Von Wright's Pragmatics Turn in Deontic Logic

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Modalities, Conditionals, and Values

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Overview

1. Von Wright's full circle in deontic logic
   - Some interpreted quotations illustrating the path
2. Formal explication
   - A set-theoretical model of normative system: the norm-set for obligation-norms and the counter-set for permission norms
   - Translation from the language of modal deontic logic to the language of set-theoretical approach
   - Translated theorems of deontic logic give definitions of perfection properties
3. Applications and consequences
   - The problem of completeness
     - Is it rational to complete a normative system using the metanormative principle *everything not permitted is forbidden*?
   - Normative reasoning in face of an inconsistent normative system
4. Concluding remarks
   - The theoretical turn of the deontic logic towards logical pragmatics

Abstract In his later works on deontic logic von Wright put forward a programmatic statement on its nature: it is "a study of conditions which must be satisfied in rational norm-giving activity". In this new perspective the axioms of standard deontic logic are to be read as descriptions of "perfection properties" that a normative system should have and to which there correspond second-order obligations, or requirements of rationality to which the norm-giver is subordinated in the norm-giving activity. A simple formal explication for von Wright's programmatic statement can be obtained using the set-theoretic approach proposed by many philosophical logicians according to which the existence of an obligation-norm within a normative system is represented by the membership of its propositional content in the set that represents the normative system in question. The translation from the language of standard deontic logic (without iterated operators) to the set-theoretic language confirms the claims put forward by von Wright in 1999, but the translation also suggests the extension of the meta-normative approach to other norm-related activities. Consequently, the distinction must be introduced between perfection properties of obligation-norm sets with respect to the activity of the norm-giver and the reasoning of the norm-recipient. For example, the norm-giver is under obligation to enunciate consistent normative systems but not deductively closed ones. On the other hand, the norm-recipient has the obligation to reason on the basis of the normative system and thus to treat it as a deductively closed set, while having no obligation with respect to its consistency. In this way von Wright's last remarks on deontic logic open up a new perspective which asks for the logical pragmatics. In addition to this and taking into account von Wright's thesis on only normative and not conceptual relation between permission and absence of prohibition, the translation of deontic postulates can also be extended to the description of the properties of the "counter-set", the set of that which is either forbidden or optional. The extended translation reveals that perfection properties come in pairs and provides an interesting way for understanding the relation of the norm-recipient to an inconsistent norm-set or to a normative system whose norm-set and counter-set are not disjunctive. The power of revision does not belong to the norm-recipient role. A normative vacuum does not appear if the norm-recipient is subordinated to an inconsistent normative system in which there is no way out of the normative conflict on the basis of the metanormative principles on the priority order over norms. In such a case it seems plausible to postulate the second-order norm for the reasoning of the norm-recipient according to which a shift to an inconsistency-tolerant logic is required.

Von Wright's deontic logic: a full circle

> "...I was anxious to rescue something of what deontic logicians, including myself, had been doing for more than thirty years. The notion of rationality came to my help and so I arrived at a position according to which deontic logic 'is neither a logic of norms nor a logic of norm-propositions but a study of conditions which must be satisfied in rational norm-giving activity'...Substantially, I abide by this view. But with one, perhaps important, modification. This is that the rationality conditions themselves may be regarded as belonging to logic. Logic 'has a wider reach than truth' ...This was what I thought initially to be the lesson of the coming into existence of deontic logic. Later I thought differently. In the end, it seems, I have gone full circle back to my original position. But I still think the journey was worth making. "
>
> Georg Henrik von Wright (1991).
> Is There a Logic of Norms?
> *Ratio Juris* 4:265–283.
>
> [pp.265–266]

Recently, in his talk at conference *The Human Condition* Euginio Bulygin has divided von Wright's work in deontic logic in four phases: 1) DOGMATIC phase of 1950s marked by ignoring the fact that norms do not have truth-value; 2) ECLECTIC phase of *Norm and Action* introducing the distinction between logic of norms and logic of norm propositions; 3) SCEPTIC marked by the thesis that logic of norms is impossible; 4) LOGIC WITHOUT TRUTH-phase redefines deontic logic as the study of rationality conditions of the norm-giving activity.

A circle

"As an undefined deontic category we introduce the concept of permission. It is the only undefined deontic category which we need. *If an act is not permitted, it is called forbidden*."

Georg Henrik von Wright (1951).
Deontic Logic.
Mind 60:1–15.

"The relation between permission and absence of prohibition is not a *conceptual* but a *normative* relation."

Georg Henrik von Wright (1991).
Is There a Logic of Norms?
Ratio Juris 4:265–283.

In the early phase there is only one primitive term; in the late phase there are two.

- Von Wright's "pilgrim's progress" from standard deontic logic to the position he held in his later works in 1990s may look as a circle, but the ending point is not the same. The theorems from 1950s still remain as theorems in 1990s deontic logic, but their position and character has been changed. They cease to be theorems of the "logical syntax" of deontic language, and become the theorems of the "logical pragmatics" of deontic language use. What had been previously understood as a conceptual relation, later becomes a normative relation; a norm for the norm-giving activity, and not the logic of the norms being given.

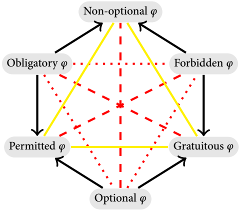
Summarizing von Wright's reinterpretation of deontic logic

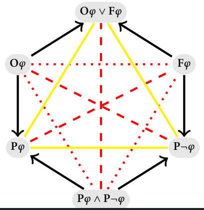
- Non-derivative character of obligations. Permission and obligation are not interdefinable, i.e., $O\varphi \leftrightarrow \neg P \neg \varphi$ is not a "conceptual truth" but a "normative demand on normative system" (von Wright, 1999).
- If permission and obligation are not interdefinable, then there must two types of consistency:
  1. External consistency relates obligation-norms and permission norms: $\neg(O\varphi \wedge P\neg\varphi)$.
  2. Internal consistency deals with obligation-norms: $\neg(O\varphi \wedge O\neg\varphi)$.
- The set of obligation-norms ought to have perfection properties. Perfection properties are "normative demands on normative systems", "rationality conditions of norm-giving activity", "second-order obligation" for the norm-giver.
- For a set-theoretical explication of von Wright's reinterpretation of deontic logic it is required to: (i) take into account perfection properties of the set corresponding to permission norms, and (ii) explicate perfection relations between obligation-norms and permission norms (e.g., external consistency).

Deontic logic as modal logic: 1950s

> "One day when I was walking along the banks of the River Cam —I was at that time living in Cambridge (England)— I was struck by the thought that the modal attributes "possible," "impossible" and "necessary" are mutually related to one another in the same way as the quantifiers "some," "no" and "all." I soon found that the formal analogy between quantifiers and modal concepts extended beyond the patterns of interdefinability...I had made another accidental observation —this time in the course of a discussion with friends— namely that the normative notions of permission, prohibition, and obligation seemed to conform to the same pattern of mutual relatedness as quantifiers and basic modalities."
>
> Georg Henrik von Wright (1999).
> Deontic logic: a personal view.
> *Ratio Juris* 12:26–38.
>
> [p.28]

The hexagon of logical relations holding in standard KD deontic logic. The red dotted line represents the contrariety relation, the red dashed line represents contradiction, the full line represents subcontrariety, and the arrows represent subalternation (implication).





The turn towards logical pragmatics

> "…deontic sentences in ordinary usage exhibit a characteristic ambiguity. Sometimes they are used as norm-formulations. We shall call this their prescriptive use. Sometimes they are used for making what we called normative statements. We call this their descriptive use. When used descriptively, deontic sentences express what we called norm-propositions. If the norms are prescriptions, norm-propositions are to the effect that such and such prescriptions 'exist', i.e. have been given and are in force."

Georg Henrik von Wright (1963).
Norm and Action : A Logical Enquiry.
London: Routledge and Kegan Paul.

Logical pragmatics takes into account the use of language. Different logics correspond to different uses of the language:

1. the *logic of norms* corresponds to the *prescriptive use of language*,
2. the *logic of norm-propositions* corresponds to the *descriptive use of language*.

Example. According to KD modal logic sentences $P\varphi$ and $P\neg\varphi$ are subcontraries, and so cannot be false together. Nevertheless, it is possible that a normative system is incomplete, not having the permission for φ nor for $\neg\varphi$. Therefore, $P\varphi \vee P\neg\varphi$ is not a theorem of the logic of norm-propositions. On the other hand, tautologies, such as $P\varphi \vee \neg P\varphi$, are theorems in the logic of norm propositions.

Deontic logic as the logic of prescriptive use of language is a study of its rationality conditions

> "Deontic logic, one could also say, is neither a logic of norms nor a logic of norm-propositions but a study of conditions which must be satisfied in rational norm-giving activity. It is strict *logic* because the conditions which it lays down are derived from *logical* relations between states in the ideal worlds which normative codes envisage."
>
> Georg Henrik von Wright (1993).
>
> A Pilgrim's Progress.
>
> In von Wright, G.H. *The Tree of Knowledge and Other Essays*, 103–113.
>
> Leiden: Brill.
>
> [p.111]

- This programmatic statement on the directions for the development of deontic logic together with the outline of the path of its realization puts once again Von Wright in the role of the "midwife" (to use his own words) of the new perspective in deontic logic.

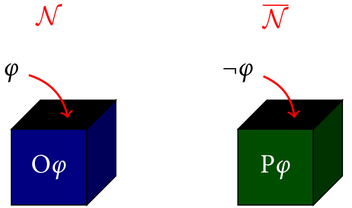
The basic idea can be summarized as follows: Thanks to the prescriptive use of language normative systems come into existence. The logical properties of normative systems are described using the language of the "logic of norm-propositions". Some logical properties are "perfection-properties". The absence of a certain perfection-property does not deprive a normative system of its normative force. In the prescriptive use of language the norm-giver ought to achieve some perfection properties of the normative system thereby produced. Deontic logic is a study of logical perfection properties; properties which act as the normative source of requirements to which the norm-giver and the norm-recipient are subordinated.

Obligation-norms and permission-norms

> "Just as possibility is the negation of the necessity of the contradictory of a proposition, permission is the negation of the obligatoriness of the contradictory. $\mathrm{P}p \leftrightarrow \neg \mathrm{O} \neg p$ is a theorem of "classical" deontic logic.
> **I think that this opinion is mistaken**. The relation between permission and absence of prohibition is not a *conceptual* but a *normative* relation. One may be able to give good reasons why such things which are not prohibited by the norms of a certain code should be regarded as permitted by the code in question. But to declare the non-prohibited permitted is a normative act. One could have a meta-norm to the effect that the not-prohibited is permitted. The well-known principles *Nulla poena sine lege* and *Nullum crimen sine lege* may be thought of as versions of this meta-norm. Or at least as closely related to it."
>
> Georg Henrik von Wright (1991).
> Is There a Logic of Norms?
> *Ratio Juris* 4:265–283.

The logically primitive character both of permissions and obligations poses a difficult problem for the semantic modelling. A possible solution will be presented here.

The metaphor: putting in boxes having different logical structure (I)



- The facts about a normative system are represented using the language of set-theory:
    - $O\varphi$ is represented by $\ulcorner\varphi\urcorner \in \mathcal{N}$,
    - $P\varphi$ is represented by $\ulcorner\neg\varphi\urcorner \in \overline{\mathcal{N}}$.

Although counter-intuitive at the first glance, the adequate metaphor for permitting is that of putting the negation of the content into the permission box. This corresponds to the standard definition "it is not obligatory that $\neg\varphi$": $\neg\varphi$ cannot go into the blue box, so, it must be placed in the green box.

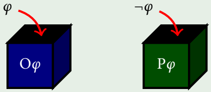
The perfection properties are different for different "boxes" since "ideal concepts" of obligation and permission have different logical structure. For example, having a contradictory pair is an imperfection property of the obligation box, but for the permission box this is neither a perfection nor an imperfection property. Similarly, completeness is a perfection property for permissions but not for obligations: it is indifferent whether $\ulcorner\varphi\urcorner \in \mathcal{N} \vee \ulcorner\neg\varphi\urcorner \in \mathcal{N}$ holds, while $\ulcorner\varphi\urcorner \in \overline{\mathcal{N}} \vee \ulcorner\neg\varphi\urcorner \in \overline{\mathcal{N}}$ ought to hold.

This model, as will be shown, can account for the fact that perfection properties come in pairs, one for obligations, another for permissions, both of which are characterized by the same theorems of standard deontic logic.

The metaphor: putting in boxes but with diverse logical structure (II)

Example

The difference in logical structure of the two "boxes" is also visible from the following facts:



- A perfect counter-set can have a contradictory pair of (negations) of permission-norm contents, which means that a certain state of affairs is optional. This fact does not cause an "explosion" since in this box the principle *ex contradictione quodlibet* does not hold.
- Presence of a disjunct for each disjunction is a perfection property only for the "counter-set", i.e., permission-norm set.
  - In a perfect system $(\varphi \vee \psi) \in \mathcal{N}$ does not require $\varphi \in \mathcal{N} \vee \psi \in \mathcal{N}$.
  - In a perfect system $(\varphi \vee \psi) \in \overline{\mathcal{N}}$ does require $\varphi \in \overline{\mathcal{N}} \vee \psi \in \overline{\mathcal{N}}$.

Metatheory of descriptive theory and normative system

- The proposed two-sets model bears resemblance to the relation between a theory T and its counter-part $\mathcal{L} - Cn(T)$. The counter part has logical properties such as "closure under the implicant" (if $\psi \in \mathcal{L} - Cn(T)$ and φ entails ψ, then $\varphi \in \mathcal{L} - Cn(T)$).
- The perfection properties of the descriptive theory have been well investigated within the logic of natural sciences. For example, the completeness of a theory, formulated in a language that cannot express its own syntax, counts as its perfection property, but the completeness of obligation-norm set is not its perfection property. The mismatch holds also on the side of "counter-sets": the completeness of the descriptive counter-part $\mathcal{L} - Cn(T)$ is an indifferent property, while in the realm of normativity it is a perfection property of the "counter-set" representing permission-norms.
- The construction is different too: there is no "exclusion" part in building a theory since rejecting a sentence equals accepting its negation. This need not be the case with normative systems, whose obligation and permission parts are separately built.
- These facts shows that deontic logic as the study of "rationality conditions of norm-giving activity" or "perfection properties of normative systems" is a sui generis logic. If one accepts, together with von Wright, the central position of the phenomenon of normativity in humanities and social sciences, then deontic logic plays the prominent role in the philosophy of the science of man by revealing the logical basis of its methodological autonomy.

The turn towards logical pragmatics

STANDARD DEONTIC LOGIC	SET-THEORETIC APPROACH	SECOND-ORDER NORMS
theorems of standard deontic logic	perfection properties of the normative system $\langle \mathcal{N}, \overline{\mathcal{N}} \rangle$	obligations of the norm-giver in the prescriptive use of language[1]
Example: internal and external consistency		
$O\varphi \to \neg F\varphi$	$\{\varphi, \neg\varphi\} \nsubseteq \mathcal{N}$	$[g : \underline{!O_r\varphi}]\mathbf{F}_g\, g : \underline{!F_r\varphi}$
$O\varphi \to \neg P\neg\varphi$	$\mathcal{N} \cap \overline{\mathcal{N}} = \varnothing$	$[g : \underline{!O_r\varphi}]\mathbf{F}_g\, g : \underline{!P_r\neg\varphi}$

[1] g := norm-giver; r := norm-recipient; $\mathbf{F}_g$:= it is forbidden for the norm giver that …; $g : \underline{!O_r\varphi}$:= the norm giver has used the sentence $O_r\varphi$ in the prescriptive way; $[C]E$:= after C it is the case that E.

Connecting the two languages of deontic logic

- A simple formal explication for von Wright's programmatic statement can be obtained using the set-theoretic approach proposed by many philosophical logicians (e.g., Carlos Alchourrón and Eugenio Bulygin, or, more recently, John Broome), according to which the existence of an obligation-norm within a normative system is represented by the membership of its propositional content in the set that represents the normative system in question.
- In the set theoretic approach the basic idea is to represent the norm by the membership relation between its content φ and "norm-set" $\mathcal{N}$: the expression 'it is obligatory that φ' is explicated as '$\ulcorner\varphi\urcorner \in \mathcal{N}$'.

On set-theoretic approach

- This highly reduced model can be made more realistic by adding variables, such as those for the source, addressee and situation and taking as elementary the expression 'by the source s it is obligatory in the situation w upon actor i that φ'. E.g., following Broome $\mathcal{N}$ would be treated as a three-place function which delivers norm-contents (requirements), $\mathcal{N}(s, i, w) \subseteq \mathcal{L}$.
- The major point of divergence within the set-theoretic approach lies in the properties one is willing to assign to norm-sets.
- It is in accord with the approach proposed by von Wright to treat norm-sets as simple sets consisting just of sentences that correspond to contents of explicitly promulgated norms and to lay the question of their logical properties aside.
- In the approach proposed here it is not assumed that a norm-set is deductively closed, or closed under equivalence. The norm-set, as understood here, is just a set and the question of its desirable properties is solved by second-order norms addressed to different actor roles.

Translations

$\langle \mathcal{N}, \overline{\mathcal{N}} \rangle$ is a normative system.
$\mathcal{L}_{pl}$ is the language of propositional logic.
Language $\mathcal{L}_{SDL}$: $\varphi ::= p \mid \mathrm{O}p \mid \mathrm{P}p \mid \neg\varphi \mid (\varphi_1 \wedge \varphi_2)$ where $p \in \mathcal{L}_{pl}$.
Language $\mathcal{L}_{\langle \mathcal{N}, \overline{\mathcal{N}} \rangle}$: $\varphi ::= p \mid p \in \mathcal{N} \mid p \in \overline{\mathcal{N}} \mid \neg\varphi \mid (\varphi_1 \wedge \varphi_2)$ where $p \in \mathcal{L}_{pl}$.

$\tau^+, \tau-, \tau* : \mathcal{L}_{SDL} \mapsto \mathcal{L}_{\langle \mathcal{N}, \overline{\mathcal{N}} \rangle}$

To norm-set properties

$$\tau^+(\mathrm{O}\varphi) = \ulcorner\varphi\urcorner \in \mathcal{N}$$
$$\tau^+(\mathrm{P}\varphi) = \ulcorner\neg\varphi\urcorner \notin \mathcal{N}$$
$$\tau^+(\mathrm{P}\neg\varphi) = \ulcorner\varphi\urcorner \notin \mathcal{N}$$

To counter-set properties

$$\tau^-(\mathrm{O}\varphi) = \ulcorner\varphi\urcorner \notin \overline{\mathcal{N}}$$
$$\tau^-(\mathrm{P}\varphi) = \ulcorner\neg\varphi\urcorner \in \overline{\mathcal{N}}$$
$$\tau^-(\mathrm{P}\neg\varphi) = \ulcorner\varphi\urcorner \in \overline{\mathcal{N}}$$

To normative system properties

$$\tau^*(\mathrm{O}\varphi) = \tau^+(\mathrm{O}\varphi)$$
$$\tau^*(\mathrm{P}\varphi) = \tau^-(\mathrm{P}\varphi)$$

$\tau^\star(\varphi) = \varphi$ if $\varphi \in \mathcal{L}_{pl}$; $\tau^\star(\neg\varphi) = \neg\tau^\star(\varphi)$; $\tau^\star(\varphi \wedge \psi) = \tau^\star(\varphi) \wedge \tau^\star(\psi)$, where $\star = +, -, *$.

An example: $\mathrm{O}\varphi \to \mathrm{P}\varphi$

Example

$$\tau^+(\mathrm{O}\varphi \to \mathrm{P}\varphi) = \tau^+(\mathrm{O}\varphi) \to \tau^+(\mathrm{P}\varphi) = \ulcorner\varphi\urcorner \in \mathcal{N} \to \ulcorner\neg\varphi\urcorner \notin \mathcal{N}$$

Perfection property:

- consistency of $\mathcal{N}$.

$$\tau^-(\mathrm{O}\varphi \to \mathrm{P}\varphi) = \tau^-(\mathrm{O}\varphi) \to \tau^-(\mathrm{P}\varphi) = \ulcorner\varphi\urcorner \notin \overline{\mathcal{N}} \to \ulcorner\varphi\urcorner \notin \overline{\mathcal{N}} = \ulcorner\varphi\urcorner \in \overline{\mathcal{N}} \vee \ulcorner\varphi\urcorner \notin \overline{\mathcal{N}}$$

Perfection property:

- completeness of $\overline{\mathcal{N}}$.

Correspondences

POSTULATES OF STANDARD DEONTIC LOGIC	NORM-SET PROPERTIES	COUNTER-SET PROPERTIES
(D) $O\varphi \to P\varphi$	consistency $\tau^+(D) = \varphi \in \mathcal{N} \to \neg\varphi \notin \mathcal{N}$	completeness $\tau^-(D) = \varphi \notin \overline{\mathcal{N}} \to \neg\varphi \in \overline{\mathcal{N}}$
(2) $(O\varphi \wedge O\psi) \to O(\varphi \wedge \psi)$	closure under conjunction $\tau^+(2) = (\varphi \in \mathcal{N} \wedge \psi \in \mathcal{N}) \to (\varphi \wedge \psi) \in \mathcal{N}$	having at least one conjunct for each conjunction contained $\tau^-(2) = (\varphi \wedge \psi) \in \overline{\mathcal{N}} \to (\varphi \in \overline{\mathcal{N}} \vee \psi \in \overline{\mathcal{N}})$
$\dfrac{\vdash_{pl} \varphi \to \psi}{O\varphi \to O\psi}$ (*cl*) alternatively, (K) axiom together with necessitation rule (with taut. in both sets)	deductive closure if $\vdash_{pl} \varphi \to \psi$, then $\tau^+(cl) = \varphi \in \mathcal{N} \to \psi \in \mathcal{N}$	"closure under implicant" if $\vdash_{pl} \varphi \to \psi$, then $\tau^-(cl) = \psi \in \overline{\mathcal{N}} \to \varphi \in \overline{\mathcal{N}}$
	RELATIONAL PROPERTIES	
(D^*) $O\varphi \to \neg P\neg\varphi$	external consistency $\tau^*(D^*) = \varphi \in \mathcal{N} \to \varphi \notin \overline{\mathcal{N}}$	
(G) $O\varphi \vee P\neg\varphi$	"gaplessness" $\tau^*(G) = \varphi \in \mathcal{N} \vee \varphi \in \overline{\mathcal{N}}$	

Imperfections in graphical presentation

- "Relational (or external) inconsistency" occurs if $\mathcal{N} \cap \overline{\mathcal{N}} \neq \emptyset$.
  - An example: $O\varphi \wedge P\neg\varphi$ is given in the picture below.
- "Inner inconsistency" occurs if $\{\psi, \neg\psi\} \subseteq \overline{\mathcal{N}} \neq \emptyset$ for some ψ.
  - An example: $O\psi \wedge F\psi$ is given in the picture below.
- Incompleteness: existence of normative gaps, $\neg O\chi \wedge \neg P\chi$

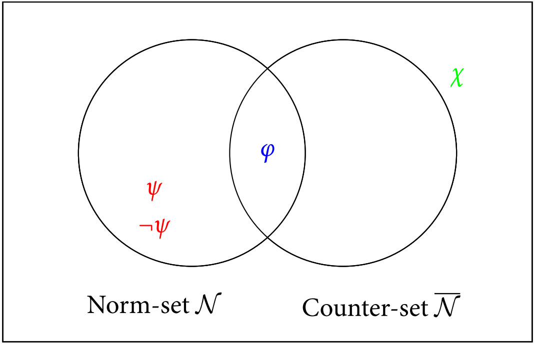


Figure : A "gapful" and "doubly inconsistent" normative system.

Relational perfections

- A normative system $\langle \mathcal{N}, \overline{\mathcal{N}} \rangle$ with "relational perfections":
  - it is gapless, $\mathcal{N} \cup \overline{\mathcal{N}} = \mathcal{L}_{\text{doable}}$,
  - it is externally consistent, $\mathcal{N} \cap \overline{\mathcal{N}} = \varnothing$.

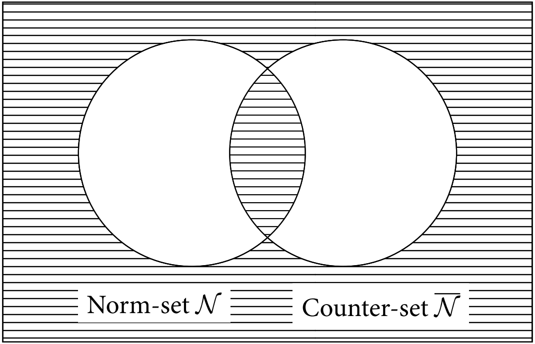


Figure : A gapless and externally consistent system.

Normative gaps

> "What is the difference 'in practice' between a state of affairs not being prohibited and its being permitted? Suppose there is a code of norms in which there is no norm P*p*. Now someone makes it so that *p*. What should be the law-giver's reaction to this, if any? Could he say: 'You were not permitted to do this and you must not do that which you are not permitted to do'? He could say this, making it a meta-norm that everything not-permitted is thereby forbidden. 'Logically' this would be just as possible, even though perhaps less reasonable, as to have a meta-norm permitting everything which is not forbidden. But one can also think of some 'middle way' between these two principles, a meta-norm to the effect that if something is not permitted by the existing norms of a code one must, as we say, 'ask permission' of the law-giver to do it."
>
> Georg Henrik von Wright.
> Is there a logic of norms?
> *Ratio Juris*, 4:265–283, 1991.

According to von Wright there are three principles by use of which a normative system can be completed: 1. $(\neg F \rhd P)$ everything not forbidden is permitted, 2. $(\neg P \rhd F)$ everything not permitted is forbidden, 3. normative gaps are filled in communication between the norm-recipient and the norm-giver.
The third principle will be left aside because of its complexity. Using the two-sets model of normative system I will try to show why the first principle is to be preferred over the second one, i.e., why the first principle is "more reasonable". In addition to this I will show that the mere "logical possibility" of the second mode $(\neg P \rhd F)$ of filling normative gaps is not a sufficient condition of its rationality, according to von Wright's own criterion of rationality of norm-giving activity.

The easy way to complete a normative system

Definition

A norm-system $\langle \mathcal{N}, \overline{\mathcal{N}} \rangle$ is gapless iff $\{\varphi, \neg\varphi\} \subseteq \mathcal{N} \cup \overline{\mathcal{N}}$ for all doable states of affairs φ and $\neg\varphi$, i.e., $\mathcal{L}_{\text{doable}} = \mathcal{N} \cup \overline{\mathcal{N}}$.

Remark

The notion of "doable state of affairs" is taken over from von Wright's works. The notion of doability introduces complex problems of logic of action. Here the set of sentences describing doable states of affairs will be simplified and identified with the set of contingent sentences,
$\mathcal{L}_{\text{doable}} = \mathcal{L}_{pl} - \{\varphi \mid \vdash_{pl} \varphi \text{ or } \vdash_{pl} \neg\varphi\}$.

The easy way of making a normative system complete is by applying the principle *everything which is not forbidden is permitted*. The way of filling in the gaps is straightforward, consisting in adding the missing sentences to the counter set and thus obtaining its extension $\overline{\mathcal{N}}^{\,*}$, as formula (1) shows.

$$\overline{\mathcal{N}}^{\,*} = \overline{\mathcal{N}} \cup \{\varphi \mid \varphi \notin Cn(\mathcal{N})\} \tag{1}$$

The long and branching road…

A completion of the normative system under the principle *everything which is not permitted is forbidden* is not a functional relation. In this mode the process is under-determined and so does not result in a unique system. The completion proceeds in two steps, each of which includes a choice.

The first step

In the first step the counter-set must be completed in the view of perfection relations and properties. Also, the perfection-relation between the obligation norm set and its counter-set ought to be preserved if present and so their intersection must remain empty. This means that it will be expanded to achieve perfection-properties of being closed under implicants and under the rule of having at least one conjunct for each member conjunction. Since the last condition has the disjunctive consequent there may be different ways of performing the closure. Therefore, the weak-ideal expansion of a counter-set results in a set of sets.

Definition

The minimal weak-ideal closure $\mathrm{WI}(\overline{N})$ of a counter-set is the set of the smallest sets a satisfying the following conditions:

1. a includes $\overline{\mathcal{N}}$: $\overline{\mathcal{N}} \subseteq a$,
2. if $\ulcorner\psi\urcorner \in \overline{\mathcal{N}}$ and φ entails ψ and $\ulcorner\varphi\urcorner \in \mathcal{L}_{\text{doable}}$, then $\ulcorner\varphi\urcorner \in a$,
3. a satisfies one of the following conditions:

   a) if $\ulcorner\varphi \wedge \psi\urcorner \in \overline{\mathcal{N}}$, $\ulcorner\psi\urcorner \notin \overline{\mathcal{N}}$, $\ulcorner\varphi\urcorner \notin Cn(\mathcal{N})$ and $\ulcorner\varphi\urcorner \in \mathcal{L}_{\text{doable}}$, then $\ulcorner\varphi\urcorner \in a$,
   b) if $\ulcorner\varphi \wedge \psi\urcorner \in \overline{\mathcal{N}}$, $\ulcorner\varphi\urcorner \notin \overline{\mathcal{N}}$, $\ulcorner\psi\urcorner \notin Cn(\mathcal{N})$ and $\ulcorner\psi\urcorner \in \mathcal{L}_{\text{doable}}$, then $\ulcorner\psi\urcorner \in a$.

Definition

Function γ picks an arbitrary member of the set $\mathrm{WI}(\overline{\mathcal{N}})$ of weak-ideal sets: $\gamma(\overline{\mathcal{N}}) \in \mathrm{WI}(\overline{\mathcal{N}})$.

Example

Let $\mathcal{L}_{\text{doable}} = \{p, q\}$. Let $\mathcal{N} = \varnothing$. Let the only norm be the norm-permission $\mathrm{P}(\neg p \vee \neg q)$. It follows that:

- $\ulcorner p \wedge q\urcorner \in \overline{\mathcal{N}}$,
- $\mathrm{WI}(\overline{\mathcal{N}}) = \{\{\ulcorner p \wedge q\urcorner, \ulcorner p\urcorner, \ulcorner p \wedge \neg q\urcorner\}, \{\ulcorner p \wedge q\urcorner, \ulcorner q\urcorner, \ulcorner \neg p \wedge q\urcorner\}\}$

The second step of a completion of normative system is under-determined and complex in itself. It consists of two phases. In each phase lists of sentence is used, which will be understood as lists of equivalence classes $[\varphi] = \{\psi \mid \vdash_{pl} \psi \leftrightarrow \varphi\}$ $[\varphi_1], \ldots, [\varphi_n], \ldots$.

❶ In the first phase the obligation norm-set and its counter-set are closed under appropriate relations by taking into account "partially placed" sentences, i.e., those where only one from a pair of contradictory sentences belongs to the closure of the system, i.e., $\ulcorner\varphi\urcorner \in Cn(\mathcal{N}) \cup \gamma(\overline{\mathcal{N}}) \leftrightarrow \ulcorner\neg\varphi\urcorner \notin Cn(\mathcal{N}) \cup \gamma(\overline{\mathcal{N}})$.

$$\mathcal{N}_0 = Cn(\mathcal{N} \cup \{\ulcorner\varphi\urcorner \mid \vdash_{pl} \varphi \leftrightarrow \neg\psi \text{ and } \ulcorner\psi\urcorner \in \gamma(\overline{\mathcal{N}})\}) \tag{2}$$

$$\overline{\mathcal{N}}_0 \in \mathrm{WI}(\gamma(\overline{\mathcal{N}}) \cup \{\ulcorner\varphi\urcorner \mid \vdash_{pl} \varphi \leftrightarrow \neg\psi \text{ and } \ulcorner\psi\urcorner \in Cn(\mathcal{N}_0)\}) \tag{3}$$

❷ In the second phase "unplaced sentences" are being added in an iterative manner to the system. "Unplaced sentences" are those where no sentence from a pair of contradictory sentences belongs to the system, i.e., $\ulcorner\varphi\urcorner \notin \mathcal{N}_0 \cup \overline{\mathcal{N}}_0 \wedge \ulcorner\neg\varphi\urcorner \notin \mathcal{N}_0 \cup \overline{\mathcal{N}}_0$.

$$\langle \mathcal{N}_{n+1}, \overline{\mathcal{N}_{n+1}} \rangle = \begin{cases} \langle Cn(\mathcal{N}_n \cup \{\ulcorner\varphi_n\urcorner\}), \overline{\mathcal{N}}_n \cup \{\ulcorner\neg\varphi_n\urcorner\} \rangle, & \text{if } \mathcal{N}_n \cup \{\ulcorner\varphi\urcorner\} \text{ is consistent,} \\ \langle Cn(\mathcal{N}_n \cup \{\ulcorner\neg\varphi_n\urcorner\}), \overline{\mathcal{N}}_n \cup \{\ulcorner\varphi_n\urcorner\} \rangle, & \text{otherwise.} \end{cases} \tag{4}$$

$$\langle \mathcal{N}^*, \overline{\mathcal{N}}^* \rangle = \langle \bigcup_{0 \geq i} \mathcal{N}_i, \bigcup_{0 \geq i} \overline{\mathcal{N}}_i \rangle \tag{5}$$

There is no preferred ordering of unplaced sentences. The outcome of the iterative process depends on the chosen ordering. In most cases the resulting systems are radically different.

The systems completed by the application of the principle *everything not permitted is forbidden* do not necessarily end in one and the same "ideal state of things".

Table : An example.

$\mathrm{O}(p \vee q)$	
$\mathcal{N} = \{p \vee q\}$	$\overline{\mathcal{N}} = \varnothing$
expansion of the counter-set by partially placed sentences and weak-ideal closure of the counter-set	
$\gamma'(\overline{\mathcal{N}}_0) = \{\neg p \wedge \neg q, \neg p, \neg p \wedge q\}$	$\gamma''(\overline{\mathcal{N}}_0) = \{\neg p \wedge \neg q, \neg q, p \wedge \neg q\}$
expansion of the obligation-set with respect to the counter sets	
$\mathcal{N}_0' = \{p \vee q, p, p \vee \neg q\}$	$\mathcal{N}_0'' = \{p \vee q, q, \neg p \vee q\}$
expansion by unplaced sentences	
list 1: $[q], \ldots$	list 2: $[\neg p], \ldots$
$\mathcal{N}_1' = Cn(\mathcal{N}_0' \cup \{q\})$	$\mathcal{N}_1'' = Cn(\mathcal{N}_0' \cup \{\neg p\})$
$(p \wedge q) \in \mathcal{N}_1'$	$\neg(p \wedge q) \in \mathcal{N}_1''$
resulting incompatible ideal states	
$\mathrm{O}(p \wedge q)$ w.r.t. $\mathcal{N}^{*'}$	$\mathrm{F}(p \wedge q)$ w.r.t. $\mathcal{N}^{*''}$

Table : Undetermined completion according to the principle *everything not permitted is forbidden*.

Possible choice?	Incomplete normative system $\langle \mathcal{N}, \overline{\mathcal{N}} \rangle$
	Weak-ideal closure of $\overline{\mathcal{N}}$: $\mathrm{WI}(\overline{\mathcal{N}})$
Yes	$\gamma(\overline{\mathcal{N}}) \in \mathrm{WI}(\overline{\mathcal{N}})$
Yes	Expansion by "partially placed" sentences $\langle \mathcal{N}_0, \overline{\mathcal{N}}_0 \rangle$
Yes	Ordering list of unplaced sentences l
	Iterative expansion by "unplaced sentences" $\langle \mathcal{N}^*, \overline{\mathcal{N}}^* \rangle$

A critique: how many ideal states?

"Generally speaking: a legal order and, similarly, any coherent code or system of norms may be said to envisage what I propose to call an ideal state of things when no obligation is ever neglected and everything permitted is sometimes the case. If this ideal state is not logically possible, i.e., could not be factual, the totality of norms and the legislating activity which has generated it do not conform to the standards of rational willing. Deviations from these standards sometimes occur — and when they are discovered steps are usually taken to eliminate them by 'improved' legislation.

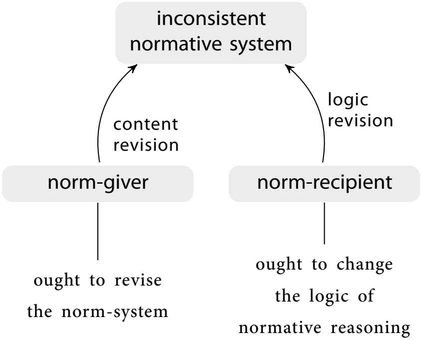
Georg Henrik von Wright.
Is and ought.
In M. C. Doeser and J. N. Kraay, editors, *Facts and Values: Philosophical Reflections from Western and Non-Western Perspectives*, pages 31–48. Springer Netherlands, Dordrecht, 1986.

If a normative system is completed under the principle *everything not permitted is forbidden*, then, if consistent, it can "envisage more then one ideal state", each equally acceptable as any other. Thus, there will be no unique ideal state with respect to obligation-norms. If intending a unique ideal state is essential to rational willing on the side of the norm-giver, then the the principle *everything not permitted is forbidden* is not only "less reasonable", as von Wright claimed, but also not (instrumentally) rational.

Social Pragmatics of Deontic Logic

- Von Wright's programmatic statement can be extended to include normative reasoning as another type of norm-related activity. Second-order norms for the roles of norm-giver and norm-recipient need not be the same. This difference most vividly appears in the relation of roles to an inconsistent normative system.

The logical pragmatics view on inconsistency



Corrective obligations with respect to an inconsitent normative-system.

Logic revision as a second-order obligation

- A normative vacuum does not appear if the norm-recipient is subordinated to an inconsistent normative system, in which there is no way out of the normative conflict on the basis of the metanormative principles on the priority order over norms.
- On the other hand, the norm-recipient cannot reason using classical logic since it would lead to the logical "explosion" (on the side of the norm-set).
- The only remaining option is logic revision. [2] So, the norm-recipient faced with an inconsistent normative system ought to adopt an inconsistency-tolerant logic under which the normative properties will be preserved, namely, closure under entailment and adjunction of the norm-set together with correlated properties of the counter-set (closure under implicants and closure under having at least one conjunct for each conjunction). Is there such a logic? Yes there is (or there are)! The deontic dialetheic logic (G. Priest, 1987) fits the need of logic revision on the side of the norm-recipient.

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[2] In the view of perfection properties, some postulates of logic revision can be outlined. The first condition that a logic change ought to satisfy is to restore coherence (=non-explosiveness) of the set whose logic is being changed. Secondly, the change of logic ought to preserve desirable logical properties. The two conditions of the logic revision, restoration condition and preservation condition, resemble the content contraction, but the difference lies in the fact that instead of consistency it is the coherence that is being restored, and, instead of maximal preservation of the content, it is the desirable logical properties that are being saved.

Why to use the term 'social pragmatics' for von Wright's reinterpretation of deontic logic

The term 'pragmatics' indicates the study of language-use: the norm-giver is engaged in the prescriptive *use of language* while constructing a normative system; the norm-recipient *uses a system constructed by language use* as the basis of her/his normative reasoning. The term 'social' indicates that more than one language-user (or social role) should be taken into account: the (role of) norm-giver, the (role of) norm-recipient, the (role of) norm-evaluator.
Social pragmatics of deontic logic studies the norms that apply to norm-related activities of social actor roles. These norms can be properly called 'second-order norms' since they cover the activities that are related to a normative-system. There is a difference between second-order norms which require construction (envisaging) a logical possible description of an ideal state (e.g., consistency norms) and second-order norms which are related to the will of the norm-giver. In the latter case if the aim is to construct a description of exactly one ideal state, then the second-order norm *everything not permitted is forbidden* is not acceptable since it might end in a multitude of equally valid ideal states.

A view on deontic *modalities* arising form von Wright's reinterpretation

- The language of modal deontic logic can be (and perhaps should be) understood as the language in which perfection properties of a normative system are being described.
- The norm-giver and the norm-recipient are related both to the actual normative-system, which may be imperfect, and to its, possibly missing, perfection properties (from which second-order norms spring).
- Logic has sometimes been understood as the *ethics of thinking*. Von Wright's reinterpretation of deontic logic prompts us to understood logic also as the *ethics of language use*. In understanding deontic logic the perspectives of different social roles of should be taken into account as well as the purpose of norm giving activity. In this way deontic logic ceases to be a "zero-actor logic" and becomes the logic of language use which requires the presence of "users". This fact redefines deontic logic as a research which necessarily includes the stance of *logical pragmatics*.[3]

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