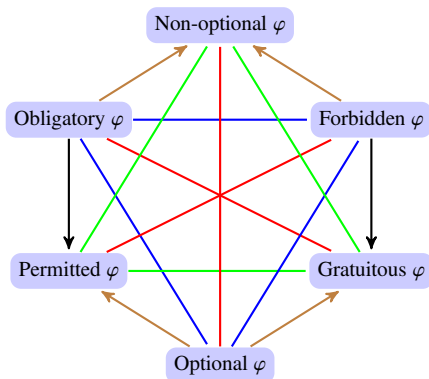


Standard deontic logic and metanormative theory

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One world is not enough

- According to the *Tractarian* criterion, modal propositions are not propositions at all since they are not truth-functions.
- The “one world” semantic theory cannot accommodate modal propositions since the truth value of their “elementary propositions” in *The World* does not determine the truth-value of the modal compound.

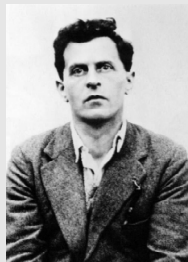
Example

The truth of what is obligatory to be (ought to be) the case is logically independent of that what is the case.

Let **O** stand for ‘It is obligatory that’.

Both $p \wedge \mathbf{O}p$ and $\neg p \wedge \mathbf{O}p$ are satisfiable.

Tractatus logico-philosophicus



5 Propositions are truth-functions of elementary propositions.

...

6.42 Hence also there can be no ethical propositions.

...

7 Whereof one cannot speak, thereof one must be silent.

What to do?



The consequence of the “one world” semantics is not just being silent about certain topics, but rather abandonment of the huge part of language.

Leibniz and modal analysis of normative concepts

Modal approach to normativity

Licetum enim est, quod viro bono possibile est.
Debitum sit, quod viro bono necessarium est.^a



Gottfried Wilhelm Leibniz.

Letter to Antoineu Arnauldu, November 1671.

Saemtliche Schriften Und Briefe. Zweite Reihe: Philosophischer Briefwechsel. Erster Band 1663–1685,

Berlin: Akademie Verlag.

^aThat is permitted what a good man possibly is.
That is obligatory what a good man necessary is.

Analysis

In Leibniz's definition normative concepts (permission **P**, obligation **O**) are defined in terms of (i) alethic modalities (possibility \diamond , necessity \square) (ii) normative properties (being a good man G_i).

$$\mathbf{O}\varphi \leftrightarrow \square(G_i \rightarrow \varphi)$$

$$\mathbf{P}\varphi \leftrightarrow \diamond(G_i \wedge \varphi)$$



Gottfried Wilhelm Leibniz
(1646.–1716.),
statue at University u Leipzig

Deontic logic as modal logic

Philosopher's recollection

One day when I was walking along the banks of the River Cam —I was at that time living in Cambridge (England)— I was struck by the thought that the modal attributes “possible,” “impossible” and “necessary” are mutually related to one another in the same way as the quantifiers “some,” “no” and “all.” I soon found that the formal analogy between quantifiers and modal concepts extended beyond the patterns of interdefinability. . . I had made another accidental observation —this time in the course of a discussion with friends— namely that the normative notions of permission, prohibition, and obligation seemed to conform to the same pattern of mutual relatedness as quantifiers and basic modalities.



Georg Henrik von Wright.

Deontic logic: a personal view.

Ratio Juris, 12:26–38, 1999.



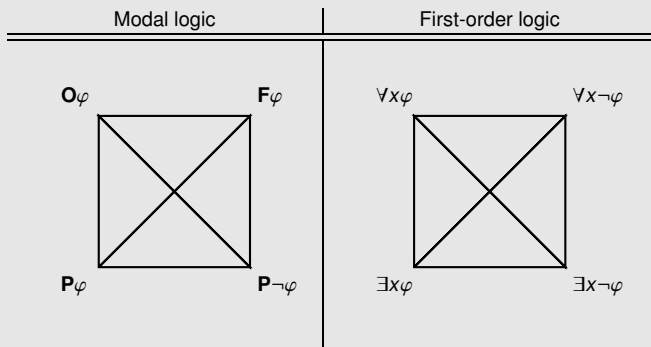
Ludwig Wittgenstein and Georg Henrik von Wright

(Photograph from April 1950.; taken in Von Wright's garden while Wittgenstein was a guest at his house.)

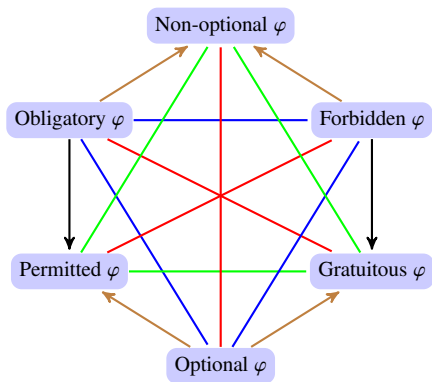
Analogy of quantification and modality

Duality; square of oppositions

Quantifiers	Alethic modalities	Deontic modalities
$\forall x\varphi$ ($\neg\exists x\neg\varphi$) ALL ...	$\Box\varphi$ ($\neg\Diamond\neg\varphi$) NECESSARY ...	$\mathbf{O}\varphi$ ($\neg\mathbf{P}\neg\varphi$) OBLIGATORY ...
$\exists x\varphi$ ($\neg\forall x\neg\varphi$) SOME ...	$\Diamond\varphi$ ($\neg\Box\neg\varphi$) POSSIBLE ...	$\mathbf{P}\varphi$ ($\neg\mathbf{O}\neg\varphi$) PERMITTED ...
$\forall x\neg\varphi$ ($\neg\exists x\varphi$) No ...	$\Box\neg\varphi$ ($\neg\Diamond\varphi$) IMPOSSIBLE ...	$\mathbf{F}\varphi$ ($\mathbf{O}\neg\varphi$, i.e. $\neg\mathbf{P}\varphi$) FORBIDDEN ...



Hexagon of “oppositions”¹

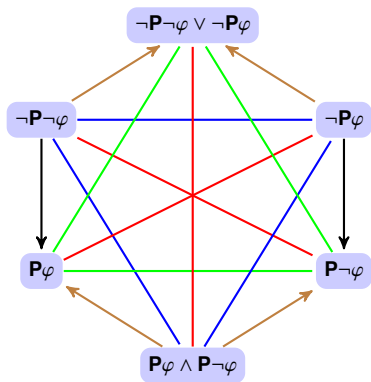


Four logical relations resulting from mutual definability (duality) of normative notions. The last one is D axiom

Name	Property	Symmetry
Contrariety	Both sentences cannot be true.	Yes.
Subcontrariety	Both sentences cannot be false.	Yes.
Contradiction	Both sentences cannot be true, and both sentences cannot be false.	Yes.
Implication	It cannot be so that the source sentence is true and target sentence is false.	No.
Implication	It cannot be so that the source sentence is true and target sentence is false.	No.

¹Some synonyms:[Permitted; Allowed][Optional; Allowed and non-obligatory][Gratuitous; Non-obligatory; Omissible][Forbidden; Prohibited; Impermissible][Non-optional; Obligatory or forbidden][Obligatory]

D axiom



Obligatory $\mathbf{O}\varphi \leftrightarrow \neg\mathbf{P}\neg\varphi \leftrightarrow \mathbf{F}\neg\varphi$

Forbidden $\mathbf{F}\varphi \leftrightarrow \neg\mathbf{P}\varphi \leftrightarrow \mathbf{O}\neg\varphi$

The “black arrow” implications $\mathbf{O}\varphi \rightarrow \mathbf{P}\varphi$ and $\mathbf{F}\varphi \rightarrow \mathbf{P}\neg\varphi$ are equivalent.^a This implication is called D axiom. It can also be read as $\neg(\mathbf{O}\varphi \wedge \mathbf{F}\varphi)$, i.e., as a claim on contrariety of $\mathbf{O}\varphi$ and $\mathbf{F}\varphi$. The analogy with alethic modalities holds since $\Box\varphi \rightarrow \Diamond\varphi$.

^aAssuming modal congruence.

Axioms and rules of standard deontic logic

- Standard deontic logic KD is a normal logic, which means that it provides:

- K axiom (schema):

$$\mathbf{O}(\varphi \rightarrow \psi) \rightarrow (\mathbf{O}\varphi \rightarrow \mathbf{O}\psi)$$

- RN necessitation rule:

$$\text{If } \vdash \varphi, \text{ then } \vdash \mathbf{O}\varphi.$$

Rule RN and axiom K define the character of modal possibilities: they obey the rules of logic (and therefore are ‘normal’). By RN, logical truths hold in any deontic possibility. By K, the consequences of truths of a deontic possibility are the truths of it.

- The only additional axiom of deontic logic is:

- D axiom (schema):

$$\mathbf{O}\varphi \rightarrow \mathbf{P}\varphi$$

- Rule RN can be deontically interpreted as “permission implies logical possibility”.²
- Deontic interpretation of K is “logical consequences of obligations are obligations themselves”.
- Axiom D roughly translates to “it is permitted to fulfil an obligation”.

²Equating provability without premises with logical necessity RN becomes $\Box\varphi \rightarrow \mathbf{O}\varphi$, and conversion gives suggested reading.

Example

The Roman Law principle *ultra posse nemo obligatur* is also known as *ought implies can* principle, and usually mistakenly attributed to Kant. The principle is translated here in its simplified form (Proposition below): (i) standard deontic logic deals with that which *ought to be* and not, as a full-blown deontic logic should, with that which *ought to be done*, (ii) the alethic modality of logical possibility will be used instead of ability modality.

Lemma

$$\vdash \mathbf{P}\varphi \rightarrow \diamond\varphi$$

Proposition

$$\vdash \mathbf{O}\varphi \rightarrow \diamond\varphi \text{ (i.e., } \vdash \neg\diamond\varphi \rightarrow \neg\mathbf{O}\varphi\text{)}.$$

Proof.

- 1 Assume $\mathbf{O}\varphi$.
- 2 $\mathbf{P}\varphi$, from (1) by D.
- 3 $\diamond\varphi$, from (2) by lemma.
- 4 Therefore, $\vdash \mathbf{O}\varphi \rightarrow \diamond\varphi$.

Unexpected results

- The introduction of relational semantics (“possible world semantics”, simultaneously and independently discovered in late 1950s by Stig Kanger and Saul Kripke) has brought some amazing insights in philosophy.
- The analogy between quantification, on the one side, and alethic and deontic modality, on the other side, has received its formal semantic explanation.

Many worlds and their relations

- **MANY WORLDS.** Modal expressions involve hidden quantification: (i) some modalities are universal, like \Box or **O**, and they talk about *all* possibilities (valuations, states, possible worlds) within the appropriate category (logical, deontic, . . .), (ii) some modalities are existential, like \Diamond or **P**, and they in the similar manner talk about *some* possibilities.
- **STRUCTURE.** The plurality of valuations is not sufficient. The distinction between modalities having the same quantificational character but validating different principles has been found in the way the possibilities are connected. The possibilities to be taken into account at the point of evaluation. Quantifiers \forall and \exists offer “bird’s eye view”: their perspective is global and covers all objects. Modalities give a local picture, a “frog’s eye view”: their perspective is located at a particular evaluation point (“the point where we stand”) and therefore covers all possibilities accessible (“visible”) from that point.

Example

Alethic logic readily accepts the principle of existential modal generalization: ‘if something is the case, that it is possible’ or $\varphi \rightarrow \Diamond\varphi$. Deontic logic readily rejects that principle: it is not valid to claim that ‘if something is the case, that it is permitted’ or $\varphi \rightarrow \mathbf{P}\varphi$.

Modal calculator:

<http://www.ffst.hr/~logika/implog/calculators/modal/modal.html>

Instructions:

http://www.ffst.hr/~logika/implog/doku.php?id=program:possible_worlds

The screenshot displays a modal logic calculator interface. On the left, a Kripke model diagram shows four possible worlds labeled 1, 2, 3, and 4. Each world contains a circle with 'p' and 'q'. World 1 has 'p' in red and 'q' in green. World 2 has 'p' in green and 'q' in red. World 3 has 'p' in green and 'q' in red. World 4 has 'p' in red and 'q' in green. Accessibility relations are shown as arrows: 1↔2, 3↔4, 1↔3, and 2↔4. A large 'X' is drawn over the diagonal arrows (1↔3 and 2↔4). On the right, a control panel is divided into sections: 'Possible worlds' with buttons for adding/removing worlds (+1, +2, -1, -2, +3, +4, -3, -4); 'Accessibility relation' with buttons for adding/removing relations (>1, >2, >3, >4) and a 'Clear' button; 'Valuation' with buttons for setting 'p' and 'q' and a 'Random V' button; and a 'Random model' button. Below the control panel is a logic formula input area with buttons for logical connectives: '(', ')', 'p', 'q', 'not', 'and', 'or', 'if, then', 'nec.', 'poss.', 'iff', 'Enter', and 'Clear'. An input field contains 'Np>p' and an 'Input' button. At the bottom, a yellow label reads 'p → ◇p'.

A new language

The researchers in philosophical logic came upon an amazing insight: for modal axioms there are corresponding properties of accessibility relation. (Axiom K and rule RN are different in category: they define the character of the worlds and say nothing about their connections.) Let's try to introduce this insight by way of a metaphor!

A metaphor

Imagine yourself being repeatedly placed in one world after another within a network of worlds. You have an axiom map: a sentential form that must come out true no matter which sentences you put into it. Your "positive task" is to test the accuracy of the map in the modal way: by looking at accessible worlds, possibly moving there and looking at accessible worlds from there, and possibly repeating the action again but in finitely number of times. It turns out that you come up with positive test results for each of successive placements. After that, you have an additional, more complicated test called "negative task": after being placed in a world you have to investigate whether it is possible to modify the world you are at and the worlds accessible from it so to make the axiom map false. If the positive task always results in affirmative (map is true) and the negative task always gives the negative answer (it is not possible to modify worlds so to falsify the map), then your axiom map is accurate and it describes some property of the paths connecting the worlds. E.g. if the map $\Box\varphi \rightarrow \Box\Box\varphi$ passes both tests in a network of worlds, i.e. if is accurate, then the following fact on the property of paths holds: if you can get from the source world to the target world via an intermediate one, then you can also get directly from the source to the target.

Geometry of meaning and expressive power

- The correspondence between axioms and properties of accessibility relations has revealed an important characteristics of the logic of the language of philosophy and science of man.
- Modal logic is not just another way to define implicitly modal terms by fixing their meaning in axioms. Rather, it is a discovery of a language.
- The language of propositional modal logic turns out to have high expressive power, different in kind from that of the language of propositional logic but lower in discriminatory power from the first-order language.³
- The “geometry of meaning” extends far beyond the square or hexagon of oppositions: the logical “space” of modal operators is structured so that different structural types correspond to different modality types.

³While the language of propositional logic has no discriminatory power, the language of propositional modal logic can discriminate between finite structures up to bisimilarity. The language of first-order logic can discriminate between finite structures up to isomorphism (a type of “picture relation” stronger than bisimilarity).

Extensional vs. intensional semantics

- In extensional semantics the truth-value of a compound sentence depends on the truth values of its constituent sentences and the rule of valuation associated with the connective. E.g. the rule associated with \wedge determines the truth-value of $\varphi \wedge \psi$ on the basis of truth-values of φ and ψ .
- Although there is a rule associated to modal operators, the truth-value of the compound sentence whose main operator is modal operator cannot be determined on the basis of the truth values of its constituent sentences. The reason for that is “hidden quantification”: rules associated with modal operators determine the truth-value of the compound by taking into account truth-value of its constituent at all (accessible) points of valuation. E.g. the rule associated with \mathbf{O} determines the truth-value of $\mathbf{O}\varphi$ on the basis of truth-value of φ at each accessible point: $\mathbf{O}\varphi$ is true in a world w if φ is true at each point (each deontic possibility, each world where that what ought to be is the fact of that world) accessible from w .
- The semantics that takes into account multiple valuations of the same syntactic object is called intensional semantics.

Geometry of meaning of modal operators

- Intensional semantics takes into account multiple valuations but the meaning of modal operators is not reducible to them. If it were, we could not distinguish the types of operators since the same rule would be associated with deontic **O** operator, alethic \Box , epistemic **K**, as well as any other universal modal operator.
- The geometry of meaning of modal operators is given by the properties of accessibility relation. Different modalities have different types of accessibility. The type of accessibility is determined by modal axioms.
E.g. what property must the relation of deontic accessibility have? The one that corresponds to the meaning of deontic operators, and that meaning is fixed by axioms.
- Thus, the meaning of modal words has two parts:
 - ① **RULE.** Quantification part. Common to all modality types.
 - ② **STRUCTURE.** Scope of quantification. Specific for given modality type. Corresponds to axioms of some regional modal logic, and, therefore, exhibits the “geometry of meaning” of the particular modal word.

Finding correspondences

For some modal formulas (Sahlqvist formulas) the corresponding property of accessibility relation can be computed using Sahlqvist-van Benthem algorithm.⁴

There exists an effective algorithm which translates all modal axioms of the form $A \rightarrow B$ into corresponding first-order properties, where A is constructed from basic formulas $\Box \dots \Box p$ using only \wedge, \vee, \diamond , B is 'positive': constructed from proposition letters with only $\wedge, \vee, \diamond, \Box$.



Johan van Benthem (2010)
Modal Logic for Open Minds.
Stanford: CSLI

⁴At <http://www.fmi.uni-sofia.bg/fmi/logic/sqema/new/sqema.jsp> there is an online calculator developed at University of Sofia implementing SQEMA algorithm <http://arxiv.org/pdf/cs/0602024.pdf> for finding first-order correspondents, invented by Willem Conradie, Valentin Goranko and Dimiter Vakarelov.

The basic idea

The basic idea is to satisfy antecedent in a minimal way.

In the antecedent:

non-modal or \diamond occurrences of propositional letters	Px_1, \dots, Px_n ST	by to	goes to	$(u = x_1 \vee \dots \vee u = x_n)$	u is the argument of P
--	-----------------------------	----------	------------	-------------------------------------	----------------------------

sequences of universal modalities of types i, \dots, j	$[i] \dots [j]p$		goes to	$R_i \circ \dots \circ R_j v u$	v is variable obtained by ST_v
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Computing correspondence for D axiom

- $\mathbf{O}p \rightarrow \mathbf{P}p$, D axiom.
- $\forall P ST_x(\mathbf{O}p \rightarrow \mathbf{P}p)$, second-order generalization of the standard translation.
- $\forall P(\forall y(R_{\mathbf{O}}xy \rightarrow Py) \rightarrow \exists z(R_{\mathbf{O}}xz \wedge Pz))$, inserting standard translation.
- $Pu := R_{\mathbf{O}}xu$, determination of a minimal valuation.
- $\forall y(R_{\mathbf{O}}xy \rightarrow R_{\mathbf{O}}xy) \rightarrow \exists z(R_{\mathbf{O}}xz \wedge R_{\mathbf{O}}xz)$, replacement of Pu with $R_{\mathbf{O}}xu$.
- $\top \rightarrow \exists z(R_{\mathbf{O}}xz \wedge R_{\mathbf{O}}xz)$, simplification.
- $\forall x \exists y(R_{\mathbf{O}}xy)$, simplification.

Poly-modal alethic-deontic

Definition

Logic $DChS5$ is defined by the rules of labelled deduction augmented with the following relational theory:

- ① $\vdash_{DChS5} R_O xfx$, D axiom, seriality, f is a Skolem-function picking a “witness” for $\exists y R_O xy$.
- ② $R_O xy \vdash_{DChS5} R_N xy$, correspondence for $\Box\varphi \rightarrow \mathbf{O}\varphi$, inclusion $R_O \subseteq R_N$.
- ③ $\vdash_{DChS5} R_N xx$, T axiom: $\Box\varphi \rightarrow \varphi$, reflexivity.
- ④ $R_N xy, R_N yz \vdash_{DChS5} R_N xz$, axiom 4: $\Box\varphi \rightarrow \Box\Box\varphi$, transitivity.
- ⑤ $R_N xy \vdash_{DChS5} R_N yx$, B axiom: $\varphi \rightarrow \Box\Diamond\varphi$, symmetry.

Exercise

Formulate the thesis of determinism in modal language!

Deriving *ought* from *is*

Lemma (Ch)

If something is necessary, then it ought to be.^a

^aCompare Spinoza's Proposition 37 (V) in *Ethics*: "There is nothing in nature, which is contrary to this intellectual love, or which can take it away."

Proof.

1		$w : \Box p$	
		—	
2		v $R_{\mathbf{O}} w v$	
3		$R_{\mathbf{N}} w v$	2/ Ch
4		$v : p$	1, 3/ Elim \Box
5		$w : \mathbf{O} p$	2–4/ Intro \mathbf{O}
6		$w : \Box p \rightarrow \mathbf{O} p$	1–5/ Intro \rightarrow

Proposition

Determinism causes deontic collapse.

Proof.

1		$w : p \rightarrow \Box p$		
2		$w : \neg p \rightarrow \Box \neg p$		
3				
3			$w : p$	
4			$w : \Box p$ 1, 3/ Elim \rightarrow	
5			$w : \mathbf{O}p$ 4/ Ch lema	
6			$w : \mathbf{O}p$	
7			$R_{\mathbf{O}} wfw$ D	
8			$R_{\mathbf{N}} wfw$ 7/ Ch	
9				
9				$w : \neg p$
10				$w : \Box \neg p$ 2, 9/ Elim \rightarrow
11				$fw : p$ 6, 7/ i O
12				$fw : \neg p$ 8, 10/ Intro \Box
13				$w : p$ 9–12/ Elim \neg
14				$w : p \leftrightarrow \mathbf{O}p$ 3–5, 6–13/ Intro \leftrightarrow

□

The fall of deontic logic

- In contrast to the fruitful development of modal logic, standard deontic logic turned out to be a blind alley because of numerous paradoxes.
- That is not to say that standard deontic logic is meaningless but, rather, that its true nature has been misunderstood.

Paradox of epistemic obligation

If the bank has been robbed (p), then Jones (the Guard) ought to know that ($\mathbf{OK}_j p$).

The bank has been robbed.

Therefore, it ought to be the case that the bank has been robbed.

We will use S5 epistemic logic. Here we will need only the rule corresponding to T axiom:

$$\vdash \mathbf{K}\varphi \rightarrow \varphi$$

i.e.

$$\vdash R_{\mathbf{K}} w w$$

1	$w : p \rightarrow \mathbf{OK}_j p$													
2	$w : p$													
3	$w : \mathbf{OK}_j p$	1, 2/ Elim \rightarrow												
4	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">v</td> <td style="border-left: 1px solid black; padding-left: 5px;">$R_{\mathbf{O}} w v$</td> <td></td> </tr> <tr> <td style="padding-right: 5px;">5</td> <td style="border-left: 1px solid black; padding-left: 5px;">$v : \mathbf{K}_j p$</td> <td style="padding-left: 20px;">3, 4/ Elim\mathbf{O}</td> </tr> <tr> <td style="padding-right: 5px;">6</td> <td style="border-left: 1px solid black; padding-left: 5px;">$R_{\mathbf{K}} w v$</td> <td style="padding-left: 20px;">$T_{\mathbf{K}}$</td> </tr> <tr> <td style="padding-right: 5px;">7</td> <td style="border-left: 1px solid black; padding-left: 5px;">$v : p$</td> <td style="padding-left: 20px;">5, 6/ Elim\mathbf{K}_j</td> </tr> </table>	v	$R_{\mathbf{O}} w v$		5	$v : \mathbf{K}_j p$	3, 4/ Elim \mathbf{O}	6	$R_{\mathbf{K}} w v$	$T_{\mathbf{K}}$	7	$v : p$	5, 6/ Elim \mathbf{K}_j	
v	$R_{\mathbf{O}} w v$													
5	$v : \mathbf{K}_j p$	3, 4/ Elim \mathbf{O}												
6	$R_{\mathbf{K}} w v$	$T_{\mathbf{K}}$												
7	$v : p$	5, 6/ Elim \mathbf{K}_j												
8	$w : \mathbf{O} p$	4–7/ Intro \mathbf{O}												

Avoiding paradoxes

- Paradoxes indicate serious problems.
- Some paradoxes are due to the simplistic character of standard deontic logic. It deals with “ought to be” type of sentences and not with “ought to do” type and thus avoids the extremely complex character of the latter. If a more complex semantic structure is introduced, the paradoxes disappear.
- Does standard deontic logic have any value besides didactic one as the propedeutics for more advanced deontic logics? The answer is definitively affirmative.
- A way of resolving paradoxes has been suggested by Von Wright. The language of standard deontic logic should be reinterpreted: it is not the language of prescriptions, not the language of norms, but a descriptive language, a language of assertions about normative systems.

Descriptive instead of prescriptive interpretation

In the classical system one can prove (derive) the formulas $\mathbf{P}p \vee \mathbf{O} \sim p$ and $\sim (\mathbf{O}p \& \mathbf{O} \sim p)$ and $\sim (\mathbf{O}p \& \mathbf{P} \sim p)$. The first says that any possible state of affairs is either permitted or forbidden. A normative order (system) which satisfies this condition is said to be complete or “gapless.” The second formula says that no state and its contradictory are both obligatory. The third says that no state is such that it is obligatory but its contradiction (“nevertheless”) permitted. A normative order which satisfies these conditions is said to be consistent or contradiction-free. Thus classic deontic logic, on the descriptive interpretation of its formulas, pictures a gapless and contradiction-free system of norms. A factual normative order *may* have these properties, and it may be thought desirable that it *should* have them.



Georg Henrik von Wright.

Deontic logic: a personal view.

Ratio Juris, 12:26–38, 1999.



Georg Henrik von Wright
(Helsinki, 1916.–Helsinki,
2003.)

A remark

Since $\mathbf{O}\varphi \leftrightarrow \neg\mathbf{P}\neg\varphi \leftrightarrow \mathbf{F}\neg\varphi$, the first and the third formulas are deontically interpreted tautologies. The second formula is D axiom: $\mathbf{O}\varphi \rightarrow \mathbf{P}\varphi$.

Normative system understood as a function

Requirements

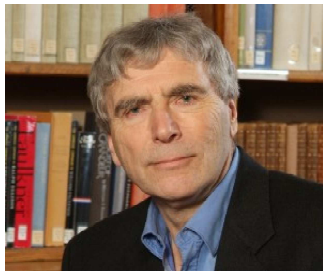
We must allow for the possibility that the requirements you are under depend on your circumstances. Here is how I shall do that formally, using possible worlds semantics. There is a set of worlds, at each of which propositions have a truth value. The values of all propositions at a particular world conform to the axioms of propositional calculus. For each source of requirements s , each person i and each world w , there is a set of propositions $k_s(i, w)$, which is to be interpreted as the set of things that s requires of i at w . Each proposition in the set is a required proposition. The function k_s from i and w to $k_s(i, w)$ I shall call s 's *code* of requirements.



John Broome.

Requirements.

u: Rønnow-Rasmussen, T., Petersson, B., Josefsson, J., and Egonsson, D. (eds.), *Homage a Wlodek: Philosophical Papers Dedicated to Wlodek Rabinowicz*, Lunds universitet, 1–41, 2007.
<http://www.fil.lu.se/hommageawlodek>



John Broome
(University of Oxford)

Von Wright's thesis on metanormative character of standard deontic logic

SDL as a metanormative theory

Thus classic deontic logic, on the descriptive interpretation of its formulas, pictures a gapless and contradiction-free system of norms. A factual normative order *may* have these properties, and it may be thought desirable that it *should* have them.



Georg Henrik von Wright.
Deontic logic: a personal view.
Ratio Juris, 12:26–38, 1999.

Methodology

Von Wright's thesis can be examined using Broome's concept of normative code in the following steps:

- 1 Development of a **TYPOLGY** of normative systems (codes).
- 2 Definition of a **TRANSLATION** (function) from the language of standard deontic logic to the language of normative requirements.
- 3 Single out the **TYPE OF NORMATIVE SYSTEM** determined by the translation of the axioms of standard deontic logic (KD).

Properties of normative system

It is amazing how many conceptual distinctions can be introduced using Broome's concept of normative system as three-place function. We will draw our attention only to few of them, to some of those whose type is determined "locally" as a property of a set of requirements that a code delivers. By quantifying over variables we get types of codes.

Deductive closure and consistency

- A set of requirements $k_s(i, w_1)$ is pl-consistent, $CS_{pl}(k_s(i, w_1))$, iff $\exists w_2 k_s(i, w_1) \subseteq w_2$.
- A set of requirements $k_s(i, w_1)$ is pl-deductively closed, $DC_{pl}(k_s(i, w_1))$, iff $k_s(i, w_1) = Cn(k_s(i, w_1))$.
- A set of requirements $k_s(i, w_1)$ is consistent in logic $l(x)$, $CS_{l(x)}(k_s(i, w_1))$, if $\exists w_2 Cn(l(x) \cup k_s(i, w_1)) \subseteq w_2$.
- A set of requirements $k_s(i, w_1)$ is a logic, $LG(k_s(i, w_1))$, iff $\exists x k_s(i, w_1) = Cn(l(x))$.



Berislav Žarnić.

A logical typology of normative systems.

Journal of Applied Ethics and Philosophy, 2:30–40, 2010.

Translation

- Let's restrict the language \mathcal{L}_{KD}° so that iterations of deontic operators are not allowed (p below stands for a formula of propositional logic):

$$\varphi ::= p \mid \mathbf{O}p \mid \mathbf{P}p \mid \neg\varphi \mid (\varphi \wedge \varphi)$$

- The translation will connect the formulas of the two language in the following manner:

\mathcal{L}_{KD}°	$\mathcal{L}_{\text{meta}}$
$\mathbf{O}\varphi$ It is obligatory that φ .	$\ulcorner \varphi \urcorner \in k_s(i, v)$ A normative source s subordinates an actor i in circumstances v to a requirement φ .

Two phases of translation

The translation function τ^0 takes the sentences of the non-modal part of $\mathcal{L}_{\text{KD}}^{\text{O}} \cap \mathcal{L}_{\text{PL}}$ and delivers (not sentences but) singular terms (sentential variables and their functions) of the metanormative language $\mathcal{L}_{\text{meta}}$. For the sake of readability sentential terms instead of functional notation, e.g. neg and $\text{conj} \dots$, will be written uols, e.g. instead of $\text{conj}(\text{neg}(p), q)$ we write $\ulcorner \neg p \wedge q \urcorner$.

Definition

$\tau^0(a) \in \{p, p_1, \dots, q, q_1, \dots\}$
for prop. letters $a \in \mathcal{L}_P$

$$\tau^0(\neg\varphi) = \neg\tau^0(\varphi)$$

$$\tau^0((\varphi \wedge \psi)) = (\tau^0(\varphi) \wedge \tau^0(\psi)).$$

Definition (The translation function $\tau^1 : \mathcal{L}_{\text{KD}}^{\text{O}} \rightarrow \mathcal{L}_{\text{meta}}$)

$$\tau^1(p) = \ulcorner \tau^0(p) \urcorner \in v$$

ako $p \in \mathcal{L}_{\text{PL}}$

$$\tau^1(\mathbf{O}\varphi) = \ulcorner \tau^0(\varphi) \urcorner \in k_s(a, v)$$

$$\tau^1(\mathbf{P}\varphi) = \neg \ulcorner \tau^0(\neg\varphi) \urcorner \in k_s(i, v)$$

$$\tau^1(\neg\varphi) = \neg\tau^1(\varphi)$$

$$\tau^1((\varphi \wedge \psi)) = (\tau^1(\varphi) \wedge \tau^1(\psi)).$$

An example and a test

The correctness of the translation will be confirmed if definitional biconditional $\mathbf{P}p \leftrightarrow \neg\mathbf{O}\neg p$ reduces to truth after translation.

Example

$$\begin{aligned} & \tau^1(\mathbf{P}p \leftrightarrow \neg\mathbf{O}\neg p) \\ & \Leftrightarrow \tau^1(\mathbf{P}p) \leftrightarrow \tau^1(\neg\mathbf{O}\neg p) \\ & \Leftrightarrow \neg \ulcorner \tau^0(\neg p) \urcorner \in k_s(i, v) \Leftrightarrow \neg \tau^1(\mathbf{O}\neg p) \\ & \Leftrightarrow \neg \ulcorner \neg \tau^0(p) \urcorner \in k_s(i, v) \Leftrightarrow \neg \ulcorner \tau^0(\neg p) \urcorner \in k_s(i, v) \\ & \Leftrightarrow \neg \ulcorner \neg p \urcorner \in k_s(i, v) \Leftrightarrow \neg \ulcorner \neg \tau^0(p) \urcorner \in k_s(i, v) \\ & \Leftrightarrow \neg \ulcorner \neg p \urcorner \in k_s(i, v) \Leftrightarrow \neg \ulcorner \neg p \urcorner \in k_s(i, v) \\ & \Leftrightarrow \top. \end{aligned}$$

Translation of KD axioms

The translation τ^1 confirms Von Wright's thesis, but also shows that it needs an amendment.

- mutual definability, $\mathbf{P}p \leftrightarrow \neg\mathbf{O}\neg p$, holds for any set of requirements (see Example above);
- the “gaplessness” condition $\mathbf{P}p \vee \mathbf{O}\neg p$ translates to $\ulcorner \neg p \urcorner \notin k_s(a, v) \vee \ulcorner \neg p \urcorner \in k_s(a, v)$ and that condition, obviously, is satisfied by any set of requirements whatsoever;
- the K axiom becomes $\ulcorner p \rightarrow q \urcorner \in k_s(a, v) \rightarrow (p \in k_s(a, v) \rightarrow q \in k_s(a, v))$ and that condition holds for any pl-deductively closed set;
- the D axiom translates to $p \in k_s(a, v) \rightarrow \ulcorner \neg p \urcorner \notin k_s(a, v)$ and that is just another way of stating pl-consistency.

Conclusion

- Deontic KD logic describes a deductively closed and consistent set of requirements, but these logical properties are defined within propositional logic.
 - ▶ It is obvious that a desirable normative system must have stronger properties.
 - ▶ Requirements address intentional states and processes of an actor. Therefore, consistency in the logic of intentionality is a desirable property. E.g., it is pl consistent to require both $B_i\varphi$ and $B_i\neg\varphi$, but these requirements are incompatible in doxastic logic.
 - ▶ Social consistency is a desirable property if a normative source delivers requirements for different actors. If $l(x)$ is the logic of the language in which requirements are stated, then a source s is **SOCIALLY CONSISTENT**:
$$\forall i_1 \forall i_2 \forall w \text{ CS}_{l(x)}(k_s(i_1, w) \cup k_s(i_2, w)).$$

Further research

- Some principles with iterated deontic operators are plausible. E.g. “A duty must be done”

$$\mathbf{O}(\mathbf{O}\varphi \rightarrow \varphi)$$

- A possible solution would be to treat iterated operators as different: $\mathbf{O}_2(\mathbf{O}_1\varphi \rightarrow \varphi)$ and to interpret \mathbf{O}_2 as being about normative properties.

Definition

An agent i at world w has an “all-or-nothing” normative property K_s that corresponds to the source s iff the set of requirements $k_s(i, w)$ is satisfied in w , i.e. $K_s(i, w) \leftrightarrow k_s(i, w) \subseteq w$.

- Let us introduce translation τ^2 and define $\tau^2(\mathbf{O}_2\varphi) = K_s(i, w) \rightarrow \tau^1(\varphi)$.
- $\tau^2(\mathbf{O}_2(\mathbf{O}_1p \rightarrow p)) = K_s(i, w) \rightarrow \tau^1(\mathbf{O}_1p \rightarrow p) = K_s(i, w) \rightarrow (\ulcorner p \urcorner \in k_s(i, w) \rightarrow \ulcorner p \urcorner \in w)$
- The formula $K_s(i, w) \rightarrow (\ulcorner p \urcorner \in k_s(i, w) \rightarrow \ulcorner p \urcorner \in w)$ saying that having a normative property implies realization of corresponding requirements, which sounds plausible, brings us back to Leibniz’s approach.