Is Unsaying Polite?

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*'Have some wine,' the March Hare said in an encouraging tone.
Alice looked all round the table, but there was nothing on it but tea. 'I don't see any wine,' she remarked.
'There isn't any,' said the March Hare.
'Then it wasn't very civil of you to offer it,' said Alice angrily.
'It wasn't very civil of you to sit down without being invited,' said the March Hare.*

— Alice's Adventures in the Wonderland [6, p. 96]

1 Two Ways of Negating a Sentence

Wittgenstein and Stenius According to the influential tradition in philosophy of language, advocated *inter alios* by Erik Stenius [23] in 1967, there are three logico-semantic moods: indicative, imperative and interrogative, and there are two main components in natural language sentences: modal element, which determines sentence's mood, and sentence radical, which carries the descriptive content. Although Wittgenstein had more permissive attitude regarding the number of sentence moods, still the conception of the twofold sentence structure, consisting in *use* of a *picture*, prevails in the later Wittgenstein's philosophy.

> But how many kinds of sentence are there? Say assertion, question, and command?—There are *countless* kinds: countless different kinds of use of what we call "symbols," "words," "sentences." And this multiplicity is not something fixed, given once for all; but new types of language, new language-games, as we may say, come into existence, and others become obsolete and get forgotten. (We can get a *rough picture* of this from the changes in mathematics.) . . .
> Imagine a picture representing a boxer in a particular stance. Now, this picture can be used to tell someone how he should stand, should hold himself; or how a particular man did stand in such-and-such situation; and so on. One might (using the language of chemistry) call that picture a proposition-radical. [29, p. 11]

Two negation positions The two-part sentence structure offers two options for placement of negation. First, it is the modal element that may be negated: *negation* [modal element] [radical]. I call this position 'external negation position.' Second, it is the sentence radical that may be negated: [modal element] *negation* [radical]. I call this position 'internal negation position.'

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Ross Alf Ross, one of the founders of imperative logic, drew a parallel distinction in 1941 [18]. According to Ross, there are imperatives with "negated theme of demand" (i.e. with negated radical) and there are imperatives with "negated factor of demand" (i.e. with negated modal element).

> [This] means that it is necessary to use linguistic expressions which distinguish between negative imperatives in two different senses, i.e. 1) imperatives with a negative theme of demand ($I(\overline{x})$ = you are (not to close the door) = you are to leave it open) and 2) imperatives with a negative factor of demand, expressing that a positive imperative with an identical theme of demand is not valid ($\overline{I}(x)$ = (you are not to) close the door = the imperative "you are to close the door" is not valid).
>
> The use of the imperative mood in colloquial language does not allow this important difference between $I(\overline{x})$ and $\overline{I}(x)$ to be clearly marked. All imperative in the grammatical sense are positive in the sense that they posses a positive factor of demand. For example, "Do not close the door!" can only mean $I(\overline{x})$ not $\overline{I}(x)$. Only by using linguistically indicative mood the difference becomes apparent. For example, "It is your duty not to close the door" ($I(\overline{x})$), and "It is not your duty to close the door" ($\overline{I}(x)$). [18, p. 63]

Ross includes "imperatives with a negative factor of demand" among imperatives, understood in a broad, nongrammatical sense. The difference between externally and internally negated imperatives is clearly visible, since their grammatical forms differ.

Searle Later, the same distinction was made in Searle's speech act theory, where illocutionary force indicator has the role similar to the role of modal element, while propositional indicator corresponds to sentence radical. Searle used the term 'illocutionary negation' for external negation, and he classified permissions as directives alongside other speech acts typically performed by uttering an imperative.

> "Permit" also has the syntax of directives, though giving permission is not strictly speaking trying to get someone to do something, rather it consists in removing antecedently existing restrictions on his doing it, and is therefore the illocutionary negation of a directive with a negative propositional content, its logical form is $\sim!(\sim p)$. [20, p. 22]

It seems that Searle needlessly narrows down permissions to those having "negative propositional content." In my opinion, the expression 'a negative propositional content' should be replaced with 'an opposite propositional content.' Then the citation could be interpreted as stating that

(i) $\sim!(\sim p)$ is illocutionary negation of $!(p)$, and
(ii) $\sim!(p)$ is illocutionary negation of $!(\sim p)$.

An example for relation (ii) is given by the pair (iii)–(iv), below.

(iii) You may close the door.
(iv) Don't close the door!

It should be noted here that "illocutionary negation" operates simultaneously at two negation positions: external negation includes internal negation.

Grice Grice proclaimed the principles of cooperative communication. If a principle is violated, an utterance becomes inappropriate for reasons other than their "truth conditions."

> In my eyes the most promising line of answer lies in building up a theory which will enable one to distinguish between the case in which an utterance is inappropriate because it is false or fails to be true, or more generally fails to correspond with the world in some favored way, and the case in which it is inappropriate for reasons of a different kind. [10, p. 4]

Horn and Tappenden Horn and Tappenden (*inter alios*) have discussed the difference between the two ways of negation in the special case of indicatives. Horn [13] has pointed out that external, i.e. metalinguistic negation erases some part of the semantic field, like presuppositions, implicatures, points of view, etc.

> I discuss the two uses of negation (descriptive and metalinguistic) in terms of what they generally negate: truth (of a proposition) vs. assertability (of an utterance).[13, p. 122]

> Metalinguistic negation, as we have seen, is used to deny or object to any aspect of a previous utterance —from the conventional or conversational implicata that may be associated with it, to its syntactic, morphological, or phonetic form.[13, p. 144]

In Horn's conception, the external negation takes some previous speech act as its object. Hence, new questions come to fore. Is external negation capable of erasing or canceling the joint effect of two or more previously performed speech acts? Can external negation also take as its object the propositional part of the negated speech act, and, in that way, incorporate internal negation?

> I will argue that negation has both a speech-act indicating and a content-modifying function, and puzzles can be generated by running them together. [24, p.263]

> [. . .] how are we to theoretically classify two distinct patterns of use exhibited by sentences containing negation? One of the uses is to be construed in terms of a speech act of denial, the other in terms of asserting a content, [. . .] So understood, the semantic (content modifying) function is given by the truth table for internal negation if it is given by a truth table at all. The speech act of denial is the commitment to the failure to obtain of the conditions that would have to obtain for S to be true. Though this speech act is correct or incorrect in just the conditions that the assertion of an external negation of S would be correct the speech act differs from the assertion of an external negation in that it bears different relations to embedded sentences. Though one can deny S, it need not be possible to define an external negation operator over the whole language. [24, p. 282]

According to Tappenden [24], the use of the word 'deny' is ambiguous. There is a use of the word which covers the cases in which "denying that S" brings in the commitment to "asserting (claiming) that $\neg S$." In the other sense, which Tappenden calls "non-derivative" sense because of its irreducibility to assertion, denial is "the commitment to the failure to obtain of the conditions that would have to obtain for S to be true."

The problem The concept of the twofold sentence structure is challenged by the fact that external, metalinguistic negation cannot be formalized if the scope of negation is restricted to sentence radical since in this way only speech acts with negated content can be expressed. There is or there should be a logical operator that acts upon the whole '[modal element][radical]' thus producing negated speech act, different from the speech act with negated content. Questions arise: Does (or would) external negation increase the pragmatic power of a language to effect changes in the mind and in the group of minds? Does external negation violate some principles of cooperative communication? The answer to these questions requires a general formal semantic model for positive speech act and its opposite speech acts. It is our thesis that within dynamic logic there are resources to build a variety of explanatory models that are able to cover the basic aspects of logical relations occurring within and between the three broad categories of speech acts: asserting, requesting and asking, which roughly correspond to the three main categories of moods: indicatives, imperatives and interrogatives.

1.1 Modeling Two Kinds of Negation in Dynamic Semantics

The second type of negation (e.g. the imperative with negated "factor of demand", the denial as irreducible speech act) is what I call 'negated speech act.' Within the framework of the dynamic semantics, negated speech act can be modeled as a semantic action that makes it logically or conversationally possible to perform the speech act of the same type but with opposite content. In the simple case, to externally negate speech act $\delta\varphi$ with radical φ means to enable the speech act $\delta\sim\varphi$ with the opposite radical $\sim\varphi$.

The language of dynamic modal logic [3, 17] provides a rich vocabulary that can be used for distinguishing types of speech acts in the way that Speaker's speech act affects Hearer's mental state.

> Let Φ be a set of proposition letters. We define the dynamic modal language $\mathcal{DML}(\Phi)$ [. . .] Its formulas and procedures (typically denoted by φ and α, respectively) are built up from proposition letters ($p \in \Phi$) according to the following rules
>
> $$\varphi ::= p \mid \bot \mid \top \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \mathsf{do}(\alpha) \mid \mathsf{ra}(\alpha) \mid \mathsf{fix}(\alpha),$$
> $$\alpha ::= \mathsf{exp}(\varphi) \mid \mathsf{con}(\varphi) \mid \alpha_1 \cap \alpha_2 \mid \alpha_1 ; \alpha_2 \mid -\alpha \mid \alpha^{\smile} \mid \varphi?.$$
>
> [17, p. 111]

The underlying idea of dynamic modal logic is to interpret procedures α as relations between valuation points, where formulas φ hold (or, in the approach of that will be followed here, relations between structures where formulas are valid or satisfiable). In this way sentences become treated as speech acts: as procedures resulting in a mental state describable by a formula. The question that interests us here is 'Which set of speech act types is expressively complete for the language of imperatives and indicatives?' Later, it will be proved that the operational part of rich vocabulary of dynamic modal logic can be reduced to few operations: testing a property of a mental state, moving towards a more informative information state, sequential composition of moves, and indeterministic choice: $\varphi?$, $\mathsf{exp}(\varphi)$, $\alpha_1;\alpha_2$, $\alpha_1 \cup \alpha_2$.

Using dynamic modal language negated speech act could be provisionally described as

$$[\![\mathsf{con}_1(\delta\varphi)]\!] = \{\langle x,y\rangle \mid y \sqsubseteq x \wedge \mathfrak{M},y \not\models \delta\varphi \wedge \neg\exists z(y \sqsubset z \sqsubseteq x \wedge \mathfrak{M},z \not\models \delta\varphi)\} \qquad (\mathsf{con}_1)$$

i.e. as a backward motion along the $\sqsubseteq$ ordering towards the nearest point y in the structure $\mathfrak{M} = \langle W, \sqsubseteq, [\![\cdot]\!], V\rangle$ where $\delta\varphi$ does not hold. The provisional description is too broad since the fact that $\delta\varphi$ is not accepted in y need not guarantee the acceptability of $\delta\sim\varphi$ there.

Example 1. According to con_1 the sentence 'It is not your duty to open the door' if used to perform negation of the directive 'Open the door' could lead to different mental states. In some of these the addressee may believe that the door has already been open or that her duty is to prevent the door from closing. Such states will neither validate the permission 'You may keep the door closed' nor the suggestion 'It might be good to keep the door closed.'

The phenomenon of contradictory relations cannot be found among imperatives since it is not the case for any future state of affairs that some imperative holds for it. A lot of future state of affairs are "imperative indifferent" and we are not obliged either to produce, sustain, destroy, or prevent them. On the other hand, no imperative holds for the state of affairs that cannot be brought about. Therefore, another type of logical opposition should be brought into the picture. The contrariety seems to be fit for the role. On the one hand, both imperatives 'Let it be the case that φ' and 'Let it be the case that $\neg\varphi$' cannot be jointly satisfied. On the other hand, it is not the case that one of them must be in force. In my pre-understanding of the matter, the abandonment of imperative opens up a logical space for another, contrary imperative. Let us denote by $\delta\varphi$ and $\delta\sim\varphi$ the speech acts of the same type but with contrary content. Using the notion of contrariety, a more precise description of negated speech act can be given in terms of relation con_2 relying on: test (?), sequential composition (;), and con_1.

$$[\![(\delta\sim\varphi)?]\!] = \{\langle x,x\rangle \mid x \models \delta\sim\varphi)\} \qquad (?)$$

$$[\![\alpha_1]\!];[\![\alpha_2]\!] = \{\langle x,y\rangle \mid \exists z(\langle x,z\rangle \in [\![\alpha_1]\!] \wedge \langle z,y\rangle \in [\![\alpha_2]\!])\} \qquad (;)$$

$$[\![\mathsf{con}_2(\delta\varphi)]\!] = [\![\mathsf{con}_1(\delta\varphi)]\!];[\![(\delta\sim\varphi)?]\!] \qquad (\mathsf{con}_2)$$

Another but equivalent way to define negated speech act of the specific type is to define it as a retreat to a mental state upon which the contrary speech act of the same type can be performed (formally, to a point in the domain do of the corresponding relation).

$$[\![\mathsf{con}_2(\delta\varphi)]\!] = [\![\mathsf{con}_1(\bot)]\!];[\![(\mathsf{do}(\delta\sim\varphi))?]]$$

Example 2. Denial of an indicative $\cdot P$ (where $\cdot$ denotes indicative mood and P is propositional content) as a "non-derivative" act receives the following dynamic interpretation: it is a token of the relation type where the second members of the relation enable update with $\cdot\neg P$.

The proposed downdate modeling for negated speech act shows that its notion depends on the notion of speech act with negated or, for the case of imperatives, with contrary content.

For the purpose of modeling denial, as negated speech act of assertion, we will introduce downdate function into Veltman's update semantics [25, 8].[1] Dynamic modal logic takes the relational approach while update semantics narrows it down to the functional one. A preliminary loose connection between dynamic modal logic and update-downdate variant of Veltman's system can be established by the following propositions: $\langle x, x[\varphi^{+}]\rangle \in [\![\mathsf{ex}(\varphi)]\!]$, $\langle x, x[\varphi^{-}]\rangle \in [\![\mathsf{con}(\varphi)]\!]$, $\langle x, x[\varphi^{?}]\rangle \in [\![\varphi?]\!]$.

A token of contractive relation must be chosen in order to apply the functional approach of update/downdate semantics. In the next section I will introduce semantic reformulation of the principles of AGM contraction and define the preferred contraction type on that basis.

2 Contraction Types?

In AGM theory [1, 12] the operation of contraction of set A by a sentence x, $A \div x$ results in maximal subset of A that does not entail x. In general there will be more than one maximal subset of A of the kind, and the set of these is called the remainder set of A by x, $A \perp x$. The remainder set $A \perp x$ contains all and only those sets B such that

(i) $B \subseteq A$,
(ii) $x \notin Cn(B)$, and
(iii) there is no B' such that $B \subset B' \subseteq A$ and $x \notin Cn(B')$.

One of the ways to define contraction $A \div x$ is to say that it is a choice operation γ picking a member of the remainder set: $A \div x \in A \perp x$ or $A \div x = \gamma(A \perp x)$. This function is called maxichoice contraction. The definition of the contraction operation is given in syntactic terms and it has three elements:

1. preservation condition — contracted set is a subset of the original set,
2. not-entailment condition — contracted set does not entail contracted sentence,
3. maximality condition — contracted set retains the maximal number of sentences from the original set.

Note that the definition of the maximality condition invokes the other two conditions, as in (iii).

The contraction operation can be defined in semantic terms.[2] First let us recursively define truth-valuation h as a binary function taking a sentence φ and a valuation w and delivering truth-value t or f.

[1] In [26] Veltman develops a semantics for counterfactuals and introduces "retraction function" that shares the same traits as the function presented here. They differ only in technical sense: since Veltman relies on use of partial valuations ("situations") while I use relation of minimal difference between full valuations.

[2] A more advanced semantics for AGM theory, but without contraction, is given in [22].

Definition 1. For a propositional language $\mathcal{L}$ built over a finite base set $\mathcal{A}$ of propositional letters, a set $w \subseteq \mathcal{A}$ is a valuation point for all $\varphi \in \mathcal{L}$.

Remark 1. The set of all valuation points for a propositional language $\mathcal{L}$ built over the finite set $\mathcal{A}$ of propositional letters will be denoted by $W = \wp\mathcal{A}$.

Definition 2. Truth valuation is a two-place function $h : \mathcal{L} \times W \to \{\mathrm{t}, \mathrm{f}\}$. Truth equals membership for atoms: $h(A, w) = \mathrm{t}$ iff $A \in w$. Compounds are standardly defined: $h(\neg\varphi, w) = \mathrm{t}$ iff $h(\varphi, w) = \mathrm{f}$; $h(\varphi \wedge \psi, w) = \mathrm{t}$ iff $h(\varphi, w) = \mathrm{t}$ and $h(\psi, w) = \mathrm{t}$, and so on as in classical propositional logic.

Definition 3. Truth set $\mathrm{tr}_X(\mathcal{T}) \subseteq W$ for a set of sentences $\mathcal{T} \subseteq \mathcal{L}$ with respect to the set X of valuation points is the set $\mathrm{tr}_X(\mathcal{T}) = \{w \in X \mid \forall\varphi(\varphi \in \mathcal{T} \to h(\varphi, w) = \mathrm{t})\}$.

Notation For the ease of reading, two notational conventions will be adopted: (i) truth set function will be written down as a one-place function whenever it takes the entire set W of valuation points as its argument, i.e. instead of $\mathrm{tr}_W(\cdot)$ I will write $\mathrm{tr}(\cdot)$; and (ii) the valuation points will be indexed by a string of its elements, e.g. w_{pq} for $\{p, q\}$, $w_\emptyset$ for $\emptyset$, etc.

2.1 Contraction defined semantically

There is a compelling way to think about contraction operation in semantic terms. First, identify each proposition x with its singleton's truth set $\mathrm{tr}_W(\{x\})$ within an exhaustive valuation space W, with the understanding that the division between truth set and its relative complement reflects the informational content of the proposition. Then, identify the fact of proposition being a member in a theory, $x \in \mathcal{T}$ with the fact that truth-set of the theory is included in the truth-set of the proposition, $\mathrm{tr}(\mathcal{T}) \subseteq \mathrm{tr}(\{x\})$. The syntactic requirement that a theory is deductively closed set, $\mathcal{T} = Cn(\mathcal{T})$ is generally satisfied in the truth-set-semantics translation since for any $\mathcal{T}$ it holds that $\mathrm{tr}(\mathcal{T}) = \mathrm{tr}(Cn(\mathcal{T}))$. Therefore, the semantic variant of the condition (1) is $\mathrm{tr}(\mathcal{T}) \subseteq \mathrm{tr}(\mathcal{T} \div x)$. Not-entailment condition (2) becomes $\mathrm{tr}(\mathcal{T} \div x) \nsubseteq \mathrm{Tr}(\{x\})$. For any set (T) that entails x, $x \in Cn(\mathcal{T})$, it holds that: if there is a valuation $w \notin \mathrm{tr}(x)$, then a minimal extension $\mathrm{tr}(\mathcal{T}) \cup \{w\}$ will meet not-entailment condition. Any valuation point v can be characterized by a conjunction of literals $\mathrm{nf}(\{v\})$ (see Definition 22 below), so the set $\mathrm{tr}(\mathcal{T} \cup \{\mathrm{nf}(\{w\})\}$ will satisfy not entailment condition. But the choice should not be made in an arbitrary manner if the conservative requirement is to be met: it may well be the case that $\mathrm{nf}(\{w\}) \notin Cn(\mathcal{T})$, violating the condition (1).

Example 3. Let $h(p, w_1) = h(q, w_1) = h(r, w_1) = \mathrm{t}$, $h(p, w_2) = h(q, w_2) = h(r, w_2) = \mathrm{f}$, $W = \wp\{p, q, r\}$, and $\mathcal{T} = Cn(\{p, q, r\})$. Then $\mathrm{tr}(\mathcal{T}) = \{w_1\}$, and $\{w_1, w_2\} \nsubseteq (\mathrm{tr}(\{p\}) \cup \mathrm{tr}(\{q\}) \cup \mathrm{tr}(\{r\}))$. Hence, there is no subset $\mathcal{T}'$ of $\mathcal{T}$ such that $\mathrm{tr}(\mathcal{T}') = \{w_1, w_2\}$. Therefore, an arbitrary enlargement of the truth set, as a semantic counterpart for syntactic contraction, cannot satisfy preservation condition (1).

The preservation condition (1) and, consequently, the maximality condition (3) cannot be given a direct semantic translation. In the syntactic version, the "size" of all theories is the same, $|\mathcal{T}| = \aleph_0$: being a deductively closed set, $\mathcal{T} = Cn(\mathcal{T})$, the cardinality of the theory $\mathcal{T}$ is infinite. Still the relation of proper inclusion of the theory $\mathcal{T}$ in $\mathcal{T}'$, $\mathcal{T} \subset \mathcal{T}'$, is informative enough, showing that there is, so to speak, more knowledge in $\mathcal{T}'$ than in $\mathcal{T}$. Maximal membership condition (3) thus meets the conservative requirement of parsimonious epistemology to preserve as much as possible of the previous knowledge content in the contracted set. On the semantic side, equinumerous truth sets are equally informative, they have the same "degree of uncertainty" but there is an important relation besides inclusion: the degree of similarity between valuations. Valuation points can assign the same truth value to some of the propositional letters: the larger the set of coincident assignments, the greater the similarity of valuations.

Definition 4. "One-way" difference relation df between members of X and Y with respect to proposition φ:

$$\mathrm{df}(\varphi, X, Y) = \{\langle w, v\rangle \in X \times Y \mid h(\varphi, w) = \mathrm{f} \wedge h(\varphi, v) = \mathrm{t}\}.$$

Notation The cardinality of a set X is denoted by $|X|$. The symbol $\triangle$ stands for operation of symmetric difference between sets: $a \triangle b = (a - b) \cup (b - a)$.

Definition 5. The relation μdf of minimal one-way difference between X and Y with respect to proposition φ:

$$\begin{aligned} &\mu\mathrm{df}(\varphi, X, Y) = \\ &\quad = \{\langle w, v\rangle \in \mathrm{df}(\varphi, X, Y) \mid \forall z \forall u(\langle z, u\rangle \in \mathrm{df}(\varphi, X, Y) \to |w \triangle v| \leq |z \triangle u|)\}. \end{aligned}$$

Example 4. For all X and Y it holds that $\mu\mathrm{df}(\top, X, Y) = \mu\mathrm{df}(\bot, X, Y) = \emptyset$.

Example 5. Let $W = \wp\{p, q\}$. Then $\mu\mathrm{df}(p \vee q, W, W) = \{\langle w_\emptyset, w_p\rangle, \langle w_\emptyset, w_q\rangle\}$.

Notation Expressions mem_1 and mem_2 stand for functions that deliver first and second members of a binary relation R, respectively; i.e. $mem_1(R) = \{x \mid \exists y\, Rxy\}$ and $mem_2(R) = \{y \mid \exists x\, Rxy\}$.

Definition 6. Set of "the closest antipodes" of X in Y with respect to φ:

$$\begin{aligned} \mathrm{ca}(\varphi, X, Y) &= \{v \mid \exists w(w \in X \wedge \langle w, v\rangle \in \mu\mathrm{df}(\varphi, X, Y))\} \\ &= mem_2(\mu\mathrm{df}(\varphi, X, Y)). \end{aligned}$$

Semantic interpretation of AGM contraction Now we can give a semantic interpretation for AGM principles of contraction by using the notion of truth set and notion of closest antipodes.

1. Preservation condition: $\mathrm{tr}(\mathcal{T}) \subseteq \mathrm{tr}(\mathcal{T} \div x)$ corresponds to $\mathcal{T} \div x \subseteq \mathcal{T}$.
2. Not-entailment condition: $\mathrm{tr}(\mathcal{T} \div x) \nsubseteq \mathrm{tr}(x)$ corresponds to $x \notin Cn(\mathcal{T} \div x)$.
3. Maximality condition: $\mathrm{tr}(\mathcal{T} \div x) = \mathrm{tr}(\mathcal{T}) \cup \mathrm{ca}(\neg x, \mathrm{tr}(\mathcal{T}), W)$ corresponds to $\mathcal{T} \div x \in \mathcal{T} \perp x$ (where $\mathcal{T} \subseteq \mathcal{L}$ and W is the set of all valuation points for $\mathcal{L}$).

Example 6. Let $\mathcal{A} = \{p,q\}$ be the base set of propositional letters. Then $\mathrm{tr}(\{p,q\} \div p) = \mathrm{tr}(\{p,q\}) \cup \mathrm{ca}(\neg p, \mathrm{tr}(\{p,q\}), W) = \{w_{pq}\} \cup \{w_q\} = \{w_{pq}, w_q\}$.

Example 7. Some vacuous contractions: $\mathrm{tr}(\{p \vee q\} \div p) = \mathrm{tr}(\{p \vee q\} \div \neg p) = \mathrm{tr}(\{p \vee q\})$.

The family of contraction operations in this semantic version is reduced to a single case, and thus underdetermination is gone. This is not to say that this image of an informational contraction is the correct one. Rather, the semantic interpretation of AGM notion of syntactic contraction unexpectedly forces upon us one instead of many candidate operations. The semantically defined operation gives preference to the most informative truth set. The semantic operation differs from Yamada's approach[3] in which contraction of truth set generated by a sequence of sentences $\varphi_1;...;\varphi_n$ with a sentence x would be equated with the truth set generated by a sequence $\varphi_1/x;...;\varphi_n/x$ (where $\varphi_i/x = \top$ if $\varphi_i = x$, and $\varphi_i/x = \varphi_i$ otherwise) in which all occurrences of x have been erased. The operation of adding "closest antipodes" may explicate the cancelation of the sentence that has only been implied although never actually uttered.

Definition 7. If φ is a sentence of a propositional language $\mathcal{L}$, then φ^+, φ^- and $\varphi^?$ are sentences of $\mathcal{L}_{act}$.

Definition 8. Function $\cdot[\cdot] : \wp W \times \mathcal{L}_{act} \to \wp W$ takes a state $\sigma \subseteq W$ and a sentence $\varphi \in \mathcal{L}_{act}$ and delivers a state σ':

$$\sigma[\varphi^+] = \sigma \cap \mathrm{tr}(\varphi) \qquad (\textit{update})$$

$$\sigma[\varphi^?] = \begin{cases} \sigma & \text{if } \sigma[\varphi^+] \neq \emptyset, \\ \emptyset & \text{otherwise.} \end{cases} \qquad (\textit{test})$$

$$\sigma[\varphi^-] = \begin{cases} \sigma \cup \mathrm{ca}(\varphi, \sigma, W) & \text{if } \sigma[\varphi^+] = \sigma, \\ \sigma & \text{otherwise.} \end{cases} \qquad (\textit{downdate})$$

The intended interpretation for functional expressions $\sigma[\varphi^\circ]$ ($\circ = +, -, ?$) is as follows: Speaker utters a sentence φ° thereby changing Hearer's mental state σ into $\sigma[\varphi^\circ]$. In this perspective, semantics cannot be divorced from pragmatics: the function $\cdot[\cdot]$ can be thought of as a 'speech act function' or as 'pragmatic interpretation function.' The natural language counterparts for speech act function presumably can be found among constatives: *update* and *test* correspond to reporting— typically performed by uttering 'It is the case that φ' and 'It might be the case that φ', respectively; *downdate* corresponds to an act of withdrawing a report that has been previously stated or implied—there does not seem to be a typical sentence that can be used to perform withdrawal, albeit perhaps 'It is not the case that φ' understood in the non-derivative sense. It is possible to conceptualize a speech act type that uses sentence φ° as one place function $\cdot[\varphi^\circ]$ thus assigning its pragmatic function to each sentence. The output of pragmatic sentence function depends on the *context* and is a *fixed* point (see Proposition 1 below).[4]

[3] See Yamada, this volume.

[4] The unbound variables are assumed to be universally quantified in all the formulas.

Proposition 1. *For* $\circ = +, -, ?$,

$$\text{If } \nvdash \varphi, \text{ then } \exists\sigma\exists\sigma' \ \sigma[\varphi^\circ] \neq \sigma'[\varphi^\circ]. \qquad (\textit{context})$$

$$\sigma[\varphi^\circ] = (\sigma[\varphi^\circ])[\varphi^\circ] \qquad (\textit{fixed})$$

Proof. Routine.

Downdate gains *success* by enabling update by contradictory sentence in a way that makes *recovery* possible. In this way the notion of non-derivative denial as a speech act of "the commitment to the failure to obtain of the conditions that would have to obtain" [24, p. 282] for φ to be true has been captured. Proposition 2 shows that upon Speaker's non-derivative denial the Hearer's state σ changes into state $\sigma[\varphi^-]$ in which φ^+ is not accepted (*cancelation*), but in which both $\neg\varphi^+$ (*success*) and φ^+ are acceptable (*recovery*).[5]

Proposition 2.

$$\sigma[\varphi^-] \neq (\sigma[\varphi^-])[\varphi^+] \qquad (\textit{cancelation})$$

$$\sigma[\varphi^-] = (\sigma[\varphi^-])[\neg\varphi^?] \qquad (\textit{success})$$

$$\sigma = (\sigma[\varphi^-])[\neg\varphi^+] \qquad (\textit{recovery})$$

Proof. Routine.

3 Imperatives, Commands and Permissions

Compared to the language of indicative sentences, imperative language shows in a more transparent way the distinction between directive speech act with positive content, directive speech act with negated content and negated directive speech act. While the first two create obligations, the third gives permission. In order to examine the possibility of dynamic modeling of the distinction, we will follow the tradition that connects the imperative semantics with action semantics. In particular, we will rely on the following ideas:

- the "propositional content" of an imperative should be syntactically represented by change expression, Lemmon [15];[6]
- the content of imperative is a prescribed action, Belnap [2], Segerberg [21];
- the semantics of action requires existence of negative condition (counter-state, "null-point," avoidability), Kanger [14], Belnap [2], Von Wright [30].

According to Von Wright [30], a minimal semantics of action should capture the following three elements:

[5] Proposition on *recovery* is stronger than acceptability claim: it shows that what has been 'undone' can also be 'redone.'

[6] Change expression syntax for imperatives was re-introduced in [32].

1. initial state, which the agent changes or which would have changed if the agent had not been active,
2. end-state, which results from the action, and
3. counter-state, which would have resulted from agent's passivity.

On these grounds Von Wright developed the fourfold classification of action types: producing $(\neg\varphi/\varphi)$, destroying $(\varphi/\neg\varphi)$, sustaining (φ/φ) and suppressing $(\neg\varphi/\neg\varphi)$ state of affairs φ. The classification of actions can be used as the basis of the twofold classification of imperatives: 1. complementary imperative, which is used for requesting production or destruction of the state of affairs: $!(\neg\varphi/\varphi)$, $!(\varphi/\neg\varphi)$, 2. symmetric imperative, which is used for requesting maintenance or suppression of the state of affairs: $!(\varphi/\varphi)$, $!(\neg\varphi/\neg\varphi)$. To those two, a third type of imperative should be added: "one-sided" imperative $!(\top/\varphi)$, $!(\top/\neg\varphi)$, which has drawn much attention in the literature, e.g. [7].

Example 8. Let C stand for 'The door is closed'. (i) 'Close the door!' and (ii) 'Don't close the door!' are complementary $!(\neg C/C)$ and symmetric imperatives $!(\neg C/\neg C)$, respectively. Pre-theoretically speaking, they are used for the same kind of speech act, for directives or requests. Their contents differ, and the aforementioned imperatives may be understood as having negated content with respect to the other. On the other hand, permission expressed by (iii) 'You don't have to close the door' or 'You may leave the door open' relate to imperative (i) as a negation of the speech act performed by uttering it.

Example 9. The meaning of complementary imperative 'Close the door!' can be depicted by its implications: (initial state) 'The door is open at the moment before;' (end-state) 'The door shall (ought to) be closed at the moment after;' (negative condition) 'It is possible that the door will not be closed at the moment after;' (positive condition) 'It is possible that the door will be closed at the moment after.'[7]

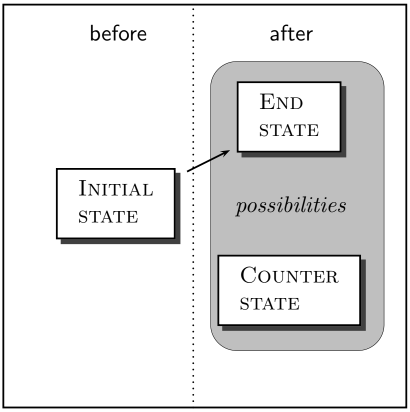

Fig. 1 Following Von Wright's action semantics, the semantics for imperatives as commanded actions should include: three valuation points— initial, end, and counter state; two moments— before and after; relation of commanded change (here represented by arrow); and set of *possible* after situations (here denoted by 'possibilities').

[7] The March Hare's suggestion in the *motto* violates positive condition.

According to the proposed approach, the speech act φ^- negates φ^+. The former is conceived as a token of semantic relation $\mathsf{con}_2(\varphi)$ which enables the acceptance of the speech act with the opposite content $(\sim\varphi)^+$. In other words, downdate with φ enables update with $\sim\varphi$. In order to apply this approach to the case of speech acts performed by uttering imperatives, one must define the relevant opposition for imperatives. As I have argued elsewhere [33],[8] a pair of imperative contraries consists of a complementary and a symmetric imperative; e.g. contrariety of $!(\neg C/C)$ is $!(\neg C/\neg C)$ and vice versa (see Example 8).

I will formalize imperatives as change expressions [15] having peculiar phenomenology concerning their "direction of fit with the world;" the left part should fit the world while it is the world that should fit the right part:

$$\overset{\textit{imperative}}{\overline{\;!(\underset{\text{initial state}}{\overset{\text{word to world fit}}{A}} \;/\; \underset{\text{end state}}{\overset{\text{world to word fit}}{B}})\;}}\;\;\underset{\text{commanded change}}{\underline{\qquad}}$$

3.1 *Language* $\mathcal{L}^{act}_{imp}$

3.1.1 Syntax

Definition 9. Let language $\mathcal{L}_{PL}$ of classical propositional logic built over finite set $\mathcal{A}$ of propositional letters be given. If $\varphi \in \mathcal{L}_{PL}$, then $\cdot(\varphi/\top)$ is indicative before-sentence in $\mathcal{L}_{imp}$ and $\cdot(\top/\boxdot\varphi)$, $\cdot(\top/\Diamond\varphi)$ are indicative after-sentences in $\mathcal{L}_{imp}$. If $\varphi \in \mathcal{L}_{PL}$ and $\psi \in L_{PL}$, then $!(\varphi/\psi)$ and $!(\top/\varphi)$ are imperative sentences in $\mathcal{L}_{imp}$. If φ is indicative before-sentence in $\mathcal{L}_{imp}$ and if ψ is imperative sentence on $\mathcal{L}_{imp}$, then $(\varphi \to \psi)$ and $(\psi \to \varphi)$ are conditional imperative sentences in $\mathcal{L}_{imp}$. Nothing else is a sentence in $\mathcal{L}_{imp}$.

Definition 10. If $\varphi \in \mathcal{L}_{imp}$, then φ^+, φ^-, $\varphi^?$ are sentences in the language $\mathcal{L}^{act}_{imp}$.

3.1.2 Semantics

Definition 11. Set Σ of cognitive motivational states is the set constructed in the following way:

- $\mathcal{A}$ is a finite set of propositional letters,
- $W = \wp\mathcal{A}$ is the set of state descriptions (valuation points),
- $\mathsf{Moments} = \{\mathsf{before}, \mathsf{after}\}$ is the set of moments,
- $Init = W \times \{\mathsf{before}\}$ is the set of initial situations,
- $Res = W \times \{\mathsf{after}\}$ is the set of resulting situations,

[8] In [33] contrariety of imperative is called 'negative imperative.'

- $Changes = Init \times Res$ is the set of changes,
- $\Sigma = \wp(Changes \times Res)$ is the set of cognitive-motivational states.

Definition 12. For $\varphi \in L_{PL}$, $X \subseteq W$ or $X \subseteq (W \times \mathsf{Moments})$, $t \in \mathsf{Moments}$, $|\varphi|^t_X$ is set of φ-state descriptions of X coupled with moment t:

$$|\varphi|^t_X = \begin{cases} \mathrm{tr}_X(\{\varphi\}) \times \{t\} & \text{if } X \subseteq W, \\ X \cap (\mathrm{tr}_W(\{\varphi\}) \times \{t\}) & \text{if } X \subseteq (W \times \mathsf{Moments}). \end{cases}$$

Definition 13. Intension $[\![\varphi/\psi]\!]$ of a change expression (φ/ψ) is the set

$$[\![\varphi/\psi]\!] = |\varphi|^{\mathsf{before}}_W \times |\psi|^{\mathsf{after}}_W .$$

Definition 14. Set $\Phi \subseteq \Sigma$ of absurd states: $\Phi = \{\langle \rho, \pi \rangle \mid \rho = \emptyset \vee \neg mem_2(\rho) \subseteq \pi\}$.
$\mathbf{1} = \langle \emptyset, \emptyset \rangle$ is a distinguished element in Φ.

Definition 15. For $\langle \rho_1, \pi_1 \rangle \in \Sigma$ and $\langle \rho_2, \pi_2 \rangle \in \Sigma$, operation $\Cup$ of merging structures is defined as: $\langle \rho_1, \pi_1 \rangle \Cup \langle \rho_2, \pi_2 \rangle = \langle \rho_1 \cup \rho_2, \pi_1 \cup \pi_2 \rangle$.

Interpretation function $\cdot[\cdot]$ for the language $\mathcal{L}^{act}_{imp}$ is function from $\Sigma \times \mathcal{L}^{act}_{imp}$ into Σ. Some of the interpretations turn out to be rather complex and not reducible to basic cases. Therefore, the definition of interpretation function will be split into several cases: text interpretation (Definition 16), interpretation of updates (Definition 17), interpretation of tests (Definition 18), interpretation of downdates (Definition 19). In the end definitions will be given for sentences definable in terms of others (Definition 20).

Definition 16.

$$\langle \rho, \pi \rangle [\varphi_1] \ldots [\varphi_n] = \langle \rho, \pi \rangle [\varphi_1; \ldots; \varphi_n] = (((\langle \rho, \pi \rangle [\varphi_1]) \ldots)[\varphi_{n-1}])[\varphi_n], \quad \text{for} \quad \varphi_1, \ldots, \varphi_{n-1}, \varphi_n \in \mathcal{L}^{act}_{imp}. \tag{1}$$

Definition 17.

$$\langle \rho, \pi \rangle [!(\top/\varphi)^+] = \begin{cases} \langle \rho \cap [\![\top/\varphi]\!], \pi \rangle & \text{if } |\varphi|^{\mathsf{after}}_\pi \neq \emptyset \text{ and } |\varphi|^{\mathsf{after}}_\pi \subset \pi, \\ \mathbf{1} & \text{otherwise.} \end{cases} \tag{2}$$

$$\langle \rho, \pi \rangle [\cdot(\varphi/\top)^+] = \langle \rho \cap [\![\varphi/\top]\!], \pi \rangle \tag{3}$$

$$\langle \rho, \pi \rangle [\cdot(\top/\boxdot\varphi)^+] = \langle \rho \cap [\![\top/\varphi]\!], \pi \cap |\varphi|^{\mathsf{after}}_{Res} \rangle \tag{4}$$

$$\langle \rho, \pi \rangle [(\cdot(\varphi/\top) \to !(\top/\psi))^+] = \\ = \begin{cases} \langle \rho, \pi \rangle [!(\top/\psi)^+] & \text{if } \langle \rho, \pi \rangle [\cdot(\varphi/\top)^+] = \langle \rho, \pi \rangle, \\ \langle \rho, \pi \rangle [\cdot(\neg\varphi/\top)^+] \Cup \langle \rho, \pi \rangle [\cdot(\varphi/\top)^+][!(\top/\psi)^+] & \text{otherwise.} \end{cases} \tag{5}$$

The intended interpretation $\langle \rho, \pi \rangle$ is Hearer's cognitive-motivational state, $mem_1(\rho)$ is the truth set for her beliefs about the facts at moment before, $mem_2(\rho)$ is the truth set for her goals at moment after, π is the truth set for her beliefs about possible facts at moment after. Regarding imperative update in clause (2), Hearer's mental state may be receptive or not for the directive or the request $!(\top/\varphi)^+$. If the first is the case, she restricts her goals to φ situations, leaving her beliefs unchanged. Hearer is not receptive to directives or requests requiring either the impossible, $|\varphi|_{\pi}^{\mathsf{after}} = \emptyset$, or the inevitable, $|\varphi|_{\pi}^{\mathsf{after}} = \pi$. Receptiveness does not guarantee success since it might be the case that new goals cannot be consistently added, i.e. if $\rho \cap [\![\top/\varphi]\!] = \emptyset$. In the clause (3), constative $\cdot(\varphi/\top)^{+}$ changes beliefs about the facts at moment before; while in (4), constantive $\cdot(\top/\boxdot\varphi)^{+}$ changes beliefs about possible facts at moment after. Conditional imperative in the clause (5) shows that the desired semantics cannot be reduced to three sets: $mem_1(\rho)$, $mem_2(\rho)$, π. If the indicative antecedent is already accepted, the goals will change. But if not, the conditional may still have an effect on cognitive-motivational state. There are two possible cases. First, if antecedent is believed not to be the case, no goal change will occur. But, if the antecedent is neither believed nor disbelieved, the conditional imperative will be "memorized:" beliefs remain the same, but the relations between situations change. Among the relations starting with a φ-situation, i.e. those from $mem_1(\rho) \cap |\varphi|_{W}^{\mathsf{before}}$, only the ones pointing to a ψ-situation, i.e. those from $mem_2(\rho) \cap |\psi|_{W}^{\mathsf{after}}$, will persist. Therefore, if sometime later Hearer learns that φ is the case at moment before, then ψ will become her goal.

Von Wright's "three points of action semantics" can be built in the update semantics for complementary and symmetric imperatives. Information on initial state is encoded into the set $mem_1(\rho)$, information on end-state is encoded into the set $mem_2(\rho)$, information on counter-state (which would have or could have resulted if the agent had refrained from performing commanded action) is encoded in the set π, which also encodes information on the possibility of end-state (π represents both avoidability and possibility of end-state).

Definition 18. For $\varphi \in \mathcal{L}_{imp}$,

$$\sigma[\varphi^{?}] = \begin{cases} \sigma & \text{if } \sigma[(\varphi)^{+}] \notin \Phi, \\ \mathbf{1} & \text{otherwise.} \end{cases} \tag{6}$$

Remark 2. The natural language expressions corresponding to test sentences, the clause (6), are those used for making suggestions: 'It might be good that you see to it that φ will be the case' for $!(\top/\varphi)^{?}$; 'It might be that φ is the case' for $\cdot(\varphi/\top)^{?}$; 'It might be that φ will be the case' for $\cdot(\top/\boxdot\varphi)^{?}$. In this approach, suggestions are seen as consistency or—using the terminology of update semantics—acceptability testing: is it so that $\varphi \in \mathcal{L}_{imp}^{act}$ can be "processed" without landing into absurd cognitive-motivational state $\sigma \in \Phi$? Extending the line of thought, the use of 'therefore' belongs to the same type of operations on mental states; the difference being now that it is validity or acceptance that is being tested. Let us use the symbol $\therefore$ for this type of testing. Then, e.g. the speech act performed upon Hearer's mental state σ by Speaker uttering the sentence 'Therefore, φ' would be formalized as:

$$\sigma[\varphi^{\therefore}] = \begin{cases} \sigma & \text{if } \sigma[\varphi] = \sigma, \\ \mathbf{1} & \text{otherwise.} \end{cases}$$

Notation The following equations hold:

$$|\varphi|^{\mathsf{before}}_{\mathrm{ca}(\varphi, mem_1(mem_1(\rho)),W)} = \mathrm{ca}(\varphi, mem_1(mem_1(\rho)), W) \times \{\mathsf{before}\}$$
$$|\varphi|^{\mathsf{after}}_{\mathrm{ca}(\varphi, mem_1(mem_2(\rho)),W)} = \mathrm{ca}(\varphi, mem_1(mem_2(\rho)), W) \times \{\mathsf{after}\}$$

For the ease of reading, the shorthand notation $[\![\varphi/\top]\!]^{\rho 1}_{ca}$ will be used for 'intension of change expression restricted to time-designated closest antipodes of first members of ρ with respect to φ, and to second members of ρ,' and $[\![\top/\varphi]\!]^{\rho 2}_{ca}$ will be used for 'intension of change expression restricted first members of ρ, and to time-designated closest antipodes of second members of ρ with respect to φ:'

$$[\![\varphi/\top]\!]^{\rho 1}_{\mathrm{ca}} = |\varphi|^{\mathsf{before}}_{\mathrm{ca}(\varphi, mem_1(mem_1(\rho)),W)} \times mem_2(\rho)$$
$$[\![\top/\varphi]\!]^{\rho 2}_{\mathrm{ca}} = mem_1(\rho) \times |\varphi|^{\mathsf{after}}_{\mathrm{ca}(\varphi, mem_1(mem_2(\rho)),W)}.$$

Example 10. Let C stand for 'The window is closed' and B for 'The window is broken,' and let

$$\sigma = \langle \underbrace{\{\langle\langle w_\emptyset, \mathsf{before}\rangle, \langle w_C, \mathsf{after}\rangle\rangle\}}_{\rho}, Res\rangle$$

be cognitive-motivational state built over the "two-letter" base, $\mathcal{A} = \{B, C\}$, $W = \wp\mathcal{A}$, etc. Agent i in the mental state σ intends to close the window without breaking it, or, in other words, i believes that the window is closed and unbroken at the moment before, wants it to be the case that the window is closed and unbroken at the moment after, and believes that the latter state of affairs is both possible, and avoidable in all respects. Now, the mental state

$$\langle \rho \cup [\![\top/\neg C]\!]^{\rho 2}_{\mathrm{ca}}, Res\rangle = \langle\{\langle\langle w_\emptyset, \mathsf{before}\rangle, \langle w_C, \mathsf{after}\rangle\rangle, \langle\langle w_\emptyset, \mathsf{before}\rangle, \langle w_\emptyset, \mathsf{after}\rangle\rangle\}, Res\rangle$$

shows that i's mind has been minimally changed with respect to his wants regarding the window: no longer i wants to close the window, but i still wants to keep it unbroken.

Definition 19.

$$\langle\rho, \pi\rangle[!(\top/\varphi)^-] = \\ = \begin{cases} \langle \rho \cup [\![\top/\neg\varphi]\!]^{\rho 2}_{\mathrm{ca}}, \pi\rangle & \text{if } \langle\rho,\pi\rangle[!(\top/\varphi)^+] = \langle\rho,\pi\rangle, \\ \sigma & \text{otherwise.} \end{cases} \tag{7}$$

$$\langle\rho,\pi\rangle[\cdot(\varphi/\top)^-] = \begin{cases} \langle \rho \cup [\![\neg\varphi/\top]\!]^{\rho 1}_{\mathrm{ca}}, \pi\rangle & \text{if } \langle\rho,\pi\rangle[\cdot(\varphi/\top)^+] = \langle\rho,\pi\rangle, \\ \sigma & \text{otherwise.} \end{cases} \tag{8}$$

$$\langle\rho,\pi\rangle[\cdot(\top/\boxdot\varphi)^-] = \\ = \begin{cases} \langle \rho, \pi \cup mem_2([\![\top/\neg\varphi]\!]^{\rho 2}_{\mathrm{ca}})\rangle & \text{if } \langle\rho,\pi\rangle[\cdot(\top/\boxdot\varphi)^+] = \langle\rho,\pi\rangle, \\ \sigma & \text{otherwise.} \end{cases} \tag{9}$$

$$\sigma[(\cdot(\varphi/\top) \rightarrow !(\top/\psi))^-] = \\ = \begin{cases} \sigma[!(\top/\psi)^-] & \text{if } \sigma[(\cdot(\varphi/\top) \rightarrow !(\top/\psi))^+][\cdot(\varphi/\top)^+] = \sigma, \\ \sigma & \text{otherwise.} \end{cases} \tag{10}$$

Justification The acceptance test in downdate semantic clauses shows that "only what is done can be undone," i.e. for a speech act to be undone it must have been previously, either explicitly or implicitly effected. Semantic clauses determine the minimal modifications needed for enabling of update by a speech act having an opposite content with respect to the speech act that is downdated. The clause (7) is the most interesting one. Downdating by $!(\top/\varphi)$ must make updating by $!(\top/\neg\varphi)$ feasible. For that to happen: (i) there must be a $\neg\varphi$ situation in the goal set, which is therefore minimally expanded, (ii) the future possibility, and (iii) the existence of counter-point are already secured by the fact that $!(\top/\varphi)^+$ is accepted in $\langle\rho,\pi\rangle$ since positive condition (possibility) for $!(\top/\varphi)^+$ is negative condition (avoidability) for $!(\top/\neg\varphi)^+$ and vice versa. In the clause (10) downdate of conditional presupposes pre-theoretical determination of the opposite sentence that is to be enabled. The act-conditional (i) $(\cdot(\varphi/\top) \rightarrow !(\top/\psi))^+$ blocks $\neg\psi$ goals for φ situations. So I take (ii) $(\cdot(\varphi/\top) \rightarrow !(\top/\neg\psi))^+$ as the opposite act-sentence. The acceptance test breaks in two subcases: if $\cdot(\neg\varphi/\top)^+$ is accepted in $\sigma[(\cdot(\varphi/\top) \rightarrow !(\top/\psi))^+]$, then there is nothing to do since then (ii) is accepted as well (second line in (10)); otherwise, downdate with $!(\top/\neg\psi)^-$ will suffice.

Finally, the "reducible" sentences will be defined, but, due to the limitations of space, their informal meaning or justification will be omitted.

Definition 20.

$$\sigma[!(\varphi/\psi)^+] = \sigma[\cdot(\varphi/\top)^+][!(\top/\psi)^+] \tag{11}$$
$$\sigma[!(\varphi/\psi)^-] = \sigma[\cdot(\varphi/\top)^-][!(\top/\psi)^-] \tag{12}$$
$$\sigma[\cdot(\top/\Diamond\varphi)^+] = \sigma[\cdot(\top/\boxdot\varphi)^?] \tag{13}$$
$$\sigma[\cdot(\top/\Diamond\varphi)^-] = \sigma[\cdot(\top/\boxdot\varphi)^-] \tag{14}$$
$$\sigma[(!(\top/\varphi) \rightarrow \cdot(\psi/\top))^+] = \sigma[(\cdot(\neg\psi/\top) \rightarrow !(\top/\neg\varphi))^+] \tag{15}$$
$$\sigma[(!(\top/\varphi) \rightarrow \cdot(\psi/\top))^-] = \sigma[(\cdot(\neg\psi/\top) \rightarrow !(\top/\neg\varphi))^-] \tag{16}$$

Example 11. Implications listed in Example 9 hold:[9]

$$\sigma[!(\neg C/C)^+] = (\sigma[!(\neg C/C)^+])[\cdot(\neg C/\top)^+; !(\top/C)^+; \cdot(\top/\Diamond C)^+; \cdot(\top/\Diamond\neg C)^+].$$

The language $\mathcal{L}^{act}_{imp}$ and interpretation function $\cdot[\cdot]$ provide an uncommon approach which drags the pragmatics into the syntax of the formal language and, consequently, it equates pragmatic effects with semantic actions. It is speech act that gets a formal translation, and not the sentence by whose utterance it is performed. If this approach is sound, then logic permeates all three branches of semiotics. Pragmatics might lie within the scope of logic.

Example 12. Command 'Close the door!' is formalized as $!(\neg C/C)^+$; permission 'You don't have to close the door' ('You may leave the door open') as $!(\neg C/C)^-$; suggestion 'Maybe you should close the door' as $!(\neg C/C)^?$.

[9] For discussion on varieties of relations of meaning inclusion that can be distinguished within dynamic semantics see [4].

3.1.3 The puzzle of permission distribution

According to Searle, permission "consists in removing antecedently existing restrictions [on] doing" [20, p. 22]. Command $!(\top/\varphi)^+$ restricts Hearer's action by making a change $\top/\neg\varphi$ forbidden for her. Downdate $!(\top/\varphi)^-$ enables update $!(\top/\neg\varphi)^+$ and, therefore, it may serve as a formal explication for "restrictions removing" notion of permission.

The puzzle of distribution of permission over disjunction has been much discussed in the literature: (i) 'You may see to it that A or B' pre-theoretically implies (ii) 'You may see to it that A,' and (iii) 'You may see to it that B.' On the proposed approach (i) is translated as $!(\top/(\neg A \wedge \neg B))^-$ and interpreted as cancelation of (iv) 'See to it that $\neg A$ and $\neg B$.' Similarly, (ii) and (iii) are translated as $!(\top/\neg A)^-$ and $!(\top/\neg B)^-$, respectively. Proposition 3 shows that if there is a restriction to be removed, then by removing the whole of restriction, all of its "parts" will be removed.

Proposition 3. *Let* $\sigma[!(\top/(\neg A \wedge \neg B))^-] \neq \sigma$. *Then*

$$\begin{aligned}\sigma[!(\top/(\neg A \wedge \neg B))^-] &= \sigma[!(\top/(\neg A \wedge \neg B))^-][!(\top/\neg A)^-]\\ &= \sigma[!(\top/(\neg A \wedge \neg B))^-][!(\top/\neg B)^-]\end{aligned}$$

Proof. The proof relies on the fact that for any x such that $x \cap \mathrm{tr}(\{\neg A \wedge \neg B\}) \neq \emptyset$, $\mathrm{ca}(A \vee B, x, W) \cap \mathrm{tr}(\{A\}) \neq \emptyset$ and $\mathrm{ca}(A \vee B, x, W) \cap \mathrm{tr}(\{B\}) \neq \emptyset$.

4 Expressive Completeness

There are several interesting questions that arise at the interface between natural language and its logical formalization. In the natural language there are three kinds of imperatives: complementary or produce imperative, symmetric or sustain imperative, and "right-side" or see-to-it-that imperative. Since the syntax of natural language restricts the range of change expressions to contradictory, identical and "truncated" pairs, the translation of natural language sentences will yield only a proper subset of imperative sentences in $\mathcal{L}_{imp}$. Namely, we find only $!(\neg\varphi/\varphi)$, $!(\varphi/\varphi)$ and $!(\top/\varphi)$ types of sentences in the subset.

1. Is the subset strong enough to generate each non-absurd cognitive-motivational state? If not, what are obstacles to communication that are inherent in the language itself?
2. Further, do negated speech acts add expressive power to the language?

Within the framework of language $\mathcal{L}^{act}_{imp}$ and its semantics, the answer to the first question is affirmative (Corollary 1) and, therefore, negative to the second. Theorem 1 shows that each non-absurd cognitive-motivational state $\sigma \in \Sigma - \Phi$ can be generated using a proper subset of language $\mathcal{L}^{act}_{imp}$ in which only "positive (i.e. non-negated) speech acts" occur.

Theorem 1. *For each* $\sigma \in \Sigma - \Phi$ *there are* $\varphi_1, \ldots, \varphi_n \in \mathcal{L}_{imp}$ *such that*

$$\langle Changes, Res\rangle[\varphi_1^+; \ldots; \varphi_n^+] = \sigma.$$

Proof. Proof is given by construction of the required text.
Let $mem_1(mem_1(\rho)) = \{w_1, \ldots, w_n\}$. The construction takes three steps. First, the first members of ρ are cut out of $\langle Changes, Res\rangle$ using the sentence

$$\cdot(\mathrm{nf}(mem_1(mem_1(\rho)))/\top)^+$$

and thus obtaining $\langle mem_1(\rho) \times Res, Res\rangle$ (see Proposition 5). Second, a sequence of sentences $s(w_1)^+; \ldots; s(w_n)^+$ is applied to $\langle mem_1(\rho) \times Res, Res\rangle$ (where each sentence $s(w_i)$ is either a conditional imperative or a tautology) yielding (Proposition 6):

$$\langle mem_1(\rho) \times Res, Res\rangle[s(w_1)^+] \ldots [s(w_n)^+] = \langle \rho, Res\rangle.$$

Third, application of $\cdot(\top / \boxdot\, \mathrm{nf}(mem_1(\pi)))^+$ gives the desired result (Proposition 7):

$$\langle \rho, Res\rangle[\cdot(\top / \boxdot\, \mathrm{nf}(mem_1(\pi)))^+] = \langle \rho, \pi\rangle.$$

The text

$$\cdot(\mathrm{nf}((mem_1(mem_1(\rho)))/\top)^+; s(w_1)^+; \ldots; s(w_n)^+; \cdot(\top / \boxdot\, \mathrm{nf}(mem_1(\pi)))^+$$

is an instance that proves that each non-absurd state can be generated by a text of $\mathcal{L}^{act}_{imp}$. □

Corollary 1. *Let* $\mathcal{L}^{act}_{\to stit} \subset \mathcal{L}^{act}_{imp}$ *be a language comprising only sentences of the form:* $\cdot(\varphi/\top)^+$, $(\cdot(\varphi/\top) \to !(\top/\psi))^+$, $\cdot(\top/\boxdot\,\varphi)^+$ *. Language* $\mathcal{L}^{act}_{\to stit}$ *is expressively complete with respect to the set* $\Sigma - \Phi$ *of non-absurd states.*

Proof. Note that text construction in the proof of Theorem 1 uses only the sentences from $\mathcal{L}^{act}_{\to stit}$.[10] □

Definition 21 (Literals λ). Given $l_1, \ldots, l_n$ list of all propositional letters in $\mathcal{A}$, $w_1, \ldots, w_m$ list of all valuation points in $X \subseteq W$, $\wp\mathcal{A} = W$, $1 \leq i \leq n$, $1 \leq j \leq m$, literals λ^i_j are defined by:

$$\lambda^{w_j}_{l_i} = \begin{cases} l_i \text{ if } l_i \in w_j, \\ \neg l_i \text{ if } l_i \notin w_j. \end{cases}$$

[10] Note that a translation for the conditional imperative in dynamic modal language can be given by:

$$((\cdot(\varphi/\top))?; \mathsf{ex}(!(\top/\psi)))$$
$$\cup$$
$$((\mathsf{do}(\mathsf{ex}(\cdot(\neg\varphi/\top)) \vee \mathsf{do}(\mathsf{ex}(!(\top/\neg\psi)))))?; \mathsf{ex}(\mathsf{do}(!(\varphi/\psi)) \wedge \neg\mathsf{do}(!(\varphi/\neg\psi)))).$$

Therefore, the claim put forward in Subsection 1.1 has been proved as well.

Definition 22 (Adequate description). Function nf delivers a disjunctive normal form for the set $X \subseteq W$ with respect to given lists of letters $l_1, \ldots, l_n$ and valuation points $w_1, \ldots, w_m$ in X:[11]

$$\begin{aligned}\mathrm{nf}(X) &= ((\lambda_{l_1}^{w_1} \wedge \ldots \wedge \lambda_{l_n}^{w_1}) \vee \ldots \vee (\lambda_{l_1}^{w_m} \wedge \ldots \wedge \lambda_{l_n}^{w_m}))\\ &= \mathrm{nf}(\{w_1\}) \vee \ldots \vee \mathrm{nf}(\{w_m\}).\end{aligned}$$

Proposition 4. *For* $X \subseteq W$, $|\mathrm{nf}(X)|_W^t = X \times \{t\}$.

Proof. The proof is straightforward and only right to left direction will be shown. Suppose for some arbitrary v that $\langle v, t\rangle \in X \times \{t\}$. Obviously, $h(\mathrm{nf}(\{v\}), v) = \mathrm{t}$. By Definition 2, $h(\mathrm{nf}(X), v) = \mathrm{t}$. By Definition 12, $\langle v, t\rangle \in |\mathrm{nf}(X)|_W^t$. □

Proposition 5.

$$\langle Changes, Res\rangle[\cdot(\mathrm{nf}(mem_1(mem_1(\rho)))/\top)^+] = \langle mem_1(\rho) \times Res, Res\rangle$$

Proof. By Definition 13,

$$[\![\mathrm{nf}(mem_1(mem_1(\rho)))/\top]\!] = |\mathrm{nf}(mem_1(mem_1(\rho)))|_W^{\mathsf{before}} \times |\top|_W^{after}.$$

The fact that $|\top|_W^{after} = Res$ together with an application of Proposition 4, i.e. $|\mathrm{nf}(mem_1(mem_1(\rho)))|_W^{\mathsf{before}} = mem_1(\rho)$, gives the desired result.

Definition 23. Function $ex_\rho^{\langle w, \mathsf{before}\rangle}$ delivers set of resulting situations "visible" from situation $\langle w, \mathsf{before}\rangle$: $ex_\rho^{\langle w, \mathsf{before}\rangle} = mem_2((\{\langle w, \mathsf{before}\rangle\} \times Res) \cap \rho)$.

Definition 24. Let $mem_2(\rho) \subseteq \pi$. For each situation $\langle w, \mathsf{before}\rangle \in mem_1(\rho)$, function s delivers a sentence from $\mathcal{L}_{imp}$:

$$s(w) = \begin{cases} \cdot(\top/\top) & \text{if } ex_\rho^{\langle w, \mathsf{before}\rangle} = \pi,\\ (\cdot(\mathrm{nf}(\{w\})/\top) \rightarrow !(\top/\mathrm{nf}(mem_1(ex_\rho^{\langle w, \mathsf{before}\rangle})))) & \text{otherwise.}\end{cases}$$

Proposition 6. *Let* $\{w_1, \ldots, w_n\} = mem_1(\rho)$. *Then*

$$\langle Changes, Res\rangle[\cdot(\mathrm{nf}(mem_1(mem_1(\rho)))/\top)^+; s(w_1)^+; \ldots; s(w_n)^+] = \langle \rho, Res\rangle.$$

Proof. By Proposition 5,

$$\begin{aligned}&\langle Changes, Res\rangle[\cdot(\mathrm{nf}(mem_1(mem_1(\rho)))/\top)^+; s(w_1)^+; \ldots; s(w_n)^+] =\\ &\qquad = \langle mem_1(\rho) \times Res, Res\rangle[s(w_1)^+; \ldots; s(w_n)^+].\end{aligned}$$

There are two cases to examine concerning the number of situations in $mem_1(\rho)$.

1. First, for $|mem_1(\rho)| = 1$ let $mem_1(\rho) = \{\langle w, \mathsf{before}\rangle\}$. Therefore,

[11] For this idea I am indebt to [28].

$$\rho = \{\langle w, \text{before}\rangle\} \times mem_2(\rho).$$

There are two subcases.

a. If $ex_\rho^{\langle w,\text{before}\rangle} = Res$, then $s(w_1) = \cdot(\top/\top)$ and obviously

$$\langle mem_1(\rho) \times Res, Res\rangle[\cdot(\top/\top)^+] = \langle \rho, Res\rangle.$$

b. In the second subcase, $ex_\rho^{\langle w,\text{before}\rangle} \subset Res$. Then

$$s(w) = (\cdot(\text{nf}(\{w\})/\top) \rightarrow !(\top/\text{nf}(mem_1(ex_\rho^{\langle w,\text{before}\rangle})))).$$

Since $|\text{nf}(\{w\})|_W^{\text{before}} = \{\langle w, \text{before}\rangle\} = mem_1(\rho)$,the conditional has the following impact:

$$\sigma[(\cdot(\text{nf}(\{w\})/\top) \rightarrow !(\top/\text{nf}(mem_1(ex_\rho^{\langle w,\text{before}\rangle}))))^+] =$$
$$= \sigma[!(\top/\text{nf}(mem_1(ex_\rho^{\langle w,\text{before}\rangle})))^+],$$

where $\sigma = \langle\{\langle w, \text{before}\rangle\} \times Res, Res\rangle$. Since

$$|\text{nf}(mem_1(ex_\rho^{\langle w,\text{before}\rangle}))|_W^{\text{after}} = mem_2(\rho),$$

we get the required result.

2. For the second case, when $|mem_1(\rho)| > 1$ we have to show that semantic impact (if any) of $s(w_i)$, $1 \leq i \leq n$ is localized to $\langle w_i, \text{before}\rangle$ generating

$$\{\langle w_i, \text{before}\rangle\} \times ex_\rho^{\langle w_i,\text{before}\rangle}$$

and leaving everything else as it is. In other words, we have to show that for each $w_i \in mem_1(\rho)$,

$$\langle mem_1(\rho) \times Res, Res\rangle[s(w_i)^+] =$$
$$= \langle((mem_1(\rho) - \{\langle w_i, \text{before}\rangle\}) \times Res) \cup (\{\langle w_i, \text{before}\rangle\} \times ex_\rho^{\langle w_i,\text{before}\rangle}), Res\rangle.$$

There are two subcases to examine. For typographic reasons symbol b will be used instead of before.

a. First, if $ex_\rho^{\langle w_i,\text{b}\rangle} = Res$, then $s(w_i) = \cdot(\top/\top)$ and

$$\langle mem_1(\rho) \times Res, Res\rangle[\cdot(\top/\top)^+] = \langle mem_1(\rho) \times Res, Res\rangle.$$

b. In the second subcase: $ex_\rho^{\langle w_i,\text{b}\rangle} \subset Res$. The assumption $|mem_1(\rho)| > 1$ guarantees that

$$\langle mem_1(\rho) \times Res, Res\rangle[\cdot(\text{nf}(\{w_i\})/\top)^+] \neq \langle mem_1(\rho) \times Res, Res\rangle$$

since $|\text{nf}(\{w_i\})|_W^{\text{b}} \neq mem_1(\rho)$. Let σ stand for $\langle mem_1(\rho) \times Res, Res\rangle$. Then the update by conditional

$$s(w_i) = (\cdot(\mathrm{nf}(\{w_i\})/\top) \to !(\top/\mathrm{nf}(mem_1(ex_\rho^{\langle w_i,\mathsf{b}\rangle}))))$$

has the following impact:

$$\begin{aligned}
&\sigma[(\cdot(\mathrm{nf}(\{w_i\})/\top) \to !(\top/\mathrm{nf}(mem_1(ex_\rho^{\langle w_i,\mathsf{b}\rangle}))))^+] = \\
&= \sigma[\cdot(\neg\mathrm{nf}(\{w_i\})/\top)^+] \Cup \sigma[\cdot(\mathrm{nf}(\{w_i\})/\top)^+][!(\top/\mathrm{nf}(mem_1(ex_\rho^{\langle w_i,\mathsf{b}\rangle})))^+] \\
&= \langle (mem_1(\rho) - \{\langle w_i,\mathsf{b}\rangle\}) \times Res, Res\rangle \Cup \langle \{\langle w_i,\mathsf{b}\rangle\} \times ex_\rho^{\langle w_i,\mathsf{b}\rangle},\ Res\rangle \\
&= \langle ((mem_1(\rho) - \{\langle w_i,\mathsf{b}\rangle\}) \times Res) \cup (\{\langle w_i,\mathsf{b}\rangle\} \times ex_\rho^{\langle w_i,\mathsf{b}\rangle}),\ Res\rangle.
\end{aligned}$$

The sequence $s(w_1)^+;\ldots;s(w_n)^+$ of update functions generates the desired:

$$\begin{aligned}
&\langle \bigcap_{1\le i\le n} (((mem_1(\rho) - \{\langle w_i,\mathsf{b}\rangle\}) \times Res) \cup (\{\langle w_i,\mathsf{b}\rangle\} \times ex_\rho^{\langle w_i,\mathsf{b}\rangle})),\ Res\rangle = \\
&= \langle \rho, Res\rangle.
\end{aligned}$$

□

Proposition 7. $\langle \rho, Res\rangle[\cdot(\top/\boxdot\,\mathrm{nf}(mem_1(\pi)))^+] = \langle \rho, \pi\rangle$.

Proof. Routine.

5 Concluding Remarks

AGM theory of contraction together with hereby proposed downdate semantics entails the fact that external denial, instead od reducing, increases the degree of uncertainty. After a sentence has been withdrawn (canceled, externally negated, unsaid,...), Hearer's mental state not only becomes less determinate but also the path of change itself is under-determinate. It may turn out that the requirement of maximal preservation springs from the normative source of cooperative communication, but it might be just one among other admissible types of contraction. The negated speech acts do not make natural language more expressive, as Theorem 1 shows. Unsaying increases "communicative entropy" and is avoidable. Therefore, we should apologize if we negate a speech act. And not for the sake of cultural convention, but for the sake of logic.

Related research Directive speech acts have been analyzed in terms of changing preferences [5] and obligation patterns [31] within the framework of "dynamic epistemic logic." The foundation for "shifting the logical perspective from valid argumentation to cooperative communication" has been laid down in [11]. A dynamic modal logic for imperatives is given in [9]; a variant of update semantics for imperatives has been developed in [16]. There is renewed interest in imperative logic in philosophy, e.g. [27] and linguistics, e.g. [19].

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