

Quantum Logic and the Question of the Empirical Nature of Logic

On the destiny of disjunction

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- The research presented is in an initial stage.
- It is an attempt to put into practice the idea of a “dialogue between natural sciences and philosophy”.
- Our first question will regard the correctness of the thesis that quantum theory challenges some postulates of metaphysics and epistemology. For that purpose, three postulates of *Tractatus* will be made explicit and confronted with some principles from quantum theory.
- Second, the question of the empirical character of logic will be addressed and a variant of logical pluralism will be sketched.
- The answers for the two questions will be sought for by reflecting on the theoretical transformations of the notion of disjunction in its ontological, epistemological and logical meaning.

OVERVIEW

- 1 Physics and Philosophy: the witness and the accused
- 2 The three presuppositions of philosophical tradition embedded in Tractatus
 - Independence of states of affairs
 - Unique mode of composition of states of affairs into complexes
 - The mereology (whole-part relation) of ideal knowledge (or complete description of reality)
- 3 External inconsistency of Tractarian theory and quantum theory
 - Exposition of failures of Tractarian postulates
- 4 Logic in confrontation with quantum theory
 - Empirical character of logic
 - Coexistence of similar logical terms: a variant of logical pluralism

THE INTERROGATION DIRECTION

- Quantum theory poses numerous challenges to philosophy.
- Instead of physics being asked by philosophy, e.g. about the reference of its terms, the interrogation changes direction: it is philosophy that ought to reflect on the logico-ontological foundations of quantum physics and not vice versa.
- This dialogue is likely to create a philosophical progress, at least in the sense of discovering restrictions inherent in allegedly self-evident postulates of logic and metaphysics.

Philosophical progress and quantum theory

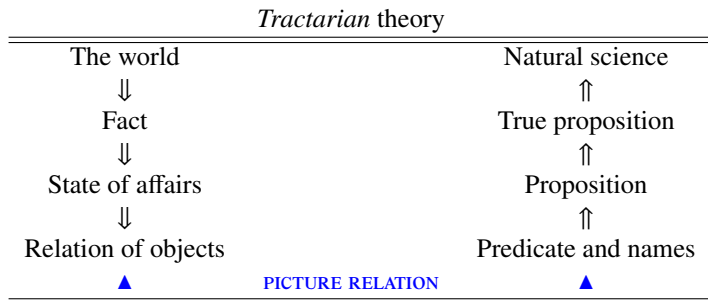
It is understandable that quantum theory provoked a debate over its interpretation. Not only is it incompatible with the world view of classical physics, but also with certain positions of classical metaphysics. [...] ... one must decide whether to consider this incompatibility as philosophical progress or a weakness of the theory. The present book is based on the conviction that we are dealing with fundamental philosophical progress. According to this conviction, it is not quantum theory that must defend itself before the court of traditional philosophies but those philosophies themselves must stand trial—in itself a philosophical process—with quantum theory in the witness stand.



Carl Friedrich von Weizsäcker
(1985) *The Structure of Physics*.
p.243, Dordrecht: Springer.

ONE WORLD AND ONE RELATION BETWEEN WORLD AND LANGUAGE

- The theory of language–world relation logic has been exposed in Wittgenstein's *Tractatus logico-philosophicus*.



⇓ shows deconstruction path.

⇑ shows construction path.

THE THREE POSTULATES

- 1 States of affairs are mutually INDEPENDENT.
- 2 States of affairs can be combined only in a “conjunctive way”.
- 3 It is not impossible to obtain a complete description of reality.

2.034 The structure of a fact consists of the structures of states of affairs.

...

2.04 The totality of existing states of affairs is the world.

2.05 The totality of existing states of affairs also determines which states of affairs do not exist.

2.06 The existence and non-existence of states of affairs is reality.

...

2.061 States of affairs are independent of one another.

2.062 From the existence or non-existence of one state of affairs it is impossible to infer the existence or non-existence of another.

2.063 The sum-total of reality is the world.

...

4.2211 Even if the world is infinitely complex, so that every fact consists of infinitely many states of affairs and every state of affairs is composed of infinitely many objects, there would still have to be objects and states of affairs.



Ludwig Wittgenstein

([1921] 1961) *Tractatus logico-philosophicus*.

Pears/McGuinness translation.

INDEPENDENCE

Example (A set-theoretical reading)

Let $W = \{s_1, s_2, \dots\}$ be the set of all possible simple states of affairs and let $atom(\mathcal{L}) = \{p_1, p_2, \dots\}$ be the set of atomic propositions of language \mathcal{L} . Picture relation \cong is a relation between atomic propositions and simple states of affairs; reality is a function whose domain is $atom(\mathcal{L}) \times \wp W$ and whose range is the set $\{\text{true}, \text{false}\}$:

$$\text{reality}(\ulcorner p \urcorner, w) = \begin{cases} \text{true} & \text{if for some } s \in W: \ulcorner p \urcorner \cong s \text{ and } s \in w, \\ \text{false} & \text{if for all } s \in W: \text{either not } \ulcorner p \urcorner \cong s \text{ or not } s \in w. \end{cases}$$

Independence principle in metaphysics and its non-exclusiveness consequence

Metaphysics Any $w \subseteq S$ is a world.

Non-exclusiveness No simple state of affairs excludes no simple state of affairs.

CONJUNCTIVE COMPOSITION

- Although there is no explicit statement in *Tractatus* on how states of affairs can be combined, it is at least consistent to assume that *Tractatus* metaphysics accepts THE PRINCIPLE OF UNIQUE MODE OF COMPOSITION OF STATES OF AFFAIRS into facts, of which a world is the maximal one.
- The unique mode of composition of states of affairs is CONJUNCTIVE MODE.
- An explicit formulation of the principle can be found in the literature. E.g. philosopher Armstrong accepts conjunctive mode and rejects negative and disjunctive modes of composition of states of affairs (cf. the quote to the right).

We require [...] a distinction between *atomic* and *molecular* states of affairs. Because negative and disjunctive states of affairs will be rejected, molecular states of affairs are all of them mere *conjunctions* of atomic states of affairs.



David Malet Armstrong
(1997) *A World of States of Affairs*.
p. 19, Cambridge University Press.

THE THIRD POSTULATE: ON MEREOLGY OF KNOWLEDGE

- The third, epistemological and linguistic postulate ascribes unlimited expressive power to the language: a complete true description of the reality is possible.
- One can safely assume that the Tractarian notion of complete description includes the thesis that knowledge of each atomic proposition is possible. If so, the postulate of possibility of complete true description implies the following epistemic proposition:
Knowledge is complete if a disjunction being known implies a disjunct being known, i.e.

$$\mathbf{K}(p \vee q) \rightarrow (\mathbf{K}p \vee \mathbf{K}q)$$

4.11 The totality of true propositions is the whole of natural science (or the whole corpus of the natural sciences).

4.12 Propositions can represent the whole of reality, but they cannot represent what they must have in common with reality in order to be able to represent it—logical form.



Ludwig Wittgenstein

([1921] 1961) *Tractatus logico-philosophicus*.

Pears/McGuinness translation.

QUANTUM THEORY AND THE THREE POSTULATES

- Neither of the three Tractarian postulates withstands confrontation with the theory of quantum mechanics.
- If the whole of knowledge is subordinated to requirements of external consistency of its parts, as I think it is, then revision either of Tractarian or quantum theory is needed.

In addition to internal consistency we require, or at least strive for, *external consistency*, i.e. for the consistency of every factual theory with every (noncompeting) theory in the same field or in the adjacent fields.



Mario Bunge

([1967] 1998) *Philosophy of Science: From Problem to Theory*. Vol. 1, . rev. ed
New Brunswick: Transaction Publishers.

ARE STATES OF AFFAIRS INDEPENDENT?

- Let us identify state of affairs with the descriptum of “basic physical propositions” of the form $m(s) = r$ or “the magnitude m has the value r in the system S ” and call it OBSERVABLE STATE OF AFFAIRS.
- The proposition $m(S) = r$ is not about, does not describe relation of m as holding between physical system s on one side and element r of measuring structure on the other.
- An observable state of affairs is not a relation between elements of two structures, empirical and abstract. Rather, it is that which is described in that way: an observable state of affairs is descriptum($m(S) = r$).

Statements of the form $m(s) = r$ — “the magnitude m has the value r in the system S ”—are the sorts of statements we shall call *basic physical propositions*...



Hilary Putnam

([1968] 1975)

The logic of quantum mechanics.

Mathematics, Matter and Method.

Philosophical Papers, vol. I. Cambridge:

Cambridge University Press

... how absurd it would be to call an “experimental proposition,” the assertion that the angular momentum ... of the earth around the sun was at a particular instant a rational number!



Garrett Birkhoff and John von Neumann

(1936)

The logic of quantum mechanics.

The Annals of Mathematics 37: 823–843 C

INDEPENDENCE AND UNCERTAINTY

- Let us denote by $\text{descriptum}(m(S) = r)$ the observable state of affairs described by basic physical proposition $m(S) = r$. Let us denote by M_i the set of all possible observable states of affairs of a system S with respect to magnitude m_i , $M_i^S = \{\text{descriptum}(m_i(S) = x) \mid \exists x m_i(S) = x\}$.
- According to [a], for any state from M_i there is a possible world to which it belongs.
- The Tractatus independence principle can be restated: for any $x \in M_i^S$ and any $y \in M_j^S$ there is a world w that contains them, i.e. $\{x, y\} \subseteq w$.
- According to uncertainty principle the Tractatus independence principle fails:

for some observable states of affairs
 $x \in M_{\text{position}}^S$ and $y \in M_{\text{momentum}}^S$ there is no
 world w that contains them.

‘Orthogonality’ between rays is an important concept for quantum mechanics: orthogonal rays refer to states that are *independent* of one another. [a] The different possible position states of a particle are all orthogonal to one another, as are all possible momentum states. But position states are not orthogonal to momentum states.
 ... The general rule **R** for a measurement (or observation) requires that the different aspects of a quantum system that can be simultaneously magnified to the classical level—and between which the system must then choose—must always be orthogonal.



Roger Penrose
 (1999)

*The Emperor's New Mind:
 Concerning Computers, Minds, and
 the Laws of Physics.*

Oxford University Press

NON-UNIQUENESS OF MODE OF COMPOSITION

- Let us denote by $\text{descriptum}(p \sqcup q \sqcup \dots)$ the unobservable state of affairs described by “quantum disjunction” of mutually exclusive (‘orthogonal’) basic physical propositions p, q, \dots .
- No such disjunctively composed state of affairs is allowed by Tractarian postulate of unique mode of composition.
- Furthermore, a different kind of picture relation becomes needed.

In quantum mechanics ... the state of a system is represented by an element in a complex Hilbert space ... In particular, this means that for any two states (e.g., for a spin-1/2 system, the states of spin-up and spin-down in direction x), an arbitrary linear combination (or ‘superposition’) is also a possible state:

$$(1) |\varphi\rangle = \alpha|+x\rangle + \beta|-x\rangle.$$



Guido Bacciagaluppi

(2009) Is logic empirical?

Handbook of Quantum Logic and Quantum Structures: Quantum Logic, K. Engesser, D.M. Gabbay and D. Lehmann (eds.).

Elsevier.

A SKETCH OF QUANTUM LANGUAGE-WORLD RELATION

- By allowing (in metaphysics) disjunctive composition of states of affairs and by allowing (in logic) picture relation between quantum disjunction and disjunctive states of affairs, we might arrive at the following definition for the new logico-metaphysical situation:

$$\text{reality}(\ulcorner p \sqcup q \urcorner, w) = \begin{cases} \text{true} & \text{if for some } s \in W: \begin{cases} \ulcorner p \urcorner \cong s \text{ and } s \in w, \\ \text{or } \ulcorner q \urcorner \cong s \text{ and } s \in w, \\ \text{or } \ulcorner p \sqcup q \urcorner \cong s \text{ and } s \in w, \end{cases} \\ \text{false} & \text{, otherwise.} \end{cases}$$

- According to quantum mechanics disjunctive states of affairs are possible: they are either explicitly permitted as in “ortodox interpretation”, or at least constitute the ontological commitments of the theory.

Suppose I do measure the position of the particle, and I find it to be at the point C . Question: Where was the particle just before I made the measurement? ... [...] 2. **The orthodox position:** The particle wasn't really anywhere. It was the act of measurement that forced the particle to “take a stand” (though how and why it decided on the point C we dare not ask). [...] This view (the so-called Copenhagen interpretation) is associated with Bohr and his followers. Among physicists it has always been the most widely accepted position.



David J. Griffiths

(1999) *Introduction to Quantum Mechanics*. Prentice Hall.

POSTULATE ON MEREOLGY OF KNOWLEDGE

- The immediate consequence of (i) accepting disjunctive mode of composition of states of affairs and of (ii) adopting a wide notion of picture relation— is that —Tractarian notion of complete true description must be rejected.
- It is possible that maximal knowledge of a disjunction does not imply knowledge of any disjunct provided that the disjunction in question is a quantum one:
Knowledge can be complete even if no disjunct of a known disjunction is known, i.e. the following situation is possible with respect to complete knowledge:

$$\mathbf{K}(p \sqcup q) \wedge \neg \mathbf{K}p \wedge \neg \mathbf{K}q$$

Schrödinger's mereological principle of ideal knowledge

Maximal knowledge of a total system does not necessarily include total knowledge of all its parts . . .



Erwin Schrödinger

([1935] 1980)

The present situation in quantum mechanics

Proceedings of the American

Philosophical Society **124**: 323–338.

IS LOGIC EMPIRICAL?

- It seems that we are required to accept empirical considerations in logic, i.e. to maintain external consistency between logic and quantum theory.
- The requirement seems self-evident from the pluralistic standpoint in philosophy of logic.
- The many varieties of quantum logic have been developed starting with Birkhoff-von Neumann algebraic quantum logic, where basic experimental proposition is interpreted as one-dimensional subspace of a Hilbert space \mathcal{H} (ray), quantum disjunction is algebraically interpreted as join operation (delivering least upper bound), $\llbracket \varphi \sqcup \psi \rrbracket = \text{lub}(\llbracket \varphi \rrbracket, \llbracket \psi \rrbracket)$; and mathematico-physically as span of subspaces, $\llbracket \varphi \sqcup \psi \rrbracket = \{|x\rangle + |y\rangle \mid |x\rangle \in \llbracket \varphi \rrbracket \text{ and } |y\rangle \in \llbracket \psi \rrbracket\}$.

All a priori systems, as soon as they are applied to reality, become natural-science hypotheses which have to be verified by facts in a similar way as is done with physical hypotheses.



Jan Łukasiewicz

[1936] *Logistic and philosophy*.

Logic is empirical as geometry. It makes as much sense to speak of ‘physical logic’ as of ‘physical geometry’. We live in a world with a non-classical logic.



Hilary Putnam

[1968] The logic of quantum mechanics.

IS QUANTUM LOGIC THE LOGIC OF QUANTUM MECHANICS?

- In its semantics Birkhoff-von Neumann quantum logic uses quantum mechanics mathematical model as the basis for semantics¹ Thus it directly accommodates the theoretical ideas of quantum mechanics.
- Applying and modifying Łukasiewicz thesis we get: *Laws of classical logic such as distributivity of conjunction over disjunction, as soon as they are applied to the model of quantum reality, become natural-science hypotheses and share the destiny of physical hypotheses they accommodate; and some of the laws, such as distributivity, fail.* In that sense, quantum logic is an empirical theory.
- Nevertheless, the claim that quantum logic is THE LOGIC turns out to be obviously false when confronted with the fact of logical pluralism.
- Restricting the scope of a logic to the theory it tries to accommodate, we may ask is the quantum logic—the logic of the language of quantum theory?
- If so, the quantum disjunction is THE DISJUNCTION of quantum theory. If not, then we must allow more than one disjunction-like connective in the logic of the language of quantum theory.

¹Basic experimental proposition are interpreted as one-dimensional subspace of Hilbert space. A proposition is true of a system iff the vector that represents the system lies in the subspace assigned to proposition by the interpretation.

HOW MANY DISJUNCTIONS FOR THE QM LANGUAGE?

- I am inclined to go along with Maudlin's remark that the language of a theory may use more than one logical term from the same category.
- In particular, the language of quantum mechanics has two disjunctive operators:
 - quantum disjunction \sqcup , and
 - classical disjunction, \vee .
- This suggests a richer notion of logical pluralism: not only (i) logical terms can differ in meaning with respect to theories or discourses, but also (ii) similar logical terms can have different meaning in the language of one and the same theory.

Two disjunction like operators

Using the usual technique for deriving predictions from quantum states, one would say that if “The particle is at $r_1 \sqcup$ the particle is at r_2 ” is true, then an experiment designed to locate the particle will be certain to “find” it either at r_1 or at r_2 , where the “or” is the classical disjunction. That is, the truth of the join of the propositions implies the truth of a classical disjunction regarding the result of a “measurement”. This is an implication from a proposition using a quantum connective to a proposition formulated with a classical connective — it is an inference that cannot even be formulated if the classical disjunction is unavailable. A fortiori it cannot be an inference which could in any way suggest that the classical connective is *expendable*.



Tim Maudlin

(2005) The tale of quantum logic.

Hilary Putnam, Y. Ben-Menahem (ed.), Cambridge University Press.

HOW MANY DISJUNCTIONS FOR THE QM LANGUAGE?

- Quantum dynamic logic is a logic that can accommodate the varieties of disjunction like operators within the same formal system.
- (Superposition of basic experimental states p and q is a weakest precondition for the success of the actions that test whether p or q holds)

Quantum dynamic logic

From the philosophical perspective, [the] approach [of dynamic quantum logic] sheds new light on the old debate about the nature of quantum logic. In contrast to common opinions (e.g. Putnam 1968), we found no empirical ground to abandon any principles of classical logic. [The approach of dynamic quantum logic] can explain all the non-classical features of the quantum behavior as a consequence of non-classical “dynamics” of quantum information. [This] setting does not require any change of the classical laws governing “static” information. It is non-classical nature of quantum-information-extracting actions (quantum tests) that explains strangeness of quantum behavior.



Alexandru Baltag and Sonja Smets

(2012) The dynamic turn in quantum logic.

[Synthese186:753–773.](#)

$$(p \sqcup q) \rightarrow [(p \vee q)?](p \vee q)$$