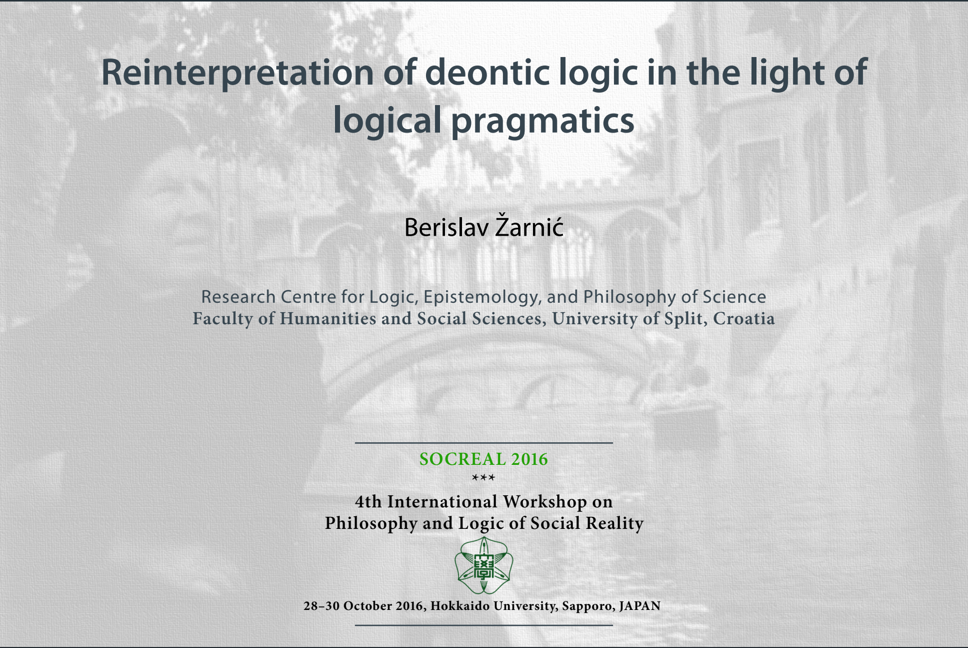

Reinterpretation of deontic logic in the light of logical pragmatics

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Overview

❶ Von Wright's reinterpretation of deontic logic
  - Prescriptive and descriptive use of deontic sentences

❷ A formal explication of Von Wright's reinterpretation
  - The norm-set for obligation-norms and the counter-set for permission norms
  - Translations from SDL language to set-theoretic language

❸ Classical deontic logic as the theory of ideal normative systems

❹ Philosophy of deontic logic from the viewpoint of logical pragmatics

❺ Application and a refinement of the two-sets model
  - Is it rational to complete a normative system using the meta-principle *everything not permitted is forbidden*?
  - Hypothetical norms

Reinterpretation of SDL

Von Wright's late work on deontic logic

Von Wright's reinterpretation of deontic logic developed gradually and has introduced important conceptual distinctions and theses, among which the following stand out:

- the distinction between prescriptive and descriptive use of deontic sentences;
- the thesis that relation between permission and absence of prohibition is not conceptual but normative in character;
- this normative relation is one among other "perfection properties" of the normative system,
- the norm-giver (in the norm-giving activity by which the normative system is produced) ought to achieve perfection properties of the system.

Georg Henrik von Wright (1916–2003), one of the founders of deontic logic. His reinterpretation of deontic logic has been presented in his invited talk at the *Fourth International Workshop on Deontic Logic in Computer Science (DEON '98)*, Bologna, Italy, 8-10 January, 1998.

Deontic logic as the logic of prescriptive use of language is a study of its rationality conditions

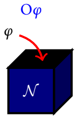
> " …deontic sentences in ordinary usage exhibit a characteristic ambiguity. Sometimes they are used as norm-formulations. We shall call this their prescriptive use. Sometimes they are used for making what we called normative statements. We call this their descriptive use. When used descriptively, deontic sentences express what we called norm-propositions. If the norms are prescriptions, norm-propositions are to the effect that such and such prescriptions 'exist', i.e. have been given and are in force."
>
> von Wright, G.H. (1963). *Norm and Action : A Logical Enquiry.* London: Routledge and Kegan Paul.

- "…two interpretations of the same symbolism …"
- How far can this parallelism go? The answer to this question is related to the problem of the possibility of compounding mood-designated sentences. For example, is there also a disjunction of imperatives, $!\varphi \vee !\psi$, alongside imperative disjunction, $!(\varphi \vee \psi)$?

formula types, where $\Delta, \Delta' \in \{P, O, F\}$	prescriptive use and interpretation	descriptive use and interpretation
$O\varphi$	yes	yes
$P\varphi$	yes (the two-sets model required)	yes
$\neg\Delta\varphi$	?	yes
$\Delta\varphi \wedge \Delta'\psi$	?	yes
$\Delta\varphi \vee \Delta'\psi$	?	yes

The simplest model of categorical norms

In Von Wright's works there is no formal semantics for the distinction between prescriptive and descriptive use of deontic sentences. Nevertheless, the simplest model for categorical obligation-norms has been given by other researchers; introduced by Alchourrón and Bulygin in 1981 and developed by Broome in 2007.

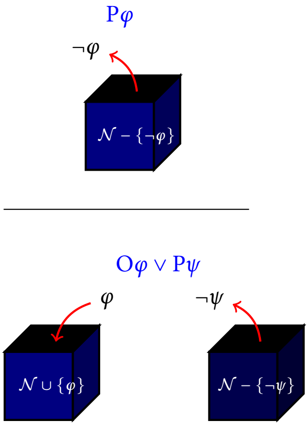


The basic metaphor for the prescriptive use is the action of adding the norm-content of obligation-norm to the "norm-box". In formal terms: if S is the set of categorical obligation-norms, i.e., norms of the form $O\varphi$, and if the "norm-content function" *nc* extracts the norm-content (i.e. $nc(O\varphi) = \varphi$, then the result of the prescriptive use of sentences from S is the obligation-norm set $\mathcal{N} = \bigcup_{x \in S} nc(x)$, i.e., $\mathcal{N} = \{\varphi \mid O\varphi \in S\}$.

On the other hand, the metaphor for the descriptive use is that of looking into the "norm-box". In the descriptive use a deontic sentence just describes the content of the "norm-box". In formal terms: if τ is a translation function from the language of (descriptively used) categorical deontic sentences x to the set-theoretical language of norm-set membership claims, then $\tau(x)$ is a description of the categorical obligation norm set. An example: $\tau(O\varphi) = \ulcorner \varphi \urcorner \in \mathcal{N}$.

Extension of box-metaphor

Figure : The approach depicted in these figures will not be followed here.



The box-metaphor can be extended to cover the prescriptive use of permission sentences and molecular sentences. For example, removal of the contradictory content from the norm-set can be used to model permission and negation of obligation sentences; duplication of obligation norm sets can be used to model the disjunction. Nevertheless, there are reasons against this enlargement of the action repertoire.

The objections that would be relevant for Von Wright's reinterpretation of deontic logic are: – the concept of a genuine "permission-norm" cannot be restricted to "removal of antecedently existing prohibitions", – if the purpose of the norm-giving activity is to coordinate actions of different actors in such a way that they do not reach incompatible conclusions while using in their normative reasoning the same consistent set of obligation norm contents, then multiplication of obligation norm sets endangers the realization of the purpose.

Descriptive use

- In the simplest model of prescriptive use of deontic sentences only atomic obligation sentences are admitted. On the other hand, there is no limitation regarding the descriptive use.
- The translations show the descriptive meaning of deontic language: it describes the content of the obligation norm set. For example, $\tau(\mathrm{P}\varphi) = \ulcorner\neg\varphi\urcorner \notin \mathcal{N}$; $\tau(\mathrm{O}\varphi \vee \mathrm{O}\psi) = \ulcorner\varphi\urcorner \in \mathcal{N} \vee \ulcorner\psi\urcorner \in \mathcal{N}$.

> “ On the descriptive interpretation of deontic sentences, their molecular compounds are themselves sentences expressing true or false propositions. The compounds are, in other words, truth-functional, and have descriptive meaning. But according to the prescriptive interpretation of deontic sentences, molecular compounds formed by them with the aid of sentential connectives do not express norms, and have no prescriptive meaning. The connectives, in other words, do not apply to norms, i.e. to prescriptively interpreted deontic sentences.”

von Wright, G.H. (1999).
Value, Norm, and Action in My Philosophical Writings. In *Actions, Norms, Values*, Georg Meggle (ed.), pp. 11–34. Berlin, Boston: De Gruyter.

The two uses in Yamada-style notation

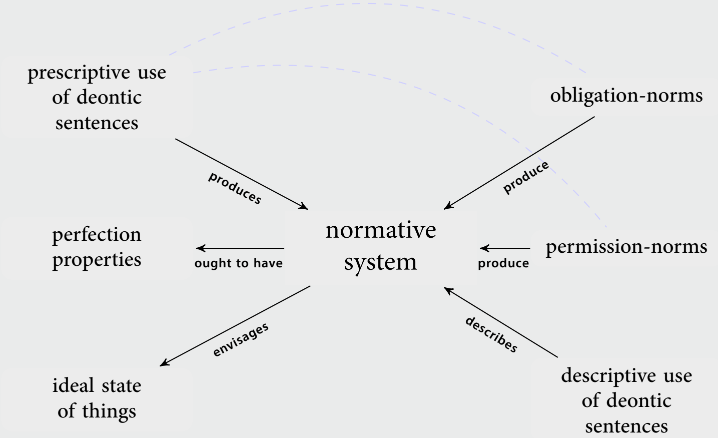
The ambiguity of deontic sentences

The two uses can be distinguished by their effects and written down in an *a*d hoc dynamic logic type of notation [act]effect, following the ideas introduced in Yamada's work:

- In the prescriptive use a directive type of speech-act is performed; the effect takes place in the "social world", i.e., a change in the normative reality possibly occurs; after *i* says/writes: "It is obligatory for *j* that φ" , it is the case that it is obligatory for *j* that φ,$[i : \underline{O_j\varphi}]\ O_j\varphi$.
- In the descriptive use an assertive type of speech-act is performed; the effect takes place in the "subjective world", i.e., a change of the recipient's beliefs possibly occurs; after *i* says/writes: "It is obligatory for *j* that φ" , it is the case that *j* believes that it is obligatory for *j* that φ, $[i : \underline{O_j\varphi}]B_jO_j\varphi$.
- It should be noted that the "prescriptive use of a deontic sentence" has double direction of fit (the world to fit the words and the words to fit the world); "it makes it so by saying it is so"; thus deontic sentences in the prescriptive use are a kind of "performatives" (or "declarations").
- The interpretational hypothesis that will be followed in this talk is that standard deontic logic deals with the second-order deontic operator O that applies to acts of prescriptive use of deontic sentences, e.g., the second-order norm of external consistency,

  second order example: $[i : \underline{O_j\varphi}]O_i\neg\ i : \underline{P_j\neg\varphi}$

The conceptual map of Von Wright's late work



prescriptive use of deontic sentences

obligation-norms

perfection properties

normative system

permission-norms

ideal state of things

descriptive use of deontic sentences

Deontic logic as the logic of prescriptive use of language is a study of its rationality conditions

> "Deontic logic, one could also say, is neither a logic of norms nor a logic of norm-propositions but a study of conditions which must be satisfied in rational norm-giving activity. It is strict *logic* because the conditions which it lays down are derived from *logical* relations between states in the ideal worlds which normative codes envisage."
>
> Georg Henrik von Wright (1993).
> A Pilgrim's Progress.
> In von Wright, G.H. *The Tree of Knowledge and Other Essays*, 103–113.
> Leiden: Brill.
>
> [p.111]

This programmatic statement laying down the directions for the development of deontic logic and outlining realization puts once again Von Wright in the role of the "midwife" (to use his own words) of the new viewpoint in deontic logic, namely, the viewpoint of logical pragmatics.

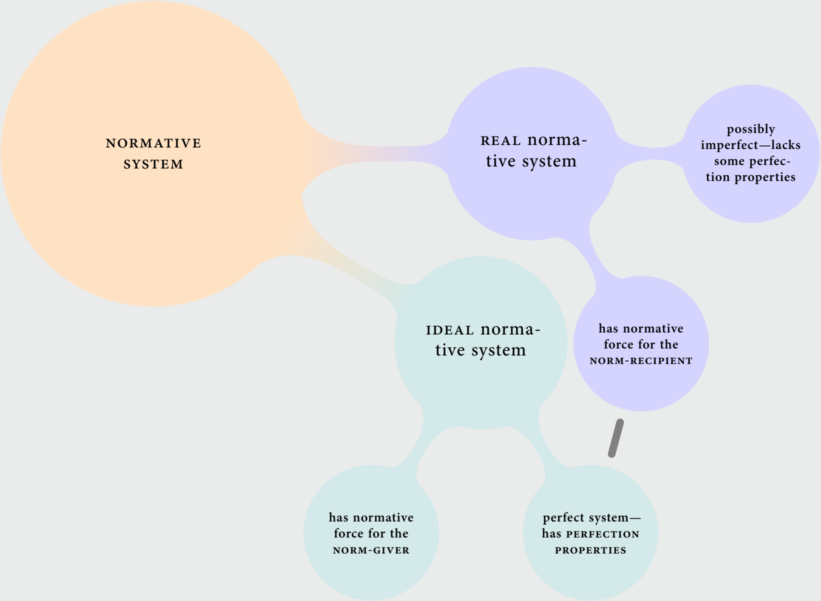
Von Wright’s turn towards logical pragmatics

- The use and users of language and of language-constructions are taken into the picture.
- A normative systems can come into existence thanks to the prescriptive *use* of language. The logical properties of real normative systems can be described using the language of the “logic of norm-propositions”. Some logical properties are “perfection-properties” of a normative-system. The absence of a certain perfection-property does not deprive a normative system of its normative force. In the *prescriptive use* of language the norm-giver *ought to achieve* some perfection properties of the normative system is subordinated. There are two types of *oughts*: the *ought* resulting from the norm-giving activity, and the *ought* to which the norm-giving activity is subordinated.
- According to Von Wright, deontic logic is a study of logical perfection properties; properties the achievement of which fulfils *rationality conditions of norm-giving activity*.
- The approach can be generalized so to include other roles, such as the role of norm-recipient, and other norm-related activities, such as normative reasoning.

Two-sets model

Set-theoretic approach in deontic logic

- Von Wright's reinterpretation of deontic logic can be formally explicated within the set-theoretic approach. The set-theoretic approach has been introduced into the logic of normative systems by Alchourrón and Bulygin in 1981. Within this framework deontic sentences are treated as claims on membership in the set of consequences $Cn(\mathcal{N})$ of "explicitly commanded propositions" $\mathcal{N}$. Thus, $\mathrm{O}\varphi$ in their approach means $\varphi \in Cn(\mathcal{N})$, while $\mathrm{P}\varphi$ is explicated as $\neg\varphi \notin Cn(\mathcal{N})$. More recently a refinement and generalization of the set-theoretic approach has been developed by Broome (e.g. *Rationality through Reasoning*, 2013) where the set of requirements is a set closed under equivalence and equated with the value of a code function, which takes as its arguments a normative source, an actor and a world.
- The point of divergence within the set-theoretic approaches lies in the properties that are assigned to sets of norms or requirements. It is in accord with the approach proposed by Von Wright to treat real norm-sets, the one corresponding to obligation-norms and the other to permission-norms, as simple sets consisting just of affirmed and negated propositional contents of explicitly promulgated norms, not presupposing any a priori given properties. Rather, it is the question of compliance with second-order normativity related to actors' roles whether a real normative system posses desirable logical properties and approximates an ideal system. In the approach of this paper it is neither assumed that a obligation norm-set is deductively closed nor that it is closed under equivalence. In the one-set model the *real normative system* is just a set of contents of "explicitly promulgated obligation norms". Similarly, in the two-sets model the *real normative system* is just a pair consisting of the set of contents of "explicitly promulgated obligation norms" and the set of contradictory contents of "explicitly promulgated permission norms".



NORMATIVE SYSTEM

REAL normative system

possibly imperfect—lacks some perfection properties

has normative force for the NORM-RECIPIENT

IDEAL normative system

has normative force for the NORM-GIVER

perfect system—has PERFECTION PROPERTIES

Norm-related activities: between the Real and the Ideal

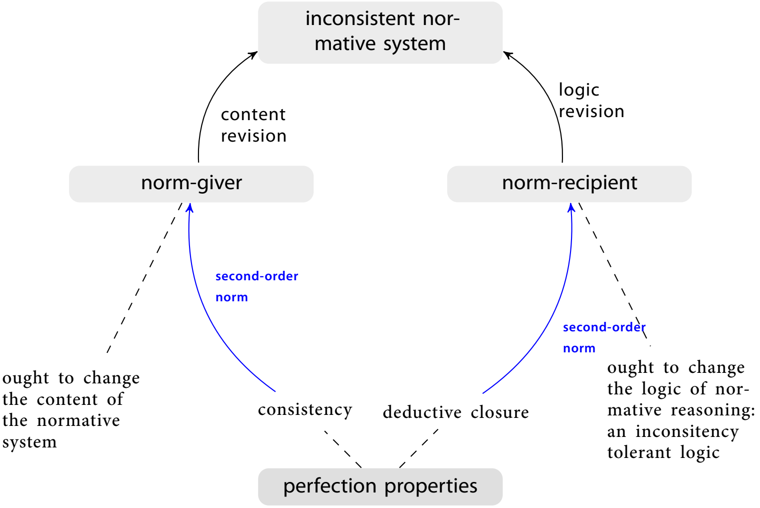


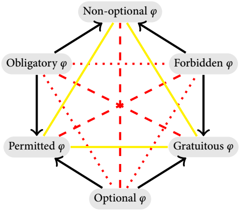
Figure : Corrective obligations with respect to an inconsitent normative-system.

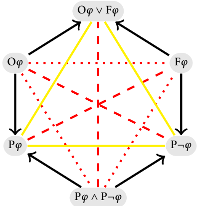
Normative relation between normative concepts

> "Just as possibility is the negation of the necessity of the contradictory of a proposition, permission is the negation of the obligatoriness of the contradictory. $\mathrm{P}p \leftrightarrow \neg\mathrm{O}\neg p$ is a theorem of "classical" deontic logic.
> I think that this opinion is mistaken. The relation between permission and absence of prohibition is not a *conceptual* but a *normative* relation.One may be able to give good reasons why such things which are not prohibited by the norms of a certain code should be regarded as permitted by the code in question. But to declare the non-prohibited permitted is a normative act. One could have a meta-norm to the effect that the not-prohibited is permitted. The well-known principles *Nulla poena sine lege* and *Nullum crimen sine lege* may be thought of as versions of this meta-norm. Or at least as closely related to it."

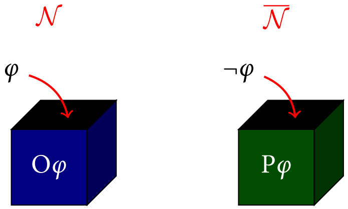
von Wright, G.H. (1991). Is There a Logic of Norms? *Ratio Juris* 4:265–283.

The hexagon of logical relations holding in standard KD deontic logic. The red dotted line represents the contrariety relation, the red dashed line represents contradiction, the full line represents subcontrariety, and the arrows represent subalternation (implication). In a REAL normative system these relations need not obtain. The hexagon depicts normative relations between normative concepts, not conceptual relations. So, only in an IDEAL system deontic hexagon holds.





The metaphor: putting in boxes having different logical structure (I)



- The facts about a normative system are represented using the language of set-theory:
  - $\mathrm{O}\varphi$ is represented by $\ulcorner \varphi \urcorner \in \mathcal{N}$,
  - $\mathrm{P}\varphi$ is represented by $\ulcorner \neg\varphi \urcorner \in \overline{\mathcal{N}}$.

Although counter-intuitive at the first glance, the adequate metaphor for permitting is that of putting the negation of the content into the permission box. This corresponds to the standard definition “permitted that φ = not obligatory that $\neg\varphi$”: $\neg\varphi$ cannot go into the blue box, so, it must be placed in the green box.

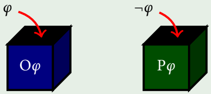
The perfection properties are different for different “boxes” since “ideal concepts” of obligation and permission have different logical structure. For example, having a contradictory pair is an imperfection property of the obligation box, but for the permission box this is neither a perfection nor an imperfection property. Similarly, completeness is a perfection property for permissions but not for obligations: it is indifferent whether $\ulcorner \varphi \urcorner \in \mathcal{N} \vee \ulcorner \neg\varphi \urcorner \in \mathcal{N}$ holds, while $\ulcorner \varphi \urcorner \in \overline{\mathcal{N}} \vee \ulcorner \neg\varphi \urcorner \in \overline{\mathcal{N}}$ ought to hold.

This model, as will be shown, can account for the fact that perfection properties come in triads, one for obligation “box”, another for permission “counter-box”, the third for the relation between boxes, and all of these are characterized by the same theorems of standard deontic

The metaphor: putting in boxes but with diverse logical structure (II)

Example

The difference in logical structure
of the two “boxes” is visible from the following fact:



- A perfect counter-set can have a contradictory pair of (negated) permission-norm contents, which means that a certain state of affairs is optional. This fact does not cause an “explosion” since in this box the principle *ex contradictione quodlibet* does not hold.

Normative system

According to the extended Von Wright's reinterpretation of deontic logic the norm-giver and the norm-recipient relate to the ideal concepts of obligation and permission.

Definition

Let $\mathcal{L}_{pl}$ be the language of propositional logic. A set $\mathcal{N} \subseteq \mathcal{L}_{pl}$ is called obligation norm set and contains contents of obligation-norms. A set $\overline{\mathcal{N}} \subseteq \mathcal{L}_{pl}$ is called permission norm counter-set and contains contradictory contents of permission-norms. A normative system is the pair $\langle \mathcal{N}, \overline{\mathcal{N}} \rangle$.

A technical remark

The ideal concepts of obligation and permission can be explicated by pointing out the "perfection properties" of their corresponding sets, namely, of the norm-set and the counter-set. Since the filter structure and the weak-ideal structure of the respective sets will be later recognized as responsible for their perfection properties these terms must be introduced. The first one is a well-know concept while the second will be introduced here.

Definition

A set X of truth-sets is a weak ideal iff (i) $[\![\varphi]\!] \subseteq [\![\psi]\!]$ and $[\![\psi]\!] \in X$, then $[\![\varphi]\!] \in X$, and (ii) if $[\![\varphi]\!] \cap [\![\psi]\!] \in X$, then $[\![\varphi]\!] \in X$ or $[\![\psi]\!] \in X$.

ideal theoretical type	set $X \subseteq \wp S$	its relative complement $Y = \wp S - X$
descriptive type: consistent and complete X	$[\![\varphi]\!] = \{v \mid v(\varphi) = \mathrm{t}\}$ is the truth-set of φ; $\{[\![\varphi]\!] \mid \varphi \in Cn(T)\}$ is a maximal-filter (not containing $\varnothing = [\![\bot]\!]$; containing $\{v \mid v \text{ is a valuation}\} = [\![\top]\!]$; being closed under intersection and superset relation, having a member for each pair of a set and its complement)	$\{[\![\varphi]\!] \mid \varphi \notin Cn(T)\}$ is an (algebraic) ideal (containing $[\![\bot]\!]$, not containing $[\![\top]\!]$, being closed under union and subset relation)
normative type: consistent X	$\{[\![\varphi]\!] \mid \varphi \in Cn(T)\}$ is a filter	$\{[\![\varphi]\!] \mid \varphi \notin Cn(T)\}$ is a "weak ideal"

Metatheory of descriptive theory and normative system

- The proposed two-sets model bears resemblance to the relation between a theory T and its counter-part $\mathcal{L} - Cn(T)$. The counter part has logical properties such as "closure under implicant" (if $\psi \in \mathcal{L} - Cn(T)$ and φ entails ψ, then $\psi \in \mathcal{L} - Cn(T)$).
- The perfection properties of the descriptive theory have been well investigated within the logic of natural sciences. For example, the completeness of a theory counts as its perfection property. In contrast to this, the completeness of obligation-norm set is not its perfection property. The mismatch holds also on the side of "counter-sets": the completeness of the descriptive counter-part $\mathcal{L} - Cn(T)$ is an indifferent property, while in the realm of normativity it is a perfection property of the "counter-set" representing permission-norms.
- The construction is different too: there is no need to separately build the "exclusion" part for a descriptive theory since rejecting a sentence equals accepting its negation. This need not be the case with normative systems, whose obligation and permission parts can be separately built.
- These facts show that deontic logic as the study of "rationality conditions of norm-giving activity" or as the study of "perfection properties of normative systems" is a sui generis logic. If one accepts, together with von Wright, the central position of the phenomenon of normativity in humanities and social sciences, then deontic logic plays the prominent role in the philosophy of the science of man by revealing the logical basis of its methodological autonomy.

Von Wright's Conjecture

> "…classic deontic logic, on the descriptive interpretation of its formulas, pictures a gapless and contradiction-free system of norms. A factual normative order may have these properties, and it may be thought desirable that it should have them. But can it be a truth of logic that a normative order has ('must have') these 'perfection'-properties?"
>
> Georg Henrik von Wright (1999).
>
> Deontic Logic: A Personal View.
>
> *Ratio Juris* 12:26–38.

A formal explication of Von Wright's reinterpretation of deontic logic asks for the establishment of a connection between the theorems of standard deontic logic and properties of an ideal normative system.

For this purpose the translation function has been introduced, connecting theorems standard deontic logic with the properties of the norm-set, in my JAEP paper (2010) and extended to include properties of the counter-set and relations between the two sets, in my EUJAP 11(2) 2015 and in DEON 2016 Proceedings paper.

Berislav Žarnić (2010).
A logical typology of normative systems.
Journal of Applied Ethics and Philosophy 2:30–40.

The three translation functions

$\langle \mathcal{N}, \overline{\mathcal{N}} \rangle$ is a normative system.

$\mathcal{L}_{pl}$ is the language of propositional logic.

Descriptive language $\mathcal{L}_{SDL}$: $\varphi ::= p \mid \mathrm{O}p \mid \mathrm{P}p \mid \neg\varphi \mid (\varphi_1 \wedge \varphi_2)$ where $p \in \mathcal{L}_{pl}$.

Descriptive language $\mathcal{L}_{\langle \mathcal{N}, \overline{\mathcal{N}} \rangle}$: $\varphi ::= p \mid \ulcorner p \urcorner \in \mathcal{N} \mid \ulcorner p \urcorner \in \overline{\mathcal{N}} \mid \neg\varphi \mid (\varphi_1 \wedge \varphi_2)$ where $p \in \mathcal{L}_{pl}$.

"Quine quotes", $\ulcorner \ldots \urcorner$, will be omitted at most places in the subsequent text for the ease of reading and writing.

Function *ctd* is defined as follows: $ctd(\varphi) = \begin{cases} \varphi', \text{ if } \varphi = \neg\varphi' \text{for some } \varphi' \\ \neg\varphi, \text{ otherwise.} \end{cases}$

$$\tau^{+}, \tau-, \tau* : \mathcal{L}_{SDL} \mapsto \mathcal{L}_{\langle \mathcal{N}, \overline{\mathcal{N}} \rangle}$$

To norm-set properties

$$\tau^{+}(\mathrm{O}\varphi) = \ulcorner \varphi \urcorner \in \mathcal{N}$$
$$\tau^{+}(\mathrm{P}\varphi) = \ulcorner ctd(\varphi) \urcorner \notin \mathcal{N}$$

To counter-set properties

$$\tau^{-}(\mathrm{O}\varphi) = \ulcorner \varphi \urcorner \notin \overline{\mathcal{N}}$$
$$\tau^{-}(\mathrm{P}\varphi) = \ulcorner ctd(\varphi) \urcorner \in \overline{\mathcal{N}}$$

To system properties

$$\tau^{*}(\mathrm{O}\varphi) = \tau^{+}(\mathrm{O}\varphi)$$
$$\tau^{*}(\mathrm{P}\varphi) = \tau^{-}(\mathrm{P}\varphi)$$

$\tau^{\circ}(\varphi) = \varphi$ if $\varphi \in \mathcal{L}_{pl}$;
$\tau^{\circ}(\neg\varphi) = \neg\tau^{\circ}(\varphi)$;
$\tau^{\circ}(\varphi \wedge \psi) = \tau^{\circ}(\varphi) \wedge \tau^{\circ}(\psi)$, where $\circ = +, -, *$.

Perfection properties: an almost ideal NS

Theory of perfection properties

- The theory of "perfection properties" is not given in Von Wright's works, he just uses the term.
- The concept of "perfect property" is a second-order notion, a property of properties. This concept belongs to the theory of values or axiology. In my research I have encountered only one logical theory of values: the theory given by Gödel in 1970 within the axiomatic basis of the proof of the existence of God as "summum bonum" (the highest good). Gödel defines the logical structure of "positive property (in moral-aesthetic sense)" as a *maximal filter*: (Ax1) conjunction of positive properties gives a positive property; (Ax2) in the pair of property and its complement exactly one of these is positive; (Ax4) a property implied by a positive property is itself positive; and in addition (Ax2a; Ax2b) positive properties are necessarily positive and negative properties are necessarily not positive.
- Gödel's theory of "moral-aesthetic positivity" only partly corresponds to the idea of "perfection property of a normative system", which is *a perfection in the logical sense*. Although the filter structure of obligation-norm set comes close to Gödel's structure of positivity, on the other hand, the fact that the model of normative system has two sets makes inapplicable the absolute notion of perfection property. There is no perfection property "as such" but only perfection properties of the obligation norm set and permission norm counter-set. So, the conjunction relational perfection properties is not possible: from 'X is a perfection property for $\mathcal{N}$ and Y is perfection property for $\overline{\mathcal{N}}$' it does not follow that $X \wedge Y$ is a perfection property of $\mathcal{N}$ and of $\overline{\mathcal{N}}$, but, as will be shown here, the contrariety of this follows.

Relata of activities

- Perfection properties of the normative system $\langle \mathcal{N}, \overline{\mathcal{N}} \rangle$ are:
    - ❶ perfection properties of the obligation norm set $\mathcal{N}$;
    - ❷ perfection properties of the permission norm counter-set $\overline{\mathcal{N}}$;
    - ❸ perfection properties of the relation between the obligation norm set $\mathcal{N}$ and the permission norm counter-set $\overline{\mathcal{N}}$.
- The activities of actor roles relate to different perfection properties. For example, the norm-recipient relates to deductively closed obligation norm set. In other words, for the norm recipient $\mathcal{N} = Cn(\mathcal{N})$ since in normative reasoning she or he has to accept any consequence of the set. On the other hand, the norm-giver does not relate to deductively closed obligation norm set. In other words, for the norm-giver $\mathcal{N} \subset Cn(\mathcal{N})$ since in producing normative system she or he does not have to proclaim all consequences of the initial set.

Some examples

- The deductive closure of the obligation norm set $\mathcal{N}$ is not a perfection property that the norm-giver ought to achieve in norm-giving activity since it is useless and impossible to explicitly promulgate each implied obligation. The deductive closure of the obligation norm set $\mathcal{N}$ is a perfection property to which the norm-recipient relates in the activity of normative reasoning since the norm-recipient ought to accept anything that follows from explicit obligations.
- The weak ideal closure of the permission norm set $\overline{\mathcal{N}}$ is partly a perfection property for the norm recipient who can reason along the deduction lines (since reverse consequence relation between negations is just consequence relation between affirmations). On the other hand, the closure under the condition of having at least one conjunct for each conjunct contained in the permission norm counter set $\overline{\mathcal{N}}$, or equivalently permitting at least one disjunct for each permitted disjunction, is a perfection property that the norm giver ought to achieve. One of the reasons for this is connected to the indeterminacy of completing normative system under the principle “everything not permitted is forbidden”, what will be discussed later.

More examples

Independence of the obligation norm set as a perfection property

"Analogously to the contraction operation, the operation $\mathcal{N} \div p$ of pure derogation needs an additional choice operation γ to pick a member of the remainder set: $\mathcal{N} \div p = \gamma(\mathcal{N} \perp p)$. The special and neat case of pure derogation arises when the norms of the initial norm-set are independent, i.e. when no norm from the set is entailed by the rest, i.e. $p \notin Cn(\mathcal{N} - \{p\})$ for all $p \in \mathcal{N}$. Only in this special case does it hold that pure derogation imposes no need to choose since there is exactly one member in the remainder set, namely $\mathcal{N} - \{p\}$, (1).

$$\text{If } p \notin Cn(\mathcal{N} - \{p\}) \text{ for all } p \in \mathcal{N}, \text{ then } \mathcal{N} \div p = \mathcal{N} - \{p\} \tag{1}$$

In view of possible derogation, independence turns out to be another perfection property of a norm-set, one that by relieving the burden of choice from the norm-applier enables "uniformity of judicial practice". If a norm-set does not have the independence property, then pure derogation could lead to the switching of roles: by being forced to choose between the elements of the remainder set, the norm-applier actually becomes the norm-giver."

Žarnić, B. and G. Bašić (2014).
Metanormative principles and norm governed social interaction.
Revus: Journal for constitutional theory and philosophy of law 22:105120

Doable content

- In standard KD deontic logic any logical truth is obligatory because of the necessitation rule: if $\vdash \varphi$, then $\vdash O\varphi$. On the other hand, Stig Kanger (1924–1988) restricted the obligation content to those states of affairs that can come about as the result of the human action. Consequently, in Kanger's axiomatization there is no place for necessitation rule. Instead he uses congruence rule: if $\vdash \varphi \rightarrow \psi$, then $\vdash O\varphi \rightarrow O\psi$; which within the restricted, "doable" domain does the job jointly performed by K axiom and necessitation rule in standard deontic logic.
- The description of restriction to doable content requires connecting "ought" and "is" statements, what standard deontic logic does not admit because of its unimodality . Replacing contingency for doability, the following would come close to a perfection property: $(O\varphi \lor P\varphi) \rightarrow (\Diamond\varphi \land \Diamond\neg\varphi)$.

Genuine norm

A norm will be called genuine if its content is doable by all agents to which it applies. A norm with a not-doable content will be said to be spurious. An example of a spurious norm would be one which pronounces a contradictory or tautologous state of affairs obligatory.
The idea that only norms with a doable content are genuine, i. e. genuinely apply to the agents to whom they are addressed, is a variant to the well-known idea, usually associated with the name of Kant, that what one ought to do one can do. "Ought implies Can".

Georg Henrik von Wright (1997).

Ought to Be – Ought to Do.

in E. Garzón Vald´eś, W. Krawietz, G.H.von Wright, and Ruch Zimmerling (Eds.), *Normative Systems in Legal and Moral Theory*, pp. 427–435. Berlin: Duncker & Humblot.

Perfection properties

PROPERTY	OBLIGATION NORM SET	PERMISSION NORM COUNTER-SET	RELATION BETWEEN THE TWO SETS	RELATED ACTIVITY
consistency	yes (internal consistency)	no	yes (external consistency)	norm-giving
completeness (gaplessness)	no	yes	-	norm-giving
independence	yes	no	-	norm-contraction
deductive closure (filter structure)	yes	no	-	norm-giving; normative-reasoning
reverse deductive closure (weak ideal structure)	no	yes	-	normative-reasoning
“doable” content	yes	yes	yes	norm-gving

An example: $\mathrm{O}\varphi \rightarrow \mathrm{P}\varphi$

Example

$$\tau^+(\mathrm{O}\varphi \rightarrow \mathrm{P}\varphi) = \tau^+(\mathrm{O}\varphi) \rightarrow \tau^+(\mathrm{P}\varphi)$$
$$= \ulcorner\varphi\urcorner \in \mathcal{N} \rightarrow \ulcorner\neg\varphi\urcorner \notin \mathcal{N}$$

Perfection property:

- internal consistency of $\mathcal{N}$.

$$\tau^-(\mathrm{O}\varphi \rightarrow \mathrm{P}\varphi) = \tau^-(\mathrm{O}\varphi) \rightarrow \tau^-(\mathrm{P}\varphi)$$
$$= \ulcorner\varphi\urcorner \notin \overline{\mathcal{N}} \rightarrow \ulcorner\neg\varphi\urcorner \in \overline{\mathcal{N}}$$
$$= \ulcorner\varphi\urcorner \in \overline{\mathcal{N}} \vee \ulcorner\varphi\urcorner \notin \overline{\mathcal{N}}$$

Perfection property:

- completeness of $\overline{\mathcal{N}}$.

$$\tau^*(\mathrm{O}\varphi \rightarrow \mathrm{P}\varphi) = \tau^+(\mathrm{O}\varphi) \rightarrow \tau^-(\mathrm{P}\varphi)$$
$$= \ulcorner\varphi\urcorner \in \mathcal{N} \rightarrow \ulcorner\neg\varphi\urcorner \in \overline{\mathcal{N}}$$

Perfection property:

- an unnamed property of $\langle\mathcal{N}, \overline{\mathcal{N}}\rangle$ (suggestions are welcome).

What SDL axioms say about normative systems?

POSTULATES AND RULES OF STANDARD DEONTIC LOGIC	OBLIGATION NORM-SET PROPERTIES	PERMISSION COUNTER-SET PROPERTIES
(D) $O\varphi \rightarrow P\varphi$	internal consistency (consistency of the obligation-norm set) $\tau^{+}(D) = \ulcorner\varphi\urcorner \in \mathcal{N} \rightarrow \ulcorner\neg\varphi\urcorner \notin \mathcal{N}$	completeness of the permission-norm set $\tau^{-}(D) = \ulcorner\varphi\urcorner \in \overline{\mathcal{N}} \vee \ulcorner\neg\varphi\urcorner \in \overline{\mathcal{N}}$
(2*) $(O\varphi \wedge O\psi) \rightarrow O(\varphi \wedge \psi)$	closure under conjunction $\tau^{+}(2^*) = (\varphi \in \mathcal{N} \wedge \psi \in \mathcal{N}) \rightarrow (\varphi \wedge \psi) \in \mathcal{N}$	having at least one conjunct for each conjunction contained $\tau^{-}(2^*) = (\varphi \wedge \psi) \in \overline{\mathcal{N}} \rightarrow (\varphi \in \overline{\mathcal{N}} \vee \psi \in \overline{\mathcal{N}})$
(Rc) $\dfrac{\vdash_{pl} \varphi \rightarrow \psi}{O\varphi \rightarrow O\psi}$	deductive closure $\tau^{+}(O\varphi \rightarrow O\psi) = \varphi \in \mathcal{N} \rightarrow \psi \in \mathcal{N}$ if $\vdash_{pl} \varphi \rightarrow \psi$	“closure under implicants” $\tau^{-}(O\varphi \rightarrow O\psi) = \psi \in \overline{\mathcal{N}} \rightarrow \varphi \in \overline{\mathcal{N}}$ if $\vdash_{pl} \varphi \rightarrow \psi$
	RELATIONAL PROPERTIES (SYSTEM PROPERTIES)	
(D*) $O\varphi \rightarrow \neg P\neg\varphi$	external consistency $\tau^{*}(D^*) = \varphi \in \mathcal{N} \rightarrow \varphi \notin \overline{\mathcal{N}}$	
(Com) $O\varphi \vee P\neg\varphi$	“gaplessness” $\tau^{*}(Com) = \varphi \in \mathcal{N} \vee \varphi \in \overline{\mathcal{N}}$	

An example

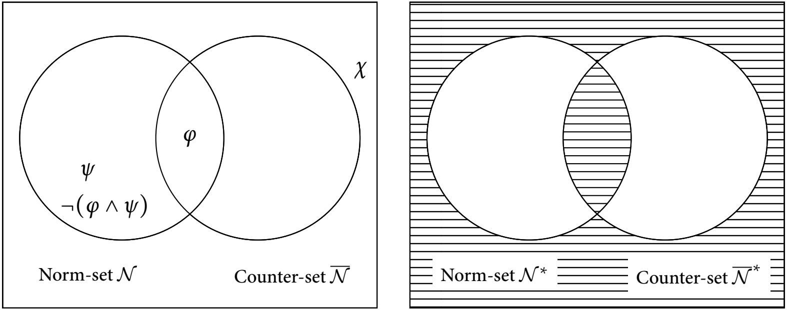


Figure : A comparison between imperfect system $\langle \mathcal{N}, \overline{\mathcal{N}} \rangle$ and system $\langle \mathcal{N}^*, \overline{\mathcal{N}}^* \rangle$, which is endowed with some perfection properties. System $\langle \mathcal{N}, \overline{\mathcal{N}} \rangle$ is internally inconsistent under deductive closure; it is externally inconsistent as the presence of φ at the intersection shows and makes both Oφ and P$\neg\varphi$ true; it is also incomplete since $\chi \notin \mathcal{N} \cup \overline{\mathcal{N}}$. System $\langle \mathcal{N}^*, \overline{\mathcal{N}}^* \rangle$ is externally consistent and complete.

Almost ideal normative system

An almost ideal normative system (*ains*) is internally (IntC) and externally consistent (ExtC), its obligation norm-set is closed under conjunction (2^*) and entailment (Rc), and it is complete (Comp). An ideal normative system is characterized by the following axiom schemata and rules:

$$\begin{array}{rl}
(2^*) & \vdash_{ains} (\ulcorner\varphi\urcorner \in \mathcal{N} \wedge \ulcorner\psi\urcorner \in \mathcal{N}) \rightarrow \ulcorner\varphi \wedge \psi\urcorner \in \mathcal{N} \\
(\text{IntC}) & \vdash_{ains} \ulcorner\varphi\urcorner \in \mathcal{N} \rightarrow \ulcorner\neg\varphi\urcorner \notin \mathcal{N} \\
(\text{ExtC}) & \vdash_{ains} \ulcorner\varphi\urcorner \in \mathcal{N} \rightarrow \ulcorner\varphi\urcorner \notin \overline{\mathcal{N}} \\
(\text{Comp}) & \vdash_{ains} \ulcorner\varphi\urcorner \in \mathcal{N} \vee \ulcorner\varphi\urcorner \in \overline{\mathcal{N}} \\
(\text{RcO}) & \dfrac{\vdash_{pl} \varphi \rightarrow \psi}{\vdash_{ains} \ulcorner\varphi\urcorner \in \mathcal{N} \rightarrow \ulcorner\psi\urcorner \in \mathcal{N}} \\
(\text{RcP}) & \dfrac{\vdash_{pl} \varphi \rightarrow \psi}{\vdash_{ains} \ulcorner\psi\urcorner \in \overline{\mathcal{N}} \rightarrow \ulcorner\varphi\urcorner \in \overline{\mathcal{N}}}
\end{array}$$

A statement of the form $\vdash_{ains} \varphi$ means ‘φ is derivable from set of axioms $\{1o, 2^\star, \text{IntC}, \text{ExtC}, \text{Comp}\}$ using rules $\vdash_{pl}$ of propositional logic and rules RcO and RcP.’

An unnecessary axiom

In order to establish the connection with standard deontic logic, which admits necessitation rule (if $\vdash \varphi$, then $\vdash O\varphi$), axiom (1) will be added.[1] Axiom (1) is not needed for establishing correspondences between sets memberships and standard deontic logic in Kanger's style.

(1o) $\vdash_{ains} \top \in \mathcal{N}$

Remark

Axiom 1o is not needed if the obligation norm set sets is not empty. Congruence rule RcO guarantees the presence of $\top$ if $\mathcal{N} \neq \varnothing$, since verum follows from anything, and by internal consistency and completeness it follows that $\bot \in \overline{\mathcal{N}}$. The non-emptiness of $\mathcal{N}$ should be counted as a perfection property, otherwise there would be no normative system. Nevertheless, the sole presence of tautologies is not a sufficient condition for the proper non-emptiness.

---

[1] Inclusion of logical truths into a normative system can hardly be understood as its perfection property. Cf. " …Principle of Deontic Contingency: *A tautologous act is not necessarily obligatory, and a contradictory act is not necessarily forbidden.*" (von Wright, 1951:11)

The theorem on standard deontic logic and the theory of ideal normative system

Translations of axioms and rules of standard deontic logic yield some truths about an ideal normative system. These truths constitute a part of the theory of ideal normative system thus confirming Von Wright's conjecture.

Theorem

If $\vdash_{sdl} \varphi$, *then* $\vdash_{ains} \tau^{+}(\varphi)$, $\vdash_{ains} \tau^{-}(\varphi)$, *and* $\vdash_{ains} \tau^{*}(\varphi)$.

Proof.

All axioms and rules of standard deontic logic can be derived in $\vdash_{ains}$. Therefore, any step of a proof in $\vdash_{sdl}$ can be reproduced within $\vdash_{ains}$. □

Language of the doable

Assume that language $\mathcal{L}_{\text{doable}}$ is the set of contingent propositions and that it also satisfies some unspecified conditions of "doability" (as a minimum condition assume closure under negation). Extension of $\mathcal{L}_{\text{doable}}$ to sets membership statements gives $\mathcal{L}^{\text{doable}}_{\langle \mathcal{N}, \overline{\mathcal{N}} \rangle}$.

Definition

Language $\mathcal{L}^{\text{doable}}_{\langle \mathcal{N}, \overline{\mathcal{N}} \rangle}$:
$\varphi ::= \ulcorner p \urcorner \in \mathcal{N} \mid \ulcorner p \urcorner \in \overline{\mathcal{N}} \mid \neg\varphi \mid (\varphi_1 \wedge \varphi_2)$ where $p \in \mathcal{L}_{\text{doable}}$.

> The variables, *p*, *q*, etc., and their molecular compounds in the deontic formulas can be understood as schematic representations either of (names of) human action or of (sentences describing) states of affairs which can come to obtain as the result of human action. I shall call them *doable* states of affairs. For example: "O*p*" could represent the expression "one ought to close the window" or, alternatively, "the window ought to be closed." The first is—to use established German terminology—a *Tun-Sollen* ("ought to do"); the second is a *Sein-Sollen* ("ought to be"). [p.29]

Georg Henrik von Wright (1999).
Deontic Logic: A Personal View.
Ratio Juris 12:26–38.

Standard deontic logic as a theory of ideal normative systems

Proposition

Let $\vdash_{ins}$ *be the system different from* $\vdash_{ains}$ *only by exclusion of axiom (1a).*
If $\{\tau^{+}(\varphi), \tau^{-}(\varphi), \tau^{*}(\varphi)\} \subseteq \mathcal{L}^{\text{doable}}_{\langle \mathcal{N}, \overline{\mathcal{N}} \rangle}$*, the the following holds:*
if $\vdash_{sdl} \varphi$*, then* $\vdash_{ins} \tau^{+}(\varphi)$*,* $\vdash_{ins} \tau^{-}(\varphi)$*, and* $\vdash_{ins} \tau^{*}(\varphi)$*.*

Proof.

The proposition is a restriction of the previous theorem. Axiom (1o) is needed solely for proving the necessitation rule and can be dropped. ☐

Some remarks

Corollary

If $\{\tau^+(\varphi), \tau^-(\varphi), \tau^*(\varphi)\} \subseteq \mathcal{L}^{\text{doable}}_{\langle \mathcal{N}, \overline{\mathcal{N}} \rangle}$*, then the following holds:*

- *if* $\vdash_{sdl} \varphi$*, then* $\tau^+(\varphi)$ *describes a perfection property of the obligation norm set;*
- *if* $\vdash_{sdl} \varphi$*, then* $\tau^-(\varphi)$ *describes a perfection property of the permission norm counter-set;*
- *if* $\vdash_{sdl} \varphi$ *and both* O *and* P *occur in* φ*, then* $\tau^*(\varphi)$ *describes a perfection relation between the two sets.*

Proof.

Axioms $\{2^\star, \text{IntC}, \text{ExtC}, \text{Comp}\}$ (but not axiom 1o) describe perfection properties. By accepting the modification of Gödel's axiom "anything implied by a perfection property is a perfection property", the corollary follows. ☐

Remark

The restriction to doable states of affairs also shows that the limitations in the expressive power of the sdl-language: no φ *such that* $\vdash_{sdl} \varphi$ *can give* $\tau^+(\varphi)$ *claiming that exists* $\varphi \in \mathcal{L}^{doable}$ *in* $\mathcal{N}$*.*

Proposition

$$\vdash_{ains} \tau^{+}((\mathrm{O}\varphi \land \mathrm{P}\psi) \to \mathrm{P}(\varphi \land \psi))$$

Proof.

$$\tau^{+}((\mathrm{O}\varphi \land \mathrm{P}\psi) \to \mathrm{P}(\varphi \land \psi)) = (\varphi \in \mathcal{N} \land \neg\psi \notin \mathcal{N}) \to \neg(\varphi \land \psi) \notin \mathcal{N}$$

1	$\varphi \in \mathcal{N} \land \neg\psi \notin \mathcal{N}$	
2	$\ulcorner\neg(\varphi \land \psi)\urcorner \in \mathcal{N}$	
3	$\varphi \in \mathcal{N}$	1/ Elim∧
4	$(\varphi \land \neg(\varphi \land \psi)) \in \mathcal{N}$	2, 3/ 2*
5	$(\varphi \land \neg(\varphi \land \psi)) \to \neg\psi$	$\vdash_{pl}$
6	$\neg\psi \in \mathcal{N}$	4, 5/ RcO
7	$\neg\psi \notin \mathcal{N}$	1/ Elim∧
8	$\ulcorner\neg(\varphi \land \psi)\urcorner \notin \mathcal{N}$	2–7/ Intro¬
9	$(\varphi \in \mathcal{N} \land \neg\psi \notin \mathcal{N}) \to \neg(\varphi \land \psi) \notin \mathcal{N}$	1–8/ Intro→

□

Proposition

$$\vdash_{ains} \tau^{-}(\mathrm{O}(\varphi \to \psi) \to (\mathrm{O}\varphi \to \mathrm{O}\psi))$$

Proof.

$$\tau^{-}(\mathrm{O}(\varphi \to \psi) \to (\mathrm{O}\varphi \to \mathrm{O}\psi)) = (\varphi \to \psi) \notin \overline{\mathcal{N}} \to (\varphi \notin \overline{\mathcal{N}} \to \psi \notin \overline{\mathcal{N}})$$

$$\begin{array}{r|l|l|ll}
1 & & & (\varphi \to \psi) \notin \overline{\mathcal{N}} & \\
\cline{3-3}
2 & & & \quad \varphi \notin \overline{\mathcal{N}} & \\
3 & & & \quad \varphi \in \mathcal{N} & \text{2/ Comp} \\
4 & & & \quad (\varphi \to \psi) \in \mathcal{N} & \text{1/ Comp} \\
5 & & & \quad (\varphi \wedge (\varphi \to \psi)) \in \mathcal{N} & \text{3, 4/ 2*} \\
6 & & & \quad (\varphi \wedge (\varphi \to \psi)) \to \psi & \vdash_{pl} \\
7 & & & \quad \psi \in \mathcal{N} & \text{5, 6/ RcO} \\
8 & & & \quad \psi \notin \overline{\mathcal{N}} & \text{7/ ExtC} \\
9 & & & \varphi \notin \overline{\mathcal{N}} \to \psi \notin \overline{\mathcal{N}} & \text{2–8/ Intro}\to \\
10 & & (\varphi \to \psi) \notin \overline{\mathcal{N}} \to (\varphi \notin \overline{\mathcal{N}} \to \psi \notin \overline{\mathcal{N}}) & & \text{1–9/ Intro}\to
\end{array}$$

□

Proposition

$\vdash_{ains} \tau^*(\mathrm{O}\varphi \to \mathrm{P}\varphi)$

Proof.

$\tau^*(\mathrm{O}\varphi \to \mathrm{P}\varphi) = \varphi \in \mathcal{N} \to \neg\varphi \in \overline{\mathcal{N}}$

1	$\varphi \in \mathcal{N}$	
2	$\ulcorner\neg\varphi\urcorner \notin \overline{\mathcal{N}}$	
3	$\neg\varphi \in \mathcal{N}$	2/ Comp
4	$\neg\varphi \notin \mathcal{N}$	1/ IntC
5	$\ulcorner\neg\varphi\urcorner \in \overline{\mathcal{N}}$	2–4/ Elim¬
6	$\varphi \in \mathcal{N} \to \ulcorner\neg\varphi\urcorner \in \overline{\mathcal{N}}$	1–5/ Intro→

□

Logical pragmatics

What can be learnt on *deontic logic* form von Wright's reinterpretation?

- Deontic logic is not the logic of actual normative systems for they can fail in being perfect. It is neither the logic of normative reasoning since its basis can be imperfect.
- Deontic logic is the logic of perfect systems and their perfection properties, the ones that the norm-giver ought to achieve in normative system (re)construction, and the ones to which the norm-recipient relates in her/his reasoning in spite of the actual imperfections.
- Von Wright's late works on deontic logic (from 1980s onwards) show the need of supplementing the research from the viewpoint of logical pragmatics.
  Consider the example of second-order norm that forbids the norm-giver g to produce an externally inconsistent normative system for the norm-recipient r: $[g : \underline{O_r \varphi}]F_g g : \underline{P_r \neg\varphi}$. What is the source of the second-order norm? According to Von Wright it is an requirement of rationality. Nevertheless, there is another explanation: The second-order norms are the norms of language use. Thus, the language of normativity is subordinated to the normativity of language (in use).

Application

Normative gaps

> "What is the difference 'in practice' between a state of affairs not being prohibited and its being permitted? Suppose there is a code of norms in which there is no norm P*p*. Now someone makes it so that *p*. What should be the law-giver's reaction to this, if any? Could he say: 'You were not permitted to do this and you must not do that which you are not permitted to do'? He could say this, making it a meta-norm that everything not-permitted is thereby forbidden. 'Logically' this would be just as possible, even though perhaps less reasonable, as to have a meta-norm permitting everything which is not forbidden. But one can also think of some 'middle way' between these two principles, a meta-norm to the effect that if something is not permitted by the existing norms of a code one must, as we say, 'ask permission' of the law-giver to do it."
>
> Georg Henrik von Wright.
> Is there a logic of norms?
> *Ratio Juris*, 4:265–283, 1991.

According to von Wright there are three principles by use of which a normative system can be completed: 1. $(\neg F \rhd P)$ everything not forbidden is permitted, 2. $(\neg P \rhd F)$ everything not permitted is forbidden, 3. normative gaps are filled in communication between the norm-recipient and the norm-giver.

The third principle will be left aside because of its complexity. Using the two-sets model of normative system I will try to show why the first principle is to be preferred over the second one, i.e., why the first principle is "more reasonable". In addition to this I will show that the mere "logical possibility" of the second mode $(\neg P \rhd F)$ of filling normative gaps is not a sufficient condition of its rationality, according to von Wright's own criterion of rationality of norm-giving activity.

The easy way of completion

Completion under the meta-principle everything which is not forbidden is permitted

The easy way of making an internally and externally consistent normative system complete is by applying the principle *everything which is not forbidden is permitted*. The way of filling in the gaps is straightforward, consisting in adding the missing sentences to the counter set and thus obtaining its extension $\overline{\mathcal{N}}^*$, as formula (2) shows.

$$\overline{\mathcal{N}}^* = \overline{\mathcal{N}} \cup \{\varphi \mid \varphi \notin Cn(\mathcal{N})\} \tag{2}$$

The long and branching road…

A completion of a consistent normative system under the principle *everything which is not permitted is forbidden* is not a functional relation. In this mode the process is under-determined and so does not result in a unique system. The completion proceeds in two steps, each of which includes a choice.

The first step

In the first step the counter-set must be completed in the view of perfection relations and properties. Also, the perfection-relation of external consistency between the obligation norm set and its counter-set ought to be preserved if present and so their intersection must remain empty. This means that it will be expanded to achieve perfection-properties of being closed under implicants and under the rule of having at least one conjunct for each member conjunction. Since the last condition has the disjunctive consequent (=at least one conjunct must be present) there may be different ways of performing the closure.

- Let us consider the case of completion of $\overline{\mathcal{N}}$ under the principle of "having at least one conjunct for each conjunction contained". Suppose that $\ulcorner \varphi \wedge \psi \urcorner \in \overline{\mathcal{N}}$. There are two cases: (i) either $\varphi \in \mathcal{N}$ or $\psi \in \mathcal{N}$ (both sentences cannot be members thanks to consistency); (ii) $\varphi \notin \mathcal{N}$ and $\psi \notin \mathcal{N}$.
- Case (i) is deterministic: if one sentence is in $\mathcal{N}$, put the other in $\overline{\mathcal{N}}$.
- Case (ii) leaves three options: the two of putting only one sentence into the counter-set and the third of putting both of them there.

Therefore, the weak-ideal expansion of a counter-set results in a set of sets.

The second step

The second step in a completion of normative system is also under-determined and complex in itself. It consists of two phases.
In the first phase the obligation norm-set and its counter-set are closed under appropriate relations by taking into account “partially placed” sentences, i.e., those where only one sentence from a pair of contradictory sentences belongs to the closure of the system.
In the second phase “unplaced sentences” are being added in an iterative manner to the system. “Unplaced sentences” are those where no sentence from a pair of contradictory sentences belongs to the system, i.e., $\ulcorner\varphi\urcorner \notin \mathcal{N}_0 \cup \overline{\mathcal{N}}_0$ and $\ulcorner\neg\varphi\urcorner \notin \mathcal{N}_0 \cup \overline{\mathcal{N}}_0$. The choices abound here:

- Again the construction choice must be made: whether to put φ or $\neg\varphi$ in the expanded norm-set at some level of construction. Given a list of pairs of unplaced sentences the choice can be made at each level provided that the pair is independent from the norm-set constructed at that level.
- Furthermore, there is no preferred ordering of unplaced sentences. The outcome of the iterative process depends on the chosen ordering. In most cases the resulting systems will be radically different.

An example

The systems completed by the application of the principle *everything not permitted is forbidden* do not necessarily give a description of one and the same “ideal state of things”. Rather there is a multitude of “ideal states of things” described by a normative systems completed under this principle.

Example

Let $\mathcal{N} = \{p \vee q\}$ and $\overline{\mathcal{N}} = \varnothing$. Expansion of the counter-set by partially placed sentences and weak-ideal closure of the counter-set together yield the following set of sets: $\{\{\neg p \wedge \neg q, \neg p, \neg p \wedge q\}, \{\neg p \wedge \neg q, \neg q, \neg p \wedge \neg q\}, \{\neg p \wedge \neg q, \neg p, \neg q, \neg p \wedge q, p \wedge \neg q\}\}$. So, a choice must be made. Suppose that the norm-recipient r' chooses the first, and the norm-recipient r'' chooses the second. The expansion of the obligation-set with respect to the counter-set depends on the set being chosen, and thus it yields for r' the set $\mathcal{N}_0' = \{p \vee q, p, p \vee \neg q\}$, while for r'' it is the set $\mathcal{N}_0'' = \{p \vee q, q, \neg p \vee q\}$. Finally, the expansion by unplaced sentences depends on the list used in the construction. Suppose that r' chooses List 1 given by: $[q], \ldots$, while r'' chooses List 2:$[\neg p], \ldots$. Then $\mathcal{N}_1' = Cn(\mathcal{N}_0' \cup \{q\})$, while $\mathcal{N}_1'' = Cn(\mathcal{N}_0'' \cup \{\neg p\})$. Therefore, $(p \wedge q) \in \mathcal{N}_1'$, and $\neg(p \wedge q) \in \mathcal{N}_1''$ The completion results in incompatible ideal states as translations show: For r' it holds that $\mathrm{O}(p \wedge q)$ w.r.t. $\langle \mathcal{N}, \overline{\mathcal{N}} \rangle$ completed under the principle $\neg\mathrm{P} \rhd \mathrm{F}$. For r'' it holds that $\mathrm{F}(p \wedge q)$ w.r.t. $\langle \mathcal{N}, \overline{\mathcal{N}} \rangle$ completed under the principle $\neg\mathrm{P} \rhd \mathrm{F}$.

A critique: how many ideal states?

> “Generally speaking: a legal order and, similarly, any coherent code or system of norms may be said to envisage what I propose to call an ideal state of things when no obligation is ever neglected and everything permitted is sometimes the case. If this ideal state is not logically possible, i.e., could not be factual, the totality of norms and the legislating activity which has generated it do not conform to the standards of rational willing. Deviations from these standards sometimes occur — and when they are discovered steps are usually taken to eliminate them by ‘improved’ legislation.
>
> Georg Henrik von Wright.
> Is and ought.
> In M. C. Doeser and J. N. Kraay, editors, *Facts and Values: Philosophical Reflections from Western and Non-Western Perspectives*, pages 31–48. Springer Netherlands, Dordrecht, 1986.

If a normative system is completed under the principle *everything not permitted is forbidden*, then, if consistent, it can “envisage more then one ideal state”, each equally acceptable as any other. Thus, there will be no unique ideal state with respect to obligation-norms. If intending a unique ideal state is essential to rational willing on the side of the norm-giver, then the the principle *everything not permitted is forbidden* is not only “less reasonable”, as von Wright claimed, but also not (instrumentally) rational.

The viewpoint of logical pragmatics

Definition

A consistent normative system $\langle \mathcal{N}, \overline{\mathcal{N}} \rangle$ supplemented with the completion meta-norm m is socially incoherent if it is possible for the two norm-recipients r and r' in their normative reasoning based on $\langle \mathcal{N}, \overline{\mathcal{N}} \rangle \cdot m$ to validly deduce incompatible conclusions.

- If viewpoints of other roles in norm-regulated activities are taken into account, then the completion principle $\neg \mathrm{P} \ \triangleright \mathrm{F}$ cannot be a rational one.
- If norm-recipients use this principle in their normative reasoning, their normative conclusions might stand in exclusive opposition (either being contrary or contradictory). Thus, a normative system closed under the principle $\neg \mathrm{P} \ \triangleright \mathrm{F}$ might be socially incoherent. Consistency (as Von Wright claimed) and social coherence (as is being claimed here) are conditions of rationality of norm-giving activity. So, the meta-norm $\neg \mathrm{P} \ \triangleright \mathrm{F}$ is not only instrumentally non-rational, but non-rational with respect to the logic of normativity.

Deontic logic and rationality wider than individual

> "Deontic logic, one could also say, is neither a logic of norms nor a logic of norm-propositions but a study of conditions which must be satisfied in rational norm-giving activity. It is strict *logic* because the conditions which it lays down are derived from *logical* relations between states in the ideal worlds which normative codes envisage."

Georg Henrik von Wright (1993).
A Pilgrim's Progress.
In von Wright, G.H. *The Tree of Knowledge and Other Essays*, 103–113. Leiden: Brill.

- The notion of social incoherence shows that rationality conditions of the norm-giving activity cannot be derived solely from consistency of a singular normative-system. If the task of completion of a normative-system is assigned to norm-recipients, the fulfilment of the task should not result in social incoherence.
- Thus, the meta-principle *everything not forbidden is permitted* is not just an option, but the second-order norm to which the norm-giver is subordinated.
- The norm-giver ought to produce normative systems endowed with perfection properties. Social coherence is a perfection property. Therefore, *everything not forbidden is permitted* might be a truth of deontic logic as the theory of ideal normative systems.

Extension of language

Definition

$\mathcal{L}_{pl}$ language of propositional logic (also, norm-content language)	
$\mathcal{L}_{ctg}$ language of categorical norms	$\psi ::= \varphi \mid O\varphi \mid P\varphi$ where $\varphi \in \mathcal{L}_{pl}$
$\mathcal{L}_{hyp}$ language of hypothetical norms	$\xi ::= \psi \mid \varphi \to \psi \mid \psi \to \varphi$ where $\psi \in \mathcal{L}_{ctg}$ and $\varphi \in \mathcal{L}_{pl}$

Functions instead sets

Definition

Valuation w is a function assigning true or false for any letter p from $\mathcal{L}_{pl}$: $w(p) \in \{\mathrm{t}, \mathrm{f}\}$. Valuation represents a possible world and will be referred to by 'world'. W is the set of all valuations. Valuations of molecular sentences are classical.

The truth-set $[\![\varphi]\!]$ of a sentence φ from $\mathcal{L}_{pl}$ is the set of verifying valuations: $[\![\varphi]\!] = \{w \mid w \in W \text{ and } w(\varphi) = \mathrm{t}\}$. The truth-sets behave classically: $[\![\neg\varphi]\!] = W - [\![\varphi]\!]$, $[\![\varphi \wedge \psi]\!] = [\![\varphi]\!] \cap [\![\psi]\!]$.

Definition

Hypothetical obligation norm set N is a function taking a world w and delivering a subset $N(w)$ of sentences from $\mathcal{L}_{pl}$. Function $N(w)$ represents requirements or obligation norm-contents binding a norm-recipient in w. Hypothetical permission norm counter-set $\overline{N}$ is a function taking a world w and delivering a subset $\overline{N}(w)$ of sentences from $\mathcal{L}_{pl}$. Function $\overline{N}(w)$ corresponds in a inverse way to permission norm-contents available to a norm-recipient in w.

Definition

Function *ctd* is defined by

$$ctd(\varphi) = \begin{cases} \varphi' \text{ if } \varphi = \neg\varphi' \text{for some } \varphi' \\ \neg\varphi \text{ otherwise} \end{cases}$$

Norm-content extraction function *nc* is a function taking a world w and a sentence ξ from $\mathcal{L}_{hyp}$, and delivering a subset $N(w)$ of sentences from $\mathcal{L}_{pl}$ in the following way:

- $nc(w, \mathrm{O}\varphi) = \varphi$
- $nc(w, \mathrm{P}\varphi) = ctd(\varphi)$
- $nc(w, \varphi \rightarrow \psi) = \begin{cases} nc(w, \psi) \text{ if } w \in [\![\varphi]\!] \\ \varnothing\text{, otherwise.} \end{cases}$
- $nc(w, \psi \rightarrow \varphi) = \begin{cases} ctd(nc(w, \psi)) \text{ if } w \in [\![\neg\varphi]\!] \\ \varnothing\text{, otherwise.} \end{cases}$

  where $\varphi \in \mathcal{L}_{pl}$ and $\psi \in \mathcal{L}_{ctg}$.

Definition

Obligation norms are:

- categorical obligation norms, $O\varphi$,
- conditionals whose consequent is a categorical obligation norm, $\varphi_1 \to O\varphi_2$,
- conditionals whose antecedent is a categorical permission norm,$P\varphi_1 \to \varphi_2$,

Permission norms are:

- categorical permission norms, $P\varphi$,
- conditionals whose consequent is a categorical permission norm, $\varphi_1 \to P\varphi_2$,
- conditionals whose antecedent is a categorical obligation norm,$O\varphi_1 \to \varphi_2$,

Definition

Set S is the set of explicitly promulgated norms. S divides into:

$$\begin{aligned} Ob &= \{\xi \in S \mid \xi \text{ an obligation norm}\} \\ Pr &= \{\xi \in S \mid \xi \text{ a permission norm}\} \end{aligned}$$

$N(w)$ is the set of norm-contents of obligation norms holding at w:

$$N(w) = \bigcup_{\xi \in Ob} nc(\xi)$$

$\overline{N}(w)$ is the set of contradictory norm-contents of permission norms holding at w:

$$\overline{N}(w) = \bigcup_{\xi \in Pr} nc(\xi)$$

Normative system reconsidered

- The extension of the two-sets approach to hypothetical norms has forced us to adopt the two-functions system. Instead of $\langle \mathcal{N}, \overline{\mathcal{N}} \rangle$ we use now a pair of functions $\langle N, \overline{N} \rangle$ to model the meaning of prescriptively used sentences constituting the normative source S, which is the “textual kernel” of $\langle N, \overline{N} \rangle$.
- Extending the line of thought we arrive at he following generalization: at each w the pair $\langle N(w), \overline{N}w) \rangle$ ought to have the same perfection properties as $\langle \mathcal{N}, \overline{\mathcal{N}} \rangle$ ought to have.
- For example, at each w the normative system $\langle N(w), \overline{N}w) \rangle$ ought to be internally and externally consistent.

Absurdity of normative explosion

Example

Let $\{p \to \mathrm{O}r, q \to \mathrm{P}\neg r\} \subseteq S$. Suppose that $p \wedge q$ is true in world w, i.e., $w \in [\![p \wedge q]\!]$. Obviously $\langle N(w), \overline{Ny}(w)\rangle$ is externally inconsistent. Nevertheless, it is possible for any other v such that $v \notin [\![p \wedge q]\!]$, that $\langle N(v), \overline{N}(v)\rangle$ is consistent. From the perspective of the norm-recipient any instance of a normative system is an abstract entity consisting of N closed under deduction and $\overline{N}$ closed as a weak ideal $\langle Cn(N), \gamma(\overline{N})\rangle$, where $\gamma(\overline{N}) \in WI(\overline{N})$ is a selection function choosing an element from the set of weak-ideals.
What is the range of “explosion” of $\langle N(w), \overline{N}w)\rangle$ with respect to $p \wedge q$-type of situations? Does this fact destroy the normative system in general, i.e., in respect to any world, or just the w instance of it, or it does not destroy none of them but challenges the logic of normative reasoning based on $\langle N(w), \overline{N}w)\rangle$?

Consistency of a normative system

- The notion of consistency with respect to a normative theory is not the same notion as consistency of a descriptive theory. A descriptive theory is semantically consistent if there is an interpretation (a world) that makes true every sentence in the set.
- The consistency of a normative system cannot be defined in the similar way. This notion is a much more demanding one. It is not sufficient to have at least one w with respect to which all realizations of a normative system are logically possible (i.e. that for all $\varphi \in \overline{N}(w)$ the set $N \cup \{ctd(\varphi)\}$ is consistent). This must hold "everywhere"!

Definition

Normative system $\langle N, \overline{N} \rangle$ is strongly consistent if all its instances are consistent, i.e., for all w, $\langle N(w), \overline{N}(w) \rangle$ is internally and externally consistent.

Two approaches to inconsistency

Option 1: "all or nothing" approach

If a normative system is not *strongly consistent*, then any act is obligatory in any situation.

Option 2: flexible approach

	for all $\varphi \in \overline{N}(w)$, $\bot \notin Cn(N(w) \cup \{ctd(\varphi)\})$	$\bot \in Cn(N(v))$, or for some $\varphi \in \overline{N}(v)$, $\bot \in Cn(N(v) \cup \{ctd(\varphi)\})$
	system $\langle N, \overline{N} \rangle$ is in a stable state at w	system $\langle N, \overline{N} \rangle$ is in an unstable state at v
Logic in use by the norm-recipient and norm-applier (judge)	classical	non-clasical

In theoretical Option 2, which seems to be the more preferable option with respect to the economy of time, the fact of inconsistency of a norm-set with respect to a certain situation is a reason for actors to engage in the revision of the normative system or its underlying logic.

Normativity and logic of sciences of social reality

> "I am inviting you to see the difference between the humanities and the natural sciences in the light of the difference between the factual and the normative, between rules which state how things in fact go and rules which ordain how they should go according to the conceptions of those who instituted the rules."
>
> Georg Henrik von Wright (1981).
>
> Humanism and the Humanities.
>
> In *Philosophy and Grammar: Papers on the Occasion of the Quincentennial of Uppsala University*, S. Kanger and S. Ōhman, Sven, eds., pp. 1–16. Dordrecht: Springer
>
> [p.12]

- Von Wright's is not isolated in understanding normativity as an essential property of the social reality.
- If we accept the invitation to see the difference between the natural sciences and the sciences of man (social sciences and humanities) in the light of the difference between the factual and the normative, then deontic logic comes to the fore and becomes the specific logic of sciences of man. So, the reach of deontic logic is wider than the theory of normative systems.

What can be learnt on philosophy and logic of social reality form von Wright's interpretation of deontic logic?

- Man, as an imperfect rational being, creates imperfect normative systems by the prescriptive use of language.
- Man can be subordinated to an imperfect normative system and compelled to reason on its basis.
- Deontic logic is not the logic of actual normative systems for they can fail in being perfect. It is neither the logic of normative reasoning since its basis can be imperfect.
- Deontic logic is the logic of perfect systems and their perfection properties, the ones that the norm-giver ought to achieve in normative system (re)construction, and the ones to which the norm-recipient relates in her/his reasoning in spite of the actual imperfections.[2]

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