by treating the sets of norm-contents as values of code functions. The relevant quotation is reproduced below with minor alterations in symbols in order to match the signature that will be used later throughout this text.

> We must allow for the possibility that the requirements you are under depend on your circumstances. Here is how I shall do that formally, using possible worlds semantics. There is a set of worlds, at each of which propositions have a truth value. The values of all propositions at a particular world conform to the axioms of propositional calculus. For each source of requirements s, each person i and each world w, there is a set of propositions $\mathrm{k}_{\mathsf{s}}(i,w)$, which is to be interpreted as the set of things that s requires of i at w. Each proposition in the set is a required proposition. The function k_{s} from i and w to $\mathrm{k}_{\mathsf{s}}(i,w)$ I shall call s's *code* of requirements. (Broome, 2007b, 14)

Broome's approach bears significant resemblance to the concept of the normative system proposed by Alchourrón and Bulygin (1998).

> We can now define the concept of a normative system as the set of all the propositions that are consequences of the explicitly commanded propositions. (Alchourrón and Bulygin, 1998, 391)

Broome's concept of a code of requirements is more general in several respects. First, codes are ternary functions (taking as arguments a normative source, an agent and a world) and sets of requirements are their values. So, one can quantify over variables in the code function and obtain new concepts on that basis. Second, sets of requirements can be related to Alchourrón and Bulygin's normative system as their special case, namely as values of a deductively closed absolute code. The significant resemblance between the two notions consists in the fact that in both cases the force of a requirement (or a norm) is represented by the membership of the requirement-content (or the norm-content) in some set (in the code of requirements and in the normative system, respectively). Therefore, propositions, and not requirements, make a set of requirements, and, similarly, propositions, and not norms, constitute a normative system.

Remark 1 *Broome does not explicate the notion of normative source but introduces it by way of examples (survival, prudence, and rationality). I will not give an explication of the notion of normative source either, but will give a sketch of the distinction that was implicit in my thoughts and that was used for an explication of the relation between the normative and the logical (see Definitions 2). Normative sources are: formal and material. Formal normative sources regulate relations between intentional states, either within one category (e.g. theoretical rationality) or between categories (e.g. practical rationality). Material normative sources are those that require a specific content to be present in an intentional state. I posit the theoretical type of normative source as requiring certain beliefs, and the practical type of normative source as requiring certain desires and decisions or intentions.*

2 The Language of Norm Contents

In order to give a first order translation for Broome's functional approach, some preliminary steps must be taken. Metanormative theory speaks about a language in which norms are stated. Therefore, my starting point is $\mathcal{L}_\mathrm{n}$, the language in which the norms and conditions of their application are expressed. By $\mathcal{L}_\mathrm{n}$ I will denote a language of propositional modal logic with the following modalities: B_i for 'i believes that', D_i for 'i desires that', I_i for 'i intends that'. Later, I will give reasons for reducing the "language of intentionality" to only three modalities.

The normative language $\mathcal{L}_\mathrm{n}$ is built over the base language of propositional logic $\mathcal{L}_{PL}$ with modalities added.

Definition 1 *Let $i \in A$, $\mathrm{X} = \mathrm{B}, \mathrm{D}, \mathrm{I}$, and $p \in \mathcal{L}_{PL}$. The formulas of language $\mathcal{L}_\mathrm{n}$ are:*

$$\varphi ::= p \mid [\mathrm{X}_i]\varphi \mid \neg\varphi \mid (\varphi_1 \wedge \varphi_2).$$

The definitions of truth-functional connectives are standard.

Considered in isolation, language $\mathcal{L}_\mathrm{n}$ is not committed to any particular logic. Still, if a subset of $\mathcal{L}_\mathrm{n}$ has a logical property definable within some particular logic l, then that property will be noted as 'l-property'.

Remark 2 *The sentences of $\mathcal{L}_\mathrm{n}$ whose main operator is $[\mathrm{B}_i]$, $[\mathrm{D}_i]$, or $[\mathrm{I}_i]$ will be termed 'modals'.*

Definition 2 *The set $\mathrm{lit}(\mathcal{L}_\mathrm{n})$ of quasi-literals with respect to propositional logic is the smallest subset of $\mathcal{L}_\mathrm{n}$ containing the set of propositional letters and their negations, and the set of modals and their negations.*

Let us extend language $\mathcal{L}_\mathrm{n}$, itself a standard modal propositional language, to language $\mathcal{L}_{\mathrm{n}(\omega_1)}$ of a variant of infinitary logic, which has the same symbols as $\mathcal{L}_\mathrm{n}$, but in $\mathcal{L}_{\mathrm{n}(\omega_1)}$ the infinitary conjunction symbol $\bigwedge$ is applied to countably infinite subsets of the set of quasi-literals $\mathrm{lit}(\mathcal{L}_\mathrm{n})$. See (Keisler, 1971) for a full-blown infinitary logic.

Definition 3 *Let $p \in \mathcal{L}_\mathrm{n}$ and $x \subseteq \mathrm{lit}(\mathcal{L}_\mathrm{n})$. The formulas of language $\mathcal{L}_{\mathrm{n}(\omega_1)}$ are:*

$$\varphi ::= p \mid \bigwedge x \mid \neg\varphi \mid (\varphi_1 \wedge \varphi_2).$$

Let us also extend the deductive system $\vdash_{\mathrm{pl}}$ of propositional logic to an ad hoc variant of infinitary propositional logic $\vdash_{\mathrm{pl}(\omega_1)}$ containing the rules of $\vdash_{\mathrm{pl}}$ and the additional rules for the countable conjunctions of quasi-literals. According to the grammar of $\mathcal{L}_{\mathrm{n}(\omega_1)}$, the introduction and elimination rules for $\bigwedge$ are applicable to the sets of quasi-literals only. For $x \subseteq \mathrm{lit}(\mathcal{L}_n)$,

1. $\Gamma, \bigwedge x \vdash_{\mathrm{pl}(\omega_1)} p$ for all $p \in x$,
2. if $\Gamma \vdash_{\mathrm{pl}(\omega_1)} p$ for all $p \in x$, then $\Gamma \vdash_{\mathrm{pl}(\omega_1)} \bigwedge x$.

On the side of semantics, the definition of the truth assignment h is extended in an obvious way: $h(\bigwedge x) = \mathbf{t}$ iff $h(p) = \mathbf{t}$ for all $p \in x$.

Proposition 1 shows that the ad hoc system $\vdash_{\mathrm{pl}(\omega_1)}$ is a conservative extension of $\vdash_{\mathrm{pl}}$.

Proposition 1 *For $x \cup \{p\} \subseteq \mathcal{L}_\mathrm{n}$, if $x \vdash_{\mathrm{pl}(\omega_1)} p$, then $x \vdash_{\mathrm{pl}} p$.*

Proof The proof will be sketched. Assume $x \vdash_{\mathrm{pl}(\omega_1)} p$. The deductive system $\vdash_{\mathrm{pl}(\omega_1)}$ is sound, as can be easily checked. Therefore, $x \models_{\mathrm{pl}(\omega_1)} p$. Then also $x \models_{\mathrm{pl}} p$ thanks to the coincidence of the semantic definitions for sentences in $\mathcal{L}_\mathrm{n}$. Finally, $x \vdash_{\mathrm{pl}} p$ by the completeness of the propositional logic. □

3 Metanormative Language

In order to achieve technical clarity, I will define a first-order metanormative language $\mathcal{L}_{\mathrm{meta}}$ in which variables of different sorts range over different objects in the domain.

$\mathcal{L}_{\mathrm{meta}}$ has the following extra-logical vocabulary:

individual constants for normative sources, for agents and for worlds: $\mathrm{s}, \mathrm{s}_1, \ldots$, $\mathrm{a}, \mathrm{a}_1, \ldots$, $\mathrm{v}, \mathrm{v}_1, \ldots$;

symbols for the code of requirements function, for the propositional logic consequence function, and for the axiomatic basis of a modal logic function: $\mathrm{k}^3, \mathrm{Cn}^1, \mathrm{l}^1$;

symbols for functions generating sentential forms of the object language: neg^1, conj^2, $\mathrm{infconj}^1$ and a set of symbols $\mathrm{mod}^1_{\mathrm{B}i}$, $\mathrm{mod}^1_{\mathrm{D}i}$, $\mathrm{mod}^1_{\mathrm{I}i}$ for each $i \in \{\mathrm{a}, \mathrm{a}_1, \ldots\}$;

symbol for the function extracting quasi-literals from a given set: lt^1;

a ternary predicate symbol for the relation of an agent i having a property corresponding to a source s in a world w (normative property predicate): K_{s};

a binary predicate symbol for the relation of membership: $\in^2$.

Additionally, we may introduce a dispensable part of vocabulary containing monadic predicate symbols expressing properties of being a normative source, of being an agent, of being a sentence in $\mathcal{L}_{\mathrm{n}}$, of being a possible world: Source^1, Ag^1, Sen^1, W^1.

Variables comprise:

general variables ranging over everything: $x, x_1, ..., y, y_1, ..., z, z_1, ...$;

sorts of variables:

$\mathsf{s}, \mathsf{s}_1, ...$ ranging over $\{x \in \mathrm{D} \mid \mathrm{Source}(x)\}$

$i, i_1, ...$ ranging over $\{x \in \mathrm{D} \mid Ag(x)\}$

$p, p_1, ..., q, q_1, ...$ ranging over $\{x \in \mathrm{D} \mid \mathrm{Sen}(x)\}$

$w, w_1, ...$ ranging over $\{x \in \mathrm{D} \mid W(x)\}$.

The shorthand notations for $\mathrm{neg}(p)$, $\mathrm{conj}(p,q)$, $\mathrm{mod}_{\mathrm{B}i}(p)$, $\mathrm{mod}_{\mathrm{D}i}(p)$, $\mathrm{mod}_{\mathrm{I}i}(p)$, $\mathrm{infconj}(x)$ are $\ulcorner \neg p \urcorner$, $\ulcorner (p \wedge q) \urcorner$, $\ulcorner [\mathrm{B}_i]p \urcorner$, $\ulcorner [\mathrm{D}_i]p \urcorner$, $\ulcorner [\mathrm{I}_i]p \urcorner$, $\ulcorner \bigwedge x \urcorner$. For ease of reading, Quine quotes will be used also for the standardly defined connectives.

Example 1 $\ulcorner p \to q \urcorner$ *stands for* $\mathrm{neg}(\mathrm{conj}(p, \mathrm{neg}(q)))$.

A sole variable written between Quine quotes is the same as the variable itself. Sometimes this redundant notation will be (ab)used in order to highlight sentence variables and sentence functions within a formula.

Definition 4 *Let* c *stand for an individual constant,* v *for any variable,* f^n *for a function symbol and* P^n *for a predicate symbol.*
The terms are:

$$t ::= \mathrm{c} \mid \mathrm{v} \mid f^n(t_1, ..., t_n).$$

The atomic formulas are:

$$p ::= P^n(t_1, ..., t_n).$$

The formulas of $\mathcal{L}_{\mathrm{meta}}$ *are:*

$$\varphi ::= p \mid \neg\varphi \mid (\varphi_1 \wedge \varphi_2) \mid \forall \mathrm{v}\, \varphi.$$

Definition 5 *Sentences of* $\mathcal{L}_{\mathrm{meta}}$ *are formulas of* $\mathcal{L}_{\mathrm{meta}}$ *with all the variables bound.*

The purpose of metanormative language is to enable talking: about the syntax of sentences in $\mathcal{L}_{\mathrm{n}(\omega_1)}$, about the properties that the sentences and their sets can have in different logics (most notably "world logic" and "intentionality logics"), about the semantics of sentences in $\mathcal{L}_{\mathrm{n}(\omega_1)}$, i.e. about sentence-world relations. The basic ontology for the code functions requires: normative sources, agents, worlds and sets of sentences. Besides the set of agents and the set of normative sources, all other objects in the domain are constructed using sentences of the normative language: the worlds are theoretically identified with pl-maximal consistent sets of $\mathcal{L}_{\mathrm{n}(\omega_1)}$ (see Definition 6); code values are logic free sets of sentences; axiomatic bases of logics are sets of substitutional instances of the sentences in a given set; and sentences **are** sentences.

Definition 6 *A set* x *is maximally consistent in the logic* $\vdash_{\mathrm{pl}(\omega_1)}$ *iff* $x \subseteq \mathcal{L}_{\mathrm{n}(\omega_1)}$, *and* $x \nvdash_{\mathrm{pl}(\omega_1)} \bot$, *and for all* $y \in \mathcal{L}_{\mathrm{n}(\omega_1)}$ *it holds that if* $y \notin x$, *then* $x \cup \{y\} \vdash_{\mathrm{pl}(\omega_1)} \bot$.
The set of possible worlds is the set

$$\mathrm{MaxCon}(\mathcal{L}_{\mathrm{n}(\omega_1)}) = \{x \mid x \textit{ is max. consistent in } \vdash_{\mathrm{pl}(\omega_1)}\}.$$

Modelling constraints This kind of modelling imposes several constraints. The modal axioms for belief, desire or intention do not hold in some possible worlds, and so any kind and any measure of failure in their logics may occur.

> What sets a limit to the amount of irrationality we can make psychological sense of is a purely conceptual or theoretical matter—the fact that mental states and events are the states and events they are by their location in a logical space. (Davidson, 2004, 183)

The worlds characterized by an extreme "amount of irrationality" on the side of an agent i are admitted in the modelling. This fact should not be interpreted as a violation of Davidson's thesis, but rather as an unrealistic but harmless and dispensable theoretical possibility.

The T axiom ($\Box p \to p$) poses a more serious threat to the modelling that keeps modality and the world apart. If modalities obeying reflexive axiom T are allowed, then possible worlds, being defined as maximal consistent sets in propositional logic, would become intuitively impossible. For example, although $\{p,\ [\mathrm{K}]_i\neg p\}$ is a $\mathrm{pl}(\omega_1)$-consistent set, we do not want to have it included in any world since no false proposition may be known as a true proposition. Since the corresponding T axioms seem to constitute an important part of the meaning of verbs of knowledge and of action, epistemic and praxeological modalities must be excluded from the language of norms $\mathcal{L}_{\mathrm{n}(\omega_1)}$. The forthcoming analysis does not depend on the inclusion of "T modalities", so this strategy may be adopted as a provisional method.

Von Wright (von Wright, 1963) defined the content of a norm as "that which ought to or may or must not be or be done". Normative language $\mathcal{L}_{\mathrm{n}(\omega_1)}$ departs from von Wright's definition by taking norm-content to be *the psychological state or relation of psychological states that ought to or may or must not be present in the mind of the norm addressee on a particular occasion.* The reduction and the switch may seem drastic, but there is a rationale for it: the requirement that agent i knows that p could be replaced by $p \to [\mathrm{B}_i]p$, and a required action to see to it that p could be replaced by the required intention, i.e. $[\mathrm{I}_i]p$.

Logical Properties of Sets of Requirements

Broome (2007b, 35) claims that code values are closed under pl-equivalence, i.e. if p and q are equivalent in propositional logic, then p is a member of a set of requirements just in case q is a member. He seems to tacitly hold that this congruence property constitutes the whole of the logic of "source requirements". Broome is not isolated in adopting the congruence rule (i.e. closure under equivalence): a recent proponent is Lou Goble (2009). Broome (2008, 129) bases the acceptability of the congruence principle on the argument of the absence of contrary evidence, while Goble (2009, 483) takes it for granted since: "[it] seems [to be] a minimum requirement for a logic of ought." On the other hand, Alchourrón and Bulygin (1998) propose an approach that is both more restrictive and more permissive. First, contrary to Broome's weak congruence logic, Alchourrón and Bulygin argue that there is no logic of norms since the existence of a norm depends on the empirical fact of promulgation. Second, they claim that there is a logic of normative systems since the set of norm-contents is deductively closed. By contrast, in Broome's approach there is no general logic for a set of requirements except congruence, while deductive closure is merely a special case. Then again, following Alchourrón and Bulygin, one may think about a set of requirements as void of any logic and only later introduce the set closed under congruence as a special type. In this respect, I will follow Alchourrón and Bulygin's proposal because of its higher level of generality.

If rationality is a normative source or if rationality is presupposed by some normative sources, then some logic for rational relations between intentional states will be needed. Being restricted in no way, a code function may also deliver sets having particular logical properties. So, it is convenient to introduce sets of sentences in $\mathcal{L}_{\mathrm{n}(\omega_1)}$ which obey or contain some modal logic. By doing so, one can explicate the rational relations in terms of logic and define the type codes whose output has certain logical properties with respect to some logic of the modal operators (Definition 9).

Definition 7 *Any function g from $\mathcal{L}_{\mathrm{n}} \subset \mathcal{L}_{\mathrm{n}(\omega_1)}$ to $\mathcal{L}_{\mathrm{n}(\omega_1)}$ is a restricted substitution function iff*

- $g(p) \in \mathcal{L}_{\mathrm{n}(\omega_1)}$ *if p is a propositional letter*
- $g(\neg p) = \neg g(p)$
- $g(p \wedge q) = (g(p) \wedge g(q))$
- $g([\mathrm{X}_i]p) = [\mathrm{X}_i]g(p)$ *for* $\mathrm{X} = \mathrm{B}, \mathrm{D}, \mathrm{I},\ i \in \mathrm{A}$.

The set Sb *is the set of all restricted substitution functions.*

Remark 3 *The restriction in the domain of substitution functions is due to the fact that infinite conjunctions are not allowed to embed.*

Definition 8 *The set of all substitutional instances of sentences in a given set $x \subseteq \mathcal{L}_{\mathrm{n}}$ is the set* $\mathrm{l}(x) = \{q \mid \exists p \exists f (p \in x \wedge f \in \mathrm{Sb} \wedge f(p) = q)\}$.

Definition 9 *The set* $\mathrm{Cn}(\mathrm{l}(x))$ *is the logic for axiomatic basis x.*

Definition 10 *Let $\top_{[\mathrm{X}_i]}$ denote*

$$((p \vee \neg p) \leftrightarrow q) \to [\mathrm{X}_i]q,$$

and let $\mathrm{K}_{[\mathrm{X}_i]}$ denote

$$[\mathrm{X}_i](p \to q) \to ([\mathrm{X}_i]p \to [\mathrm{X}_i]q).$$

A set $\mathrm{Cn}(\mathrm{l}(x))$ *is a normal logic for a set of modal operators* $\mathrm{o}/x \subseteq \{[\mathrm{X}_i] \mid \mathrm{X} = \mathrm{B}, \mathrm{D}, \mathrm{I},\ i \in \mathrm{A}$, *and* $[\mathrm{X}_i]$ *occurs in some* $p \in x\}$ *iff*

$$\mathrm{Cn}(\mathrm{l}(\{\top_y \mid y \in \mathrm{o}/x\} \cup \{\mathrm{K}_y \mid y \in \mathrm{o}/x\})) \subseteq \mathrm{Cn}(\mathrm{l}(x)).$$

First order structure for metanormative language

The domain for metanormative language $\mathcal{L}_{\text{meta}}$ comprises the following objects: normative sources, $x \in \mathsf{S}$; agents, $x \in \mathrm{A}$; sentences, $x \in \mathcal{L}_{\mathrm{n}(\omega_1)}$; sets of sentences (code values, and axiomatic bases for logics), $x \subseteq \mathcal{L}_{\mathrm{n}(\omega_1)}$; worlds, $x \in \mathrm{MaxCon}(\mathcal{L}_{\mathrm{n}(\omega_1)})$.

Definition 11 $\mathrm{D} = \mathsf{S} \cup \mathrm{A} \cup \mathcal{L}_{\mathrm{n}(\omega_1)} \cup \wp\mathcal{L}_{\mathrm{n}(\omega_1)}$ *where* $\mathsf{S} \neq \emptyset$, $\mathrm{A} \neq \emptyset$, $\mathsf{S} \cap \mathrm{A} = \emptyset$.

Definition 12

$$\mathcal{I}(f)(x_1,..,x_n) = \begin{cases} y, \text{ if } \langle x_1,..,x_n,y\rangle \in \mathcal{I}(f), \\ \text{undefined, otherwise.} \end{cases}$$

Definition 13 *Function* $\mathcal{I}$ *gives the following interpretation for the vocabulary of* $\mathcal{L}_{\text{meta}}$*:*

(interpretation of names of sources) $\mathcal{I}(\mathrm{s}_i) \in \mathsf{S}$*;*

(interpretation of the code function symbol) $\mathcal{I}(\mathrm{k})$ *is a function:*

$$\mathsf{S} \times \mathrm{A} \times \mathrm{MaxCon}(\mathcal{L}_{\mathrm{n}(\omega_1)}) \to \wp\mathcal{L}_{\mathrm{n}(\omega_1)};$$

(interpretation of the function symbol for an axiomatic basis) $\mathcal{I}(\mathrm{l})$ *is a function:* $\wp\mathcal{L}_{\mathrm{n}(\omega_1)} \to \wp\mathcal{L}_{\mathrm{n}(\omega_1)}$*, such that for any* $x \subseteq \mathcal{L}_{\mathrm{n}(\omega_1)}$

$$\mathcal{I}(\mathrm{l})(x) = \{f(p) \mid p \in x \wedge f \in \mathrm{Sb}\};$$

(interpretation of the pl*-consequence function symbol)* $\mathcal{I}(\mathrm{Cn})$ *is a function:* $\wp\mathcal{L}_{\mathrm{n}(\omega_1)} \to \wp\mathcal{L}_{\mathrm{n}(\omega_1)}$*, such that for any* $x \subseteq \mathcal{L}_{\mathrm{n}(\omega_1)}$

$$\mathcal{I}(\mathrm{Cn})(x) = \{y \in \mathcal{L}_{\mathrm{n}(\omega_1)} \mid x \vdash_{\mathrm{pl}(\omega_1} y\};$$

(interpretation of sentence form function symbols) $\mathcal{I}(\mathrm{neg})$, $\mathcal{I}(\mathrm{conj})$, $\mathcal{I}(\mathrm{mod_X})$ *for* $\mathrm{X} = \mathrm{B}_i,\ \mathrm{D}_i,\ \mathrm{I}_i$, $\mathcal{I}(\mathrm{infconj})$ *are functions:* $\mathcal{L}_{\mathrm{n}(\omega_1)} \to \mathcal{L}_{\mathrm{n}(\omega_1)}$ *, such that*

$$\mathcal{I}(\mathrm{neg}) = \{\langle x,y\rangle \mid y = \neg^\frown x\}$$
$$\mathcal{I}(\mathrm{conj}) = \{\langle x,y,z\rangle \mid z = (^\frown x^\frown \wedge^\frown y^\frown)\}$$
$$\mathcal{I}(\mathrm{mod_X}) = \{\langle x,y\rangle \mid y = [\mathrm{X}]^\frown x\}$$
$$\mathcal{I}(\mathrm{infconj}) = $$
$$= \left\{ \langle x,y\rangle \;\middle|\; \begin{array}{l} x \subseteq \mathrm{lit}(\mathcal{L}_{\mathrm{n}}) \wedge \\ y = \bigwedge^\frown\{^\frown seq(x)(1)^\frown,^\frown \\ ...^\frown,^\frown seq(x)(n)^\frown,^\frown ...^\frown\} \end{array} \right\}$$

where $^\frown$ *is a concatenation operation, and where* $seq \in \prod x$*, while* $\prod x$ *denotes the set of functions* $f : \mathbb{N} \to x$*, such that* $f(i) \neq f(j)$ *for each* $i, j \in \mathbb{N}$*;*

(interpretation of the function symbol for the extraction of quasi-literals) lt *is the function:* $\wp\mathcal{L}_{\mathrm{n}(\omega_1)} \to \wp\mathcal{L}_{\mathrm{n}(\omega_1)}$*, such that for any* $x \subseteq \mathcal{L}_{\mathrm{n}(\omega_1)}$, $\mathcal{I}(\mathrm{lt})(x) = \{y \mid y \in x \wedge y \in \mathrm{lit}(\mathcal{L}_{\mathrm{n}(\omega_1)})\}$*;*

(interpretation of "normative property predicate")

$$\mathcal{I}(\mathrm{K_S}) \subseteq \mathrm{A} \times \mathrm{MaxCon}(\mathcal{L}_{\mathrm{n}(\omega_1)});$$

(interpretation of membership predicate)

$$\mathcal{I}(\in) = \{\langle x,y\rangle \mid x,y \in \mathrm{D},\ x \in y\};$$

(interpretation for "superfluous predicates")

$$\mathcal{I}(\mathrm{Source}) = \mathsf{S}$$
$$\mathcal{I}(A) = \mathrm{A}$$
$$\mathcal{I}(\mathrm{Sen}) = \mathcal{L}_{\mathrm{n}(\omega_1)}$$
$$\mathcal{I}(\mathrm{W}) = \mathrm{MaxCon}(\mathcal{L}_{\mathrm{n}(\omega_1)}).$$

Definition 14 $\mathfrak{M}_{\mathrm{mn}} = \langle \mathrm{D}, \mathcal{I}\rangle$.

Definition 15 *Variable assignment* g *in* $\mathfrak{M}_{\mathrm{mn}} = \langle \mathrm{D}, \mathcal{I}\rangle$ *is a (possibly partial) function* g*, such that for any variable* v

$$g(v) \in \mathrm{D} \textit{ iff } v \in domain(g).$$

For sorts of variables: (source variables) $g(\mathsf{s}) \in \mathsf{S}$ *if* $v = \mathsf{s}, \mathsf{s}_1, ...$*; (world variables)* $g(v) \in \mathrm{MaxCon}(\mathcal{L}_{\mathrm{n}(\omega_1)})$ *if* $v = w, w_1, ...$*; (sentence variables)* $g(v) \in \mathcal{L}_{\mathrm{n}(\omega_1)}$ *if* $v = p, p_1, ..., q, q_1, ...$*; (agent variables)* $g(v) \in \mathrm{A}$ *if* $v = i, i_1, ...$
The variable assignment g is appropriate for formula p iff all free variables of p are in the domain of g .

Notation 1 *The empty variable assignment* $g_\emptyset$ *is undefined for any variable:* $range(g_\emptyset) = \emptyset$*.*
By $g_{[x/d]}$ *we denote the variable assignment that differs from g at most by assigning* d *for x:*

$$g_{[x/\mathrm{d}]}(v) = \begin{cases} \mathrm{d}, \text{ if } x = v \\ g(v), \text{ otherwise.} \end{cases}$$

Definition 16

$$[\![t]\!]_g^{\mathfrak{M}_{\mathrm{mn}}} = $$
$$= \begin{cases} \mathcal{I}(t), \text{ if } t \text{ is an individual constant} \\ g(t), \text{ if } t \text{ is an individual variable} \\ \mathcal{I}(f)([\![t_1]\!]_g^{\mathfrak{M}_{\mathrm{mn}}}, \ldots, [\![t_n]\!]_g^{\mathfrak{M}_{\mathrm{mn}}}), \text{ if } t \text{ is } f(t_1, ..., t_n). \end{cases}$$

Definition 17 *(Satisfaction) Let g be an assignment in $\mathfrak{M}_{\text{mn}}$ which is appropriate for p. Suppose, successively, that p is $P(t_1,...,t_n)$, $\neg\varphi$, $(\varphi \wedge \psi)$, and $\forall v\varphi$.*

$$\mathfrak{M}_{\text{mn}} \models P(t_1,...,t_n)\ [g]$$
$$\textit{iff}\ \langle [\![t_1]\!]_g^{\mathfrak{M}_{\text{mn}}}, \ldots, [\![t_n]\!]_g^{\mathfrak{M}_{\text{mn}}} \rangle \in \mathcal{I}(P)$$
$$\mathfrak{M}_{\text{mn}} \models \neg\varphi\ [g]$$
$$\textit{iff not}\ \ \mathfrak{M}_{\text{mn}} \models \varphi\ [g]$$
$$\mathfrak{M}_{\text{mn}} \models (\varphi \wedge \psi)\ [g]$$
$$\textit{iff}\ \ \mathfrak{M}_{\text{mn}} \models \varphi\ [g]\ \textit{and}\ \mathfrak{M}_{\text{mn}} \models \psi\ [g]$$
$$\mathfrak{M}_{\text{mn}} \models \forall v\varphi\ [g]$$
$$\textit{iff for all}\ \ \mathrm{d} \in \mathrm{D},\ \mathfrak{M}_{\text{mn}} \models \varphi\ [g_{[v/\mathrm{d}]}].$$

Definition 18 *(Truth in a metanormative model) Formula φ is true in $\mathfrak{M}_{\text{mn}}$ iff $g_\emptyset$ satisfies φ in $\mathfrak{M}_{\text{mn}}$:*

$$\mathfrak{M}_{\text{mn}} \models \varphi\ \ \textit{iff}\ \ \mathfrak{M}_{\text{mn}} \models \varphi \quad [g_\emptyset].$$

4 Typology of sets of requirements and code functions

The use of code functions enriches the discriminative power of the logical theory of normative systems. On the one hand, in the functional approach, one may define the properties and relations of sets of requirements as in other set theoretic approaches. On the other hand, unlike other set theoretic approaches, quantifying over different argument positions in the code function makes it possible for the functional approach to introduce a number of interesting type distinctions.

First, I will give definitions for some interesting logical properties that are "local", i.e. properties of sets of requirements. In each definition, the *definiendum* introduces both an informal expression and a new predicate of language $\mathcal{L}_{\text{meta}}$. The unbound variables are assumed to be universally quantified.

Definitions 1 *A set of requirements* $\mathrm{k}_\mathsf{s}(i, w_1)$ *is* pl-*congruent,* $\mathrm{CG}_{\mathrm{pl}}(\mathrm{k}_\mathsf{s}(i, w_1))$*, iff*

$$\forall p \forall q \begin{pmatrix} \ulcorner p \leftrightarrow q \urcorner \in \mathrm{Cn}(\varnothing) \rightarrow \\ (p \in \mathrm{k}_\mathsf{s}(i, w_1) \leftrightarrow q \in \mathrm{k}_\mathsf{s}(i, w_1)) \end{pmatrix}.$$

A set of requirements $\mathrm{k}_\mathsf{s}(i, w_1)$ *is* pl-*consistent,* $\mathrm{CS}_{\mathrm{pl}}(\mathrm{k}_\mathsf{s}(i, w_1))$*, iff* $\exists w_2\, \mathrm{k}_\mathsf{s}(i, w_1) \subseteq w_2$.

A set of requirements $\mathrm{k}_\mathsf{s}(i, w_1)$ *is* pl-*deductively closed,* $\mathrm{DC}_{\mathrm{pl}}(\mathrm{k}_\mathsf{s}(i, w_1))$*, iff* $\mathrm{k}_\mathsf{s}(i, w_1) = \mathrm{Cn}(\mathrm{k}_\mathsf{s}(i, w_1))$.

A set of requirements $\mathrm{k}_\mathsf{s}(i, w_1)$ *is consistent in logic* $\mathrm{l}(x)$*,* $\mathrm{CS}_{\mathrm{l}(x)}(\mathrm{k}_\mathsf{s}(i, w_1))$*, iff*

$$\exists w_2\, \mathrm{Cn}(\mathrm{l}(x) \cup \mathrm{k}_\mathsf{s}(i, w_1)) \subseteq w_2.$$

A set of requirements $\mathrm{k}_\mathsf{s}(i, w_1)$ *is a logic,* $\mathrm{LG}(\mathrm{k}_\mathsf{s}(i, w_1))$*, iff* $\exists x\, \mathrm{k}_\mathsf{s}(i, w_1) = \mathrm{Cn}(\mathrm{l}(x))$.

A set of requirements $\mathrm{k}_\mathsf{s}(i, w_1)$ *is deductively closed with respect to logic* $\mathrm{l}(x)$*,* $\mathrm{DC}_{\mathrm{l}(x)}(\mathrm{k}_\mathsf{s}(i, w_1))$*, iff*

$$\exists y\, \mathrm{k}_\mathsf{s}(i, w_1) = \mathrm{Cn}(\mathrm{l}(x) \cup y)).$$

A set of requirements $\mathrm{k}_\mathsf{s}(i, w_1)$ *is material (not formal) in logic* $\mathrm{l}(x)$*,* $\mathrm{MT}_{\mathrm{l}(x)}(\mathrm{k}_\mathsf{s}(i, w_1))$*, iff*

$$\exists y (y \neq \emptyset \wedge y \neq \mathrm{l}(x) \wedge \mathrm{k}_\mathsf{s}(i, w_1) = \mathrm{Cn}(\mathrm{l}(x) \cup y)).$$

Second, more "global" properties are obtained through universal generalization over agents and worlds. In this way, the corresponding properties of normative sources may be defined. Such a list of the logical properties of normative sources follows with the focus on more general logical properties. Therefore, pl-properties of the sources will be omitted. Additionally, I will use existential generalization to introduce the notion of an achievable source, a notion that is critical to the theory that separates normative sources from normative properties, since only an achievable source can define a property.

Definitions 2 *A normative source* s *issues an* $\mathrm{l}(x)$*-consistent code iff* $\forall i \forall w\, \mathrm{CS}_{\mathrm{l}(x)}(\mathrm{k}_\mathsf{s}(i, w))$.

A normative source s *is formal iff* $\forall i \forall w\, \mathrm{LG}(\mathrm{k}_\mathsf{s}(i, w))$.

A normative source s *issues an* $\mathrm{l}(x)$*-deductively closed code iff* $\forall i \forall w\, \mathrm{DC}_{\mathrm{l}(x)}(\mathrm{k}_\mathsf{s}(i, w))$.

A normative source s *is material with respect to logic* $\mathrm{l}(x)$ *iff* $\exists i \exists w\, \mathrm{MT}_{\mathrm{l}(x)}(\mathrm{k}_\mathsf{s}(i, w))$.

A normative source s *is achievable iff* $\exists w\, \mathrm{k}_\mathsf{s}(i, w) \subseteq w$.

Third, some of the logical properties of normative systems are not definable in terms of the properties of a sole set of requirements. A comparison between sets of requirements leads to the introduction of new conceptual distinctions. In this way, the difference between relative and absolute sources becomes visible. Finally, for the determination of the equilibrium properties of a normative system, the social logic of normative sources must be taken into account (see Section 7), and, therefore, the notion of social consistency is introduced below.

Definitions 3 *A normative source is world-relative iff* $\exists i \exists w_1 \exists w_2\, \mathrm{k}_{\mathsf{s}}(i, w_1) \neq \mathrm{k}_{\mathsf{s}}(i, w_2)$.

A normative source is agent-relative iff

$$\exists w \exists i_1 \exists i_2\, \mathrm{k}_{\mathsf{s}}(i_1, w) \neq \mathrm{k}_{\mathsf{s}}(i_2, w).$$

A normative source is world-absolute (world-invariant) iff it is not world-relative.

A normative source is agent-absolute iff it is not agent-relative.

A normative source is socially $\mathrm{l}(x)$*-consistent iff*

$$\forall i_1 \forall i_2 \forall w\, \mathrm{CS}_{\mathrm{l}(x)}(\mathrm{k}_{\mathsf{s}}(i_1, w) \cup \mathrm{k}_{\mathsf{s}}(i_2, w)).$$

Fourth, thanks to quantification over sources, the relations between codes issued by different sources can be defined. I will give only two definitions of the kind, namely those that will be used in the rest of this article.

Definitions 4 *Normative sources* s_1 *and* s_2 *are realization-equivalent iff*

$$\forall i \forall w (\mathrm{k}_{\mathsf{s}_1}(i, w) \subseteq w \leftrightarrow \mathrm{k}_{\mathsf{s}_2}(i, w) \subseteq w).$$

Normative sources s_1 *and* s_2 *are* $\mathrm{l}(x)$*-compatible iff*

$$\forall w\, \mathrm{CS}_{\mathrm{l}(x)}(\mathrm{k}_{\mathsf{s}_1}(i, w) \cup \mathrm{k}_{\mathsf{s}_2}(i, w)).$$

The typology put to work

The terms defined above, or ones constructed in a similar fashion, can be applied in the interpretation of philosophical texts. Let us begin with antique philosopher, Epictetus (c. 50–c. 120).

> [...] instruction consists precisely in learning to desire each thing just as it happens. (Epictetus, 1925, 93)

The $\mathcal{L}_{\text{meta}}$ translation gives:

$$\forall i \forall w (\ulcorner \mathrm{D}_i p \urcorner \in \mathrm{k}_{\mathsf{inst}}(i, w) \rightarrow \ulcorner p \urcorner \in w)$$

where inst names the normative source of instruction and where modal operator D_i stands for 'agent i desires that'.

Let us consider a modern text in which the author treats rationality as a normative source that issues a world-absolute logical code.

> It is obvious enough that there are norms of rationality that apply to thoughts. If we believe certain things, logic tells us there are other things we ought or ought not to believe at the same time; decision theory gives us an idea of how the beliefs and values of a rational man must be related to each other; [...] (Davidson, 2004, 97)

Let ratio refer to the normative source of rationality. A likely $\mathcal{L}_{\text{meta}}$ translation for the first clause of the second quoted sentence states that the normative source of rationality is deductively closed with respect to the doxastic D axiom:

$$\forall i \forall w\, \mathrm{DC}_{\mathrm{l}(\{\ulcorner \mathrm{B}_i p \rightarrow \neg \mathrm{B}_i \neg p \urcorner\})}(\mathrm{k}_{\mathsf{ratio}}(i, w)).$$

Another plausible $\mathcal{L}_{\text{meta}}$ translation is a stronger one that maintains that rationality is a formal normative source which includes the doxastic D axiom:

$$\forall i \forall w$$
$$(\mathrm{LG}(\mathrm{k}_{\mathsf{ratio}}(i, w)) \wedge \mathrm{l}(\{\ulcorner \mathrm{B}_i p \rightarrow \neg \mathrm{B}_i \neg p \urcorner\}) \subseteq \mathrm{k}_{\mathsf{ratio}}(i, w)).$$

In the next example there is an interplay between the world logic, pl-logic, and some logic of intentionality, some $\mathrm{l}(x)$ logic (such as the one in the previous example requiring consistency of belief contents).

> Rationality is principally concerned with coherence among your attitudes such as your beliefs and intentions, whereas morality, prudence and other sources of normativity are rarely concerned with those things. Rationality has a domain of application where it is pretty much on its own. Examples of conflict between rationality and other sources of requirements tend to be farfetched... (Broome, 2007a, 164)

The last sentence of the citation could be interpreted as a claim that any consistent normative source issues a code that is compatible with one issued by rationality; or, stated more concisely, that the normative source of rationality is maximally compatible:

$$\forall \mathsf{s} \forall i \forall w (\mathrm{CS}_{\mathrm{pl}}(\mathrm{k}_{\mathsf{s}}(i, w)) \rightarrow \mathrm{CS}_{\mathrm{l}(x)}(\mathrm{k}_{\mathsf{s}}(i, w) \cup \mathrm{k}_{\mathsf{ratio}}(i, w))).$$

Metanormative interpretation also reveals the hidden thesis implied by the claim on the maximally compatible character of rationality. Rationality, at least in the "horizontal sense" (Zangwill, 2005) of the word, deals with formal relations between intentional states and therefore it cannot be maximally compatible unless other sources are consistent in the logic defined by the axiomatic bases for the modal part of their language. In other words, normative sources must obey the logic of the language in which their requirements are stated. This claim is rather strong, as the next section will show, for it holds only for ideal normative sources.

5 Deontic logic and the typology of normative systems

Consistent and deductively closed codes seem to play an important role in the philosophical understanding of basic normative concepts. For example, deontic KD logic without iterated deontic modalities may be conceived as the theory of a specific type of code, namely of a consistent pl-deductively closed code. This type has been discussed in the literature. For example, Alchourrón and Bulygin define "the concept of a normative system as the set of all the propositions that are consequence of the explicitly commanded propositions" (Alchourrón and Bulygin, 1998, 391), and that concept corresponds to the concept of a deductively closed set of requirements (see section 4). Although Alchourrón and Bulygin allow for a normative system to be inconsistent, they consider inconsistency as a serious defect that needs to be cured. So, inconsistent normative systems are only transient states in the development of the system. To Alchourrón and Bulygin's concept of a consistent normative system there corresponds the concept of a set of requirements that is both deductively closed and consistent.

Von Wright has pointed out the connection between deontic logic and the set-theoretical approach:

> ...classic deontic logic, on the descriptive interpretation of its formulas, pictures a gapless and contradiction-free system of norms. (von Wright, 1999, 32)

In order to investigate von Wright's thesis, a translation between metanormative language and the language of classical deontic KD logic will be introduced and used for a precise determination of the relationship between KD logic and the typology of sets of requirements.

Definition 19 *Let $p \in \mathcal{L}_{PL}$ be a formula of propositional logic.*
Formulas of restricted language $\mathcal{L}^{\mathrm{O}}_{\mathrm{KD}}$:

$$\varphi ::= p \mid \mathrm{O}p \mid \mathrm{P}p \mid \neg\varphi \mid (\varphi_1 \wedge \varphi_2).$$

Let us introduce the translation τ^1 from the restricted language $\mathcal{L}^{\mathrm{O}}_{\mathrm{KD}}$ to the metanormative language $\mathcal{L}_{\mathrm{meta}}$, with $\mathrm{O}p$ and $\mathrm{P}p$ standing for 'i in v has an s-obligation (s-permission) to p'.

Definition 20 *Function τ^0 maps sentences from the fragment $\mathcal{L}^{\mathrm{O}}_{\mathrm{KD}} \cap \mathcal{L}_{PL}$ to the set of sentential variables and sentential function terms of $\mathcal{L}_{\mathrm{meta}}$:*

$$\begin{aligned} \tau^0(a) &\in \{p, p_1, \ldots, q, q_1, \ldots\} \\ &\qquad \textit{for propositional letters } a \in \mathcal{L}_{PL} \\ \tau^0(\neg\varphi) &= \neg\tau^0(\varphi) \\ \tau^0((\varphi \wedge \psi)) &= (\tau^0(\varphi) \wedge \tau^0(\psi)). \end{aligned}$$

Definition 21 *Translation $\tau^1 : \mathcal{L}^{\mathrm{O}}_{\mathrm{KD}} \to \mathcal{L}_{\mathrm{meta}}$*

$$\begin{aligned} \tau^1(p) &= \ulcorner\tau^0(p)\urcorner \in \mathrm{v} \qquad \textit{if } p \in \mathcal{L}_{PL} \\ \tau^1(\mathrm{O}\varphi) &= \ulcorner\tau^0(\varphi)\urcorner \in \mathrm{k_s}(\mathrm{a}, \mathrm{v}) \\ \tau^1(\mathrm{P}\varphi) &= \neg \ulcorner\tau^0(\neg\varphi)\urcorner \in \mathrm{k_s}(\mathrm{a}, \mathrm{v}) \\ \tau^1(\neg\varphi) &= \neg\tau^1(\varphi) \\ \tau^1((\varphi \wedge \psi)) &= (\tau^1(\varphi) \wedge \tau^1(\psi)). \end{aligned}$$

Example 2

$$\begin{aligned} &\tau^1(\mathrm{P}p \leftrightarrow \neg\mathrm{O}\neg p) \\ &\Leftrightarrow \tau^1(\mathrm{P}p) \leftrightarrow \tau^1(\neg\mathrm{O}\neg p) \\ &\Leftrightarrow \neg \ulcorner\tau^0(\neg p)\urcorner \in \mathrm{k_s}(\mathrm{a}, \mathrm{v}) \leftrightarrow \neg\, \tau^1(\mathrm{O}\neg p) \\ &\Leftrightarrow \neg \ulcorner\neg\tau^0(p)\urcorner \in \mathrm{k_s}(\mathrm{a}, \mathrm{v}) \leftrightarrow \neg \ulcorner\tau^0(\neg p)\urcorner \in \mathrm{k_s}(\mathrm{a}, \mathrm{v}) \\ &\Leftrightarrow \neg \ulcorner\neg p\urcorner \in \mathrm{k_s}(\mathrm{a}, \mathrm{v}) \leftrightarrow \neg \ulcorner\neg\tau^0(p)\urcorner \in \mathrm{k_s}(\mathrm{a}, \mathrm{v}) \\ &\Leftrightarrow \neg \ulcorner\neg p\urcorner \in \mathrm{k_s}(\mathrm{a}, \mathrm{v}) \leftrightarrow \neg \ulcorner\neg p\urcorner \in \mathrm{k_s}(\mathrm{a}, \mathrm{v}) \\ &\Leftrightarrow \top. \end{aligned}$$

There are two interpretations of conditional obligation in standard deontic logic. *N-scope* interpretation (narrow scope interpretation) reads conditional obligation as 'if p is the case, then q ought to be the case', i.e. $p \to \mathrm{O}q$. *W-scope* interpretation (wide scope interpretation) puts the entire conditional within the obligation range: 'it ought to be the case that: if p is the case, then q is the case', i.e.

$O(p \to q)$. The narrow scope formula, i.e. $p \to Oq$, is translated by τ_1 as $p \in \mathrm{v} \to q \in \mathrm{k_s(a,v)}$. The wide scope formula, i.e. $O(p \to q)$, is translated by τ_1 as $\ulcorner p \to q \urcorner \in \mathrm{k_s(a,v)}$. There is a tendency for a natural language speaker to regard N-scope and W-scope expressions as equivalent. The impression of equivalence in meaning is justified by two theoretically derived facts. First, any code $\mathrm{k_s}(i,w)$ has its conditionalized variant $\mathrm{k_s^{cond}}(i,w)$ and the following proposition holds (the unbound variables are assumed to be universally quantified):

$$\forall w \left(\begin{array}{c} \underbrace{(p \in w \to q \in \mathrm{k_s}(i,w))}_{\text{N-scope}} \\ \leftrightarrow \\ \underbrace{\ulcorner \bigwedge \mathrm{lt}(w) \to q \urcorner \in \mathrm{k_s^{cond}}(\mathrm{a},w)}_{\text{W-scope (generalized)}} \end{array} \right).$$

In other words, for any code function requiring consequent of an obligation conditional, there is a coordinated code function that requires the entire conditional. Second, a code and its conditionalized variant are realization equivalent (see Section 6 for a more detailed exposition). Therefore, from the behaviouristic point of view or from the perspective of the normative properties being realized, there is no difference between the two codes.

Standard deontic logic translated into metanormative language

The principles of standard deontic logic hold under the translation τ^1:

- mutual definability, $Pp \leftrightarrow \neg O \neg p$, holds for any set of requirements (see Example 2);
- the "gaplessness" condition $Pp \lor O\neg p$ translates to $\ulcorner \neg p \urcorner \notin \mathrm{k_s(a,v)} \lor \ulcorner \neg p \urcorner \in \mathrm{k_s(a,v)}$ and that condition, obviously, is satisfied by any set of requirements whatsoever;
- the K axiom becomes $\ulcorner p \to q \urcorner \in \mathrm{k_s(a,v)} \to (p \in \mathrm{k_s(a,v)} \to q \in \mathrm{k_s(a,v)})$ and that condition holds for any pl-deductively closed set;
- the D axiom translates to $p \in \mathrm{k_s(a,v)} \to \ulcorner \neg p \urcorner \notin \mathrm{k_s(a,v)}$ and that is just another way of stating pl-consistency.

According to our translation scheme, von Wright's claim that classical deontic logic "pictures a gapless and contradiction-free system of norms" should be appended: classical deontic logic pictures a system of norms that is deductively closed, too, while gaplessness condition is vacuously satisfied.

One may ask whether these properties provide an adequate description of a formally sound set of requirements or whether the description provided by, some or other, deontic logic is sufficiently fine grained. For example, the τ^1 translation for D does not allow $[\mathrm{B}_i]p \land \neg[\mathrm{B}_i]p$ to enter the set of requirements, but it does allow $[\mathrm{B}_i]p \land [\mathrm{B}_i]\neg p$. So, the question arises whether the consistency property of a set of requirements is a property that is connected to the world logic, or rather a property that a set inherits when it obeys the logic of its contents, i.e. the logic of intentionality.

Although iterated deontic operators receive no translation in the scheme proposed above, one may extend the line of thought by giving additional translation rules for language of standard deontic logic restricted to a maximum of two iterations of deontic operators, treating iterated deontic modalities as a sequence of heterogeneous operators and introducing the distinction into the syntax.

Definition 22 *Let $p \in \mathcal{L}^{\mathrm{O}}_{\mathrm{KD}}$. The formulas of $\mathcal{L}^{\mathrm{O_2O}}_{\mathrm{KD}}$ are:*

$$\varphi ::= p \mid O_2 p \mid P_2 p \mid \neg\varphi \mid (\varphi_1 \land \varphi_2).$$

Definition 23 *Let $Sub(\varphi)_{\left[\frac{c_1}{x_1} \cdots \frac{c_n}{x_n}\right]}$ denote the substitutional instance of $\varphi \in \mathcal{L}_{\mathrm{meta}}$ in which the constants $c_1,...,c_n$ are replaced by the variables $x_1,...,x_n$. Translation $\tau^2 : \mathcal{L}^{\mathrm{O_2O}}_{\mathrm{KD}} \to \mathcal{L}_{\mathrm{meta}}$ is defined as follows:*

$$\begin{aligned} \tau^2(O_2 p) &= \forall i \forall w\, Sub(\tau^1(p))_{\left[\frac{\mathrm{a}}{i}\ \frac{\mathrm{v}}{w}\right]} \quad \textit{for } p \in \mathcal{L}^{\mathrm{O}}_{\mathrm{KD}} \\ \tau^2(P_2 p) &= \exists i \exists w\, Sub(\tau^1(p))_{\left[\frac{\mathrm{a}}{i}\ \frac{\mathrm{v}}{w}\right]} \quad \textit{for } p \in \mathcal{L}^{\mathrm{O}}_{\mathrm{KD}} \\ \tau^2(\neg\varphi) &= \neg\tau^2(\varphi) \\ \tau^2(\varphi \land \psi) &= (\tau^2(\varphi) \land \tau^2(\psi)). \end{aligned}$$

Such an approach to iterated deontic modalities departs from von Wright's (1999) "second order descriptive interpretation" where e.g. O_2 would stand for the existence of "normative demands on normative systems" ("norms for the norm givers"). The "first order" translation τ^1 as well as the "second order" translation τ^2 give us statements in metanormative language $\mathcal{L}_{\mathrm{meta}}$, both of which may picture some type of normative system. The difference lies in the fact that τ^1 gives a local picture of a set of requirements (for a particular source, agent and world) while τ^2 gives a more global picture of a code function. In the second case, the properties depicted are the properties of a normative source.

Let us consider KD45 deontic logic! The τ^2 translations of the reinterpreted axioms 4, $O_1 p \to O_2 O_1 p$ and 5, $P_1 p \to O_2 P_1 p$ amount to stating that any s-obligation and any s-permission hold universally. So, the reinterpreted axioms will hold only if the s-code is absolute.

Following Broome's approach (2007b; 2008), a metanormative theory must take into consideration both normative sources and normative properties since the interaction between the normative and the real takes place on the level of agent properties. A straightforward definition of the "all-or-nothing" normative property has been proposed in (Broome, 2007b, 11) and its $\mathcal{L}_{\text{meta}}$ reformulation is given below.

Definition 24 *An agent i at world w has an "all-or-nothing" normative property* K_{s} *that corresponds to the source* s *iff the set of requirements* $\mathrm{k}_{\mathsf{s}}(i,w)$ *is satisfied in w, i.e.* $\mathrm{K}_{\mathsf{s}}(i,w) \leftrightarrow \mathrm{k}_{\mathsf{s}}(i,w) \subseteq w$.

If the only way to satisfy some relative code and some absolute code is to satisfy them simultaneously, then these codes define the same normative property. The question arises as to whether the (non)absoluteness of a code function introduces a difference with respect to normative properties. The next theorem provides a negative answer.

6 A theorem on the absolute and the relative

There is a number of ways to define a conditionalized variant of a code. Definition 25, below, introduces one of the variants by using an infinite conjunction of quasi-literals to single out a world, and by assigning a conditional for each requirement.

In order to justify the negative resolution of the question posed above (i.e. is the normative property dependent on the relative-absolute character of the normative source that defines it?), several propositions will be needed. First, Lemma 1 will be established and used in the proof of Proposition 2 which shows that an adequately chosen set of quasi-literals is sufficient to determine a world. After that, the function that assigns to each code function its conditionalized variant will be introduced in Definition 25, and Theorem 1 on the existence of a realization equivalent absolute code for any relative code will be proved. The theorem is equivalent to the claim that relative and absolute codes do not generate different normative properties.

Lemma 1 *For all* $p \in \mathcal{L}_{\mathrm{n}(\omega_1)}$, $\ulcorner p \urcorner \in \mathrm{Cn}(\mathrm{lt}(w))$ *or* $\ulcorner \neg p \urcorner \in \mathrm{Cn}(\mathrm{lt}(w))$.

Proof Transfinite induction on the pl-complexity of formulas will be used. Let the complexity of modal formulas and propositional letters be 0; the complexity of $\neg p$ — one greater than the complexity of p; the complexity of $(p \wedge q)$ — one greater than the maximum of that of p and q; the complexity of $\bigwedge x$ — ω. Let us consider only the cases of limit ordinals, 0 and ω. (0) The lemma holds for propositional letters and modal formulas in virtue of the pl-maximality of w. (ω) Suppose p is $\bigwedge x$. According to the definition, any $p_i \in x$ is a quasi-literal, and by inductive hypothesis the lemma holds for each p_i. Either all the quasi-literals in x are consequences of $\mathrm{lt}(w)$, and therefore $\ulcorner \bigwedge x \urcorner \in \mathrm{Cn}(\mathrm{lt}(w))$, or some of the quasi-literals are not consequences of $\mathrm{lt}(w)$, and therefore $\ulcorner \neg \bigwedge x \urcorner \in \mathrm{Cn}(\mathrm{lt}(w))$. □

Proposition 2 $\mathrm{Cn}(\mathrm{lt}(w)) = w$

Proof First, suppose $p \in \mathrm{Cn}(\mathrm{lt}(w))$. Then, $p \in w$ since w is deductively closed. Second, suppose $p \in w$. By Lemma 2, $\ulcorner p \urcorner \in \mathrm{Cn}(\mathrm{lt}(w)) \vee \ulcorner \neg p \urcorner \in \mathrm{Cn}(\mathrm{lt}(w))$, and so $\ulcorner p \urcorner \in \mathrm{Cn}(\mathrm{lt}(w))$ since w is consistent. □

Definition 25 *A code* $\mathrm{k}_{\mathsf{s}}^{cond}$ *is the conditionalized variant of a code* k_{s} *iff*

$$\forall p \forall w_1 \left(\begin{array}{l} p \in \mathrm{k}_{\mathsf{s}}^{cond}(i,w_1) \\ \leftrightarrow \\ \exists q \exists w_2 (q \in \mathrm{k}_{\mathsf{s}}(i,w_2) \wedge p = \ulcorner \bigwedge \mathrm{lt}(w_2) \to q \urcorner) \end{array} \right).$$

Lemma 2 *Any conditionalized code is absolute.*

Proof Let w_1 and w_2 be arbitrary worlds. Assume $p \in \mathrm{k}_{\mathsf{s}}^{cond}(i,w_1)$. By Definition 25, $\exists q \exists w_3 (q \in \mathrm{k}_{\mathsf{s}}(i,w_3) \wedge p = \ulcorner \bigwedge \mathrm{lt}(w_3) \to q \urcorner)$. Then, by universal instantiation of the same definition, $p \in \mathrm{k}_{\mathsf{s}}^{cond}(i,w_2)$. Obviously, the same holds in the opposite direction. □

Theorem 1 *For each world-relative code there is a realization equivalent world-absolute code.*

Proof We show that conditionalization generates a realization equivalent absolute code. By Lemma 2, each conditionalized code is absolute. It remains to prove that:

$$k_{\mathsf{s}}(i,w_1) \subseteq w_1 \leftrightarrow k_{\mathsf{s}}^{cond}(i,w_1) \subseteq w_1.$$

For the left to right direction, assume that $k_{\mathsf{s}}(i,w_1) \subseteq w_1$. Further, assume for an arbitrary p that $p \in k_{\mathsf{s}}^{cond}(i,w_1)$. Then, by Definition 25, there is some w_2 and some $q \in k_{\mathsf{s}}(i,w_2)$, such that $p = \ulcorner \bigwedge \mathrm{lt}(w_2) \rightarrow q \urcorner$. By *tertium non datur*, either $\mathrm{Cn}(\mathrm{lt}(w_2)) = w_1$ or $\mathrm{Cn}(\mathrm{lt}(w_2)) \neq w_1$. If $\mathrm{Cn}(\mathrm{lt}(w_2)) = w_1$, then by Proposition 2 $w_1 = w_2$. So, $q \in k_{\mathsf{s}}(i,w_1)$, and therefore $q \in w_1$ by the initial assumption. Since, w_1 is a deductively closed set, $\underbrace{\ulcorner \bigwedge \mathrm{lt}(w_2) \rightarrow q \urcorner}_{p} \in w_1$. If $\mathrm{Cn}(\mathrm{lt}(w_2)) \neq w_1$, then $\ulcorner \bigwedge \mathrm{lt}(w_2) \urcorner \notin w_1$. Therefore, by completeness of w_1, $\ulcorner \neg \bigwedge \mathrm{lt}(w_2) \urcorner \in w_1$. Then, $\underbrace{\ulcorner \bigwedge \mathrm{lt}(w_2) \rightarrow q \urcorner}_{p} \in w_1$ by deductive closure.

For the right to left direction, assume $k_{\mathsf{s}}^{cond}(i,w_1) \subseteq w_1$. Further, assume for an arbitrary p that $p \in k_{\mathsf{s}}(i,w_1)$. Then, by Definition 25, $\ulcorner \bigwedge \mathrm{lt}(w_1) \rightarrow p \urcorner \in k_{\mathsf{s}}^{cond}(i,w_1)$. By the initial assumption, $\ulcorner \bigwedge \mathrm{lt}(w_1) \rightarrow p \urcorner \in w_1$. Set w_1 is deductively closed, so, given the fact that $\ulcorner \bigwedge \mathrm{lt}(w_1) \urcorner \in w_1$, we get $p \in w_1$ as desired. □

Remark 4 *Theorem 1 can be easily generalized to the claim that for any relative code, either world or agent relative, there is a realization equivalent world and agent absolute code.*

7 Glimpses beyond

It seems that a generalized set theoretic approach opens up a number of interesting topics of historical, social-theoretical, philosophical and ethical interest.

In historical terms (as the quotation below shows), Leibniz's way of understanding the connection between normative properties and normative requirements comes very close to the approach developed here, and differences and similarities should be more closely investigated:

> That is permitted what a good man possibly is. That is obligatory what a good man necessary is. (Leibniz, 2006, 280)[1]

[1] Leibniz's letter to Antoine Arnauld, November 1671: "Licitum enim est, quod viro bono possibile est. Debitum sit, quod viro bono necessarium est." [My translation]

The research presented in this paper should be extended towards the development of a typology of normative properties and a determination of the deontic logic that describes the structure of the property requirements. The typology of normative systems seems to need a supplementary typology of normative properties, most notably of those that are defined in terms of partial satisfaction.

The motivation for the AGM theory of belief revision came from a legal context. The AGM theory, inter alia, has described the logical ways in which the consistency of a theory should be maintained. The logical properties that define the state of equilibrium for the homeostatic dynamics of normative codes should be determined. Prima facie, a number of other properties, besides mere pl-consistency, should be taken into account for the determination of the equilibrium state of normative systems; in particular, properties such as code compatibility, social consistency, achievability, and logicality seem to be of theoretical importance.

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