Logical Representational Space of Intentionality

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Semantically characterized logics

- A semantically characterized logic could be defined as triple:
  - language conceived as a set of sentences, Φ [Formulas]
  - set of structures (interpretations), Σ [Structures]
  - satisfaction relation, $\models$.

Definition

Logic L is a triple $\langle \Phi, \Sigma, \models \rangle$.

Satisfaction

Definition

Satisfaction relation is relation

$$\models \subseteq \Sigma \times \Phi.$$

Definition

The set of models (the intension of) $Mod\left(\Gamma, \Sigma\right)$ of a set Γ of formulas within a set Σ of structures with respect to satisfaction relation $\models$ is:

$$Mod\left(\Gamma, \Sigma\right) = \left\{\sigma \in \Sigma \mid \forall \varphi \left(\varphi \in \Gamma \rightarrow \sigma \models \varphi\right)\right\}.$$

Remark

The precise notation would require the satisfaction relation to be mentioned. For example, for $L_a = \langle \Phi_a, \Sigma_a, \models_a \rangle$, $\Gamma_a \subseteq \Phi_a$ and $\Delta \subseteq \Sigma_a$ we should write $Mod\left(\Gamma_a, \Delta_a, \models_a\right)$. From the context it will be obvious which satisfaction relation is being used, so we will write $Mod\left(\Gamma_a, \Delta_a\right)$ instead.

Satisfiability and (Tarskian) consequence

Definition

Set Γ is satisfiable in Σ iff $Mod\left(\Gamma, \Sigma\right) \neq \emptyset$.

Definition

Consequence relation $\Vdash \subseteq \wp\Phi \times \Phi$ for a logic $\langle \Phi, \Sigma, \models \rangle$ is the relation

$$\Gamma \Vdash \varphi \text{ iff } Mod\left(\Gamma, \Sigma\right) \subseteq Mod\left(\{\varphi\}, \Sigma\right)$$

Remark

The consequence relation defined in this way is Tarskian consequence relation. Its properties include:

1. *reflexivity,* $\{\varphi\} \Vdash \varphi$,
2. *monotonicity, if* $\Gamma \subseteq \Gamma'$ *and* $\Gamma \Vdash \varphi$, *then* $\Gamma' \Vdash \varphi$,
3. *transitivity, if for all* $\psi \in \Delta$, $\Gamma \Vdash \psi$ *and* $\Delta \Vdash \varphi$, *then* $\Gamma \Vdash \varphi$.

Sublogic in GMV definition

Definition (Garcia-Matos and Vaananen)

An abstract logic $L_1 = \langle \Phi_1, \Sigma_1, \models_1 \rangle$ is a sublogic of another abstract logic $L_2 = \langle \Phi_2, \Sigma_2, \models_2 \rangle$, in symbols

$$L_1 \leq L_2,$$

if there are:
(i) a sentence $\kappa \in \Phi_2$,
and functions (ii) $\pi : \Sigma_2 \longrightarrow \Sigma_1$ and (iii) $\tau : \Phi_1 \longrightarrow \Phi_2$ such that
1. $\forall \sigma_1 \left[\sigma_1 \in \Sigma_1 \rightarrow \exists \sigma_2 \left(\sigma_2 \in \Sigma_2 \wedge \pi(\sigma_2) = \sigma_1 \wedge \sigma_2 \models_2 \kappa \right) \right]$ and
2. $\forall \varphi_1 \forall \sigma_2 \left[\begin{array}{c} (\varphi_1 \in \Phi_1 \wedge \sigma_2 \in \Sigma_2) \rightarrow \\ (\sigma_2 \models_2 \kappa \rightarrow (\sigma_2 \models_2 \tau(\varphi_1) \leftrightarrow \pi(\sigma_2) \models \varphi_1)) \end{array} \right]$

Some comments on GMV definition

- GMV definition mentions, so to speak, a "translational constant" κ.
  - The other way to think about GMV π projection would be to restrict the domain to those structures (interpretations) which model κ, i.e. $\pi : Mod(\{\kappa\}, \Sigma_1) \to \Sigma_2$.
- It can be easily shown that GMV sublogic relation preserves *sequitur* and *non sequitur* semantic relations.

Some remarks on extent of coordination

For $\Gamma \subseteq \Phi$ we write $\tau(\Gamma)$ as a shorthand notation for $\{\tau(\varphi) \mid \varphi \in \Gamma\}$.

- GMV definition of sublogic relation does not enable proof of existence of an image of "horizontal" (i. e. *sequitur* and *non sequitur*) semantic relations in a target logic:

$$\Gamma_1 \models_1 \varphi_1 \nRightarrow \tau(\Gamma_1) \models_2 \tau(\varphi_1)$$

Exploiting only a part of target logic

Example

According to the definition it may well be the case that although $\Gamma_1 \models_1 \varphi_1$, still for some $\sigma_2 \in \Sigma_2$ such that $\sigma_2 \not\models_2 \kappa$ it holds that $\sigma_2 \models_2 \tau(\psi_1)$ for all $\psi_1 \in \Gamma_1$ and $\sigma_2 \not\models_1 \varphi_1$ since there is no guarantee that coordination holds for $\Sigma_2 - Mod(\kappa, \Sigma_2)$.

- On the other hand the GMV definition does guarantee the coordination for $\models_1$ and $\models_2^*$ where $\models_2^* \subseteq Mod(\{\kappa\}, \Sigma_2) \times \Phi_2$ and, therefore, the existence of an image in target logic of *sequitur* and *non sequitur* relations from source logic.
- We will write $\Vdash^*$ for consequence relation that is defined using $\models^*$relation in a obvious way.

Using GMV definition to prove *sequitur* representation

Proposition

If $L_1 \leqq L_2$*, then* $\Gamma_1 \Vdash_1 \varphi_1 \Rightarrow \tau(\Gamma_1) \Vdash_2^* \tau(\varphi_1)$.

Proof.

Assume $\Gamma_1 \Vdash_1 \varphi_1$.
Let σ_2 be any structure such that $\sigma_2 \in Mod(\tau(\Gamma_1) \cup \{\kappa\}, \Sigma_2)$.
By GMV definition, it holds that
$\sigma_2 \in Mod(\tau(\Gamma_1) \cup \{\kappa\}, \Sigma_2) \Longleftrightarrow \pi(\sigma_2) \in Mod(\Gamma_1, \Sigma_1)$.
Therefore, $\pi(\sigma_2) \in Mod(\Gamma_1, \Sigma_1)$.
Since $\Gamma_1 \Vdash_1 \varphi_1$, $\pi(\sigma_2) \in Mod(\{\varphi_1\}, \Sigma_1)$.
By GMV definition, it holds that
$\sigma_2 \in Mod(\tau(\varphi_1), \Sigma_2) \Longleftrightarrow \pi(\sigma_2) \in Mod(\varphi_1, \Sigma_1)$.
Therefore, $\sigma_2 \in Mod(\tau(\varphi_1), \Sigma_2)$. □

Proving *non sequitur* representation

- In order to prove the existence of the image of non sequitur relation we will need the fact that π is surjective function.

Proposition

If $L_1 \leqq L_2$*, then* $\Gamma_1 \nVdash_1 \varphi_1 \Rightarrow \tau(\Gamma_1) \nVdash_2^* \tau(\varphi_1)$.

Proof.

For contraposition, assume $\tau(\Gamma_1) \cup \{\kappa\} \Vdash_2^* \tau(\varphi_1)$.
Let σ_1 be any structure such that $\sigma_1 \in Mod(\Gamma_1, \Sigma_1)$.
Since π is surjective and defined for $Mod(\{\kappa\}, \Sigma_2)$,
$\exists \sigma_2 [\sigma_2 \in Mod(\{\kappa\}, \Sigma_2) \wedge \pi(\sigma_2) = \sigma_1]$.
Let σ_2 be such a structure.
By GMV definition, it holds that
$\sigma_2 \in Mod(\tau(\Gamma_1), \Sigma_2) \iff \pi(\sigma_2) \in Mod(\Gamma_1, \Sigma_1)$.
Given that $\pi(\sigma_2) = \sigma_1$, we get $\sigma_2 \in Mod(\tau(\Gamma_1), \Sigma_2)$.
Since $\tau(\Gamma_1) \Vdash_2^* \tau(\varphi_1)$, $\sigma_1 \in Mod(\tau(\varphi_1), \Sigma_2)$.
By GMV definition, it holds that
$\sigma_1 \in Mod(\tau(\varphi_1), \Sigma_2) \iff \pi(\sigma_1) \in Mod(\varphi_1, \Sigma_1)$.
Therefore, $\sigma_1 \in Mod(\{\varphi_1\}, \Sigma_1)$. □

GMV sublogic relation enables *sequitur/non sequitur* representation (in a restricted "space").

Theorem

If $L_1 \leqq L_2$, *then* $\Gamma_1 \Vdash_1 \varphi_1 \Leftrightarrow \tau(\Gamma_1) \Vdash_2^* \tau(\varphi_1)$

Proof.

Propositions 8 and 9. □

A congenial notion

Definition (Mossakowski, Diaconescu and Tarlecki)

Corridor $\langle \tau, \pi \rangle$ is a pair of functions:
(i) sentence translation function $\tau : \Phi_1 \longrightarrow \Phi_2$,
(ii) "model reduction function": $\pi : \Sigma_2 \longrightarrow \Sigma_1$ such that

$$\sigma_2 \models_2 \tau(\varphi_1) \Leftrightarrow \pi(\sigma_2) \models_1 \varphi_1$$

for logics $L_1 = \langle \Phi_1, \Sigma_1, \models_1 \rangle$ and $L_2 = \langle \Phi_2, \Sigma_2, \models_2 \rangle$.

One-way corridor

Proposition

If there is MDT corridor between logics, then τ is a translation that preserves sequitur relation.

Remark

Since MDT corridor does not require π to be surjective, non sequitur image may fail to obtain in the target logic. Consider σ_1 such that $\sigma_1 \in Mod\left(\Gamma_1, \Sigma_1\right)$, $\sigma_1 \notin Mod\left(\{\varphi_1\}, \Sigma_1\right)$ and $\sigma_1 \notin range\left(\pi\right)$. Since π is not surjective the existence of such a structure cannot be precluded.

A comparison

- In proving the existence of sequitur image MDT corridor poses a stronger requirement than GMV sublogic relation. For MDT corridor we can add any truth ψ of L_1 [*i.e.* $Mod(\{\psi\}, \Sigma_1) = \Sigma_1$] as "translation constant" and thus transform MDT translation into GMV translation.

Provability?	*sequitur*	*non sequitur*
GMV	YES using translation constant κ since π is total on $Mod(\{\kappa\}, \Sigma_1)$	YES since π is surjective
MDT	YES since π is total	NO

Pictures, representations, images etc.

- For the purpose of establishing "picture relation" between two logics it would be harmless to introduce a special translation function for the set of formulas.
  - An example: Translation function for formulas $\tau^{\Phi} : \Phi_1 \rightarrow \Phi_2$. Translation function for sets of formulas $\tau^{\wp\Phi} : \wp\Phi_1 \rightarrow \wp\Phi_2$ where

$$\tau^{\wp\Phi}(\Gamma_1) = \{\kappa\} \cup \{\tau(\varphi_1) \mid \varphi_1 \in \Gamma_1\}$$

Remark

We are interested in "one way picture relation" with the following properties: for any set $\Gamma_1 \cup \{\varphi_1\} \subseteq \Phi_1$ there is a set $\Gamma_2 \cup \{\varphi_2\} \subseteq \Phi_2$ such that $\Gamma_1 \Vdash_1 \varphi_1$ iff $\Gamma_2 \Vdash_2 \varphi_2$. From that perspective it is not important whether we take the translation function to deliver also a translational constant $\tau^{\wp\Phi}(\Gamma_1) = \{\kappa\} \cup \{\tau(\varphi_1) \mid \varphi_1 \in \Gamma_1\}$.

Weakening the requirements

- The surjection condition can be weakened for the logics with strong (classical) negation:

Definition

A logic $L = \langle \Phi, \Sigma, \models \rangle$ has a strong (classical) negation iff for any $\varphi \in \Phi$ there is a $\psi \in \Phi$ such that

$$\begin{aligned} \text{(i)}\ \mathit{Mod}\left(\{\varphi\}, \Sigma\right) \cap \mathit{Mod}\left(\{\psi\}, \Sigma\right) &= \emptyset, \text{ and} \\ \text{(ii)}\ \mathit{Mod}\left(\{\varphi\}, \Sigma\right) \cup \mathit{Mod}\left(\{\psi\}, \Sigma\right) &= \Sigma. \end{aligned}$$

The notation $\neg\varphi$ will be used for classical negation of φ.

Parsimonious projection

- The weakened condition is the one that requires that for any set of models for a satisfiable set of sentences from the source logic there is projection that picks at least one such model.

Definition

For logics $L_1 = \langle \Phi_1, \Sigma_1, \models_1 \rangle$ and $L_2 = \langle \Phi_2, \Sigma_2, \models_2 \rangle$ **parsimonious projection** π is a projection $\pi : \Sigma_2 \longrightarrow \Sigma_1$ such that for any $\Gamma_1 \subseteq \Phi_1$ it holds that

$$Mod\left(\Gamma_1, \Sigma_1\right) \neq \emptyset \rightarrow \exists \sigma_2 \left[\sigma_2 \in \Sigma_2 \wedge \pi\left(\sigma_2\right) \in Mod\left(\Gamma_1, \Sigma_1\right)\right]$$

A theorem to prove

If there is corridor $\langle \tau, \pi \rangle$ between logics L_1 and L_2, and π is a parsimonious projection restricted to models of "translational constant" κ, and source logic L_1 has strong negation, then τ is a semantic relations preserving translation.

A theorem on corridor with parsimonious projection

Theorem

Let logic L_1 be a logic with strong negation. Then for any logic L_2 it holds that,
if there are:
(i) a sentence $\kappa \in \Phi_2$, and
(ii) **parsimonious** *function $\pi : Mod\left(\{\kappa\}, \Sigma_2\right) \longrightarrow \Sigma_1$, and*
(iii) function $\tau : \Phi_1 \longrightarrow \Phi_2$
such that
(iv) $\pi\left(\sigma_2\right) \models_1 \varphi_1$ iff $\sigma_2 \models_2 \tau\left(\varphi_1\right)$ for any $\varphi_1 \in \Phi_1$ and $\sigma_2 \in Mod\left(\{\kappa\}, \Sigma_2\right)$,
then:
τ is a semantic relations preserving translation,
i. e. $\Gamma_1 \models_1 \varphi_1$ is equivalent to $\tau\left(\Gamma_1\right) \models_2^* \tau\left(\varphi_1\right)$.

Left to right part

Proof.

Assume $\Gamma_1 \Vdash_1 \varphi_1$.
Let σ_2 be any structure such that $\sigma_2 \in \mathit{Mod}\left(\tau\left(\Gamma_1\right), \mathit{Mod}\left(\{\kappa\}, \Sigma_2\right)\right)$.
By (iv), it holds that $\sigma_2 \in \mathit{Mod}\left(\tau\left(\Gamma_1\right), \mathit{Mod}\left(\{\kappa\}, \Sigma_2\right)\right)$ iff $\pi\left(\sigma_2\right) \in \mathit{Mod}\left(\Gamma_1, \Sigma_1\right)$.
Therefore, $\pi\left(\sigma_2\right) \in \mathit{Mod}\left(\Gamma_1, \Sigma_1\right)$.
Since $\Gamma_1 \Vdash_1 \varphi_1$, $\pi\left(\sigma_2\right) \in \mathit{Mod}\left(\{\varphi_1\}, \Sigma_1\right)$.
By (iv) again, $\sigma_2 \in \mathit{Mod}\left(\tau\left(\varphi_1\right), \mathit{Mod}\left(\{\kappa\}, \Sigma_2\right)\right)$ iff $\pi\left(\sigma_2\right) \in \mathit{Mod}\left(\varphi_1, \Sigma_1\right)$.
Therefore, $\sigma_2 \in \mathit{Mod}\left(\tau\left(\varphi_1\right), \mathit{Mod}\left(\{\kappa\}, \Sigma_2\right)\right)$. $\square$

Right to left part

Proof.

Assume $\tau(\Gamma_1) \models_2^* \tau(\varphi_1)$.
For *reductio ad absurdum* assume $\Gamma_1 \not\models_1 \varphi_1$.
L_1 has strong negation, so there is a sentence $\neg\varphi \in \Phi_1$.
From $\Gamma_1 \not\models_1 \varphi_1$ we get $Mod(\Gamma_1 \cup \{\neg\varphi\}, \Sigma_1) \neq \emptyset$.
Since π is a parsimonious projection, then there is $\sigma_2 \in \Sigma_2$ such that $\pi(\sigma_2) = \sigma_1$ (for some $\sigma_1 \in \Sigma_1$) and $\pi(\sigma_2) \in Mod(\Gamma_1 \cup \{\neg\varphi\}, \Sigma_1)$.
By (iv), we get $\sigma_2 \models_2^* \tau(\Gamma_1)$ and $\sigma_2 \models_2^* \tau(\neg\varphi_1)$.
Since $\sigma_2 \models_2 \tau(\Gamma_1)$, by the hypothesis of this conditional proof we get $\sigma_2 \models_2^* \tau(\varphi_1)$.
Using (iv) again we get $\pi(\sigma_2) \models_1 \neg\varphi_1$, and that is not possible for strong negation since $\pi(\sigma_2) \in Mod(\Gamma_1 \cup \{\neg\varphi\}, \Sigma_1)$.
We have arrived at the contradiction as we wanted to. □

Philosophical motivation

- Measurement theory has received considerable attention in recent times in its new role
- The new role of measurement theory is to provide another approach to the semantics of the propositional attitudes reports, or, to put it in the different terms, to semantics of the language of intentionality.

Davidson's semantic idea

The analogy with beliefs is this. Just as in measuring weight we need a collection of entities which have a structure in which we can reflect the relations between weighty objects, so in attributing states of belief (and other propositional attitudes), we need a collection of entities related in ways that will allow us to keep track of the relevant properties of and relations among the various psychological states.
What is Present to the Mind (1991)
The most conspicuous features of the individual attitudes are their basically rational structures (if someone believes that everything is white, that person has a belief that entails that snow is white), and their relations to the world (the belief that snow is white is true if and only if snow is white).
Entities that have these required properties are our sentences, and it is not clear that any other set of entities will do as well (except utterances).
Indeterminism and Antirealism (1997)

Suppes' logic of measurement

- The measurement-theoretic approach in semantics of language of intentionality is founded in theory of measurement developed by Patrick Suppes (together with his colleagues, like Dana Scott, Joseph Zinnes and others).
- According to Suppes, in measurement two systems, the one being measured $E = \langle A, R_1, ..., R_n \rangle$, with $R_i \subseteq A^m$ where m is the arity of the predicate R_i, and the other, the measuring one $M = \langle B, S_1, ..., S_n \rangle$, are being correlated by a function $f : A \longrightarrow B$ in such a way that for any $x_1, ..., x_n \in A$

$$\text{if } R_i\left(x_1, ..., x_n\right), \text{ then } S_i\left(f\left(x_1\right), ..., f\left(x_n\right)\right).$$

Different ways of doing the same

- Suppes considers the cases where representational system $M = \langle B, S_1, ..., S_m \rangle$ is (a subsystem of) some numerical system.
- In numerical representation different systems may do equally well.
- If so, then some numerical function $t : B \to N$ (where N is a set of numbers and $B \subseteq N$) can be defined in such way that it generates a new homomorphic image $M^* = \langle B^*, S_1^*, ..., S_n^* \rangle$ of E

$$[R_i (x_1, ..., x_n) \Rightarrow S_i (f (x_1), ..., f (x_n))]$$

implies

$$[R_i (x_1, ..., x_n) \Rightarrow S_i^* (t (f (x_1)), ..., t (f (x_n)))]$$

- The type of a scale $\langle E, M, f \rangle$ is determined by its "admissible transformations".

Measurement, representation and representational type

- For the measurement to be logically sound, two theorems must be proved.
- First, representation theorem must show that empirical system is homomorphic to a chosen numerical representational system.
- Second, uniqueness theorem must show that scale type is well chosen so that admissible transformations do create homomorphic images of empirical system.

Meaningfulness

- In the end there is the principle of meaningfulness of numerical statements in measurement.

  A numerical statement is meaningful if and only if its truth (or falsity)is constant under admissible scale transformations of any of its numerical assignments...
  Basic Measurement Theory (1963)

Using the approach

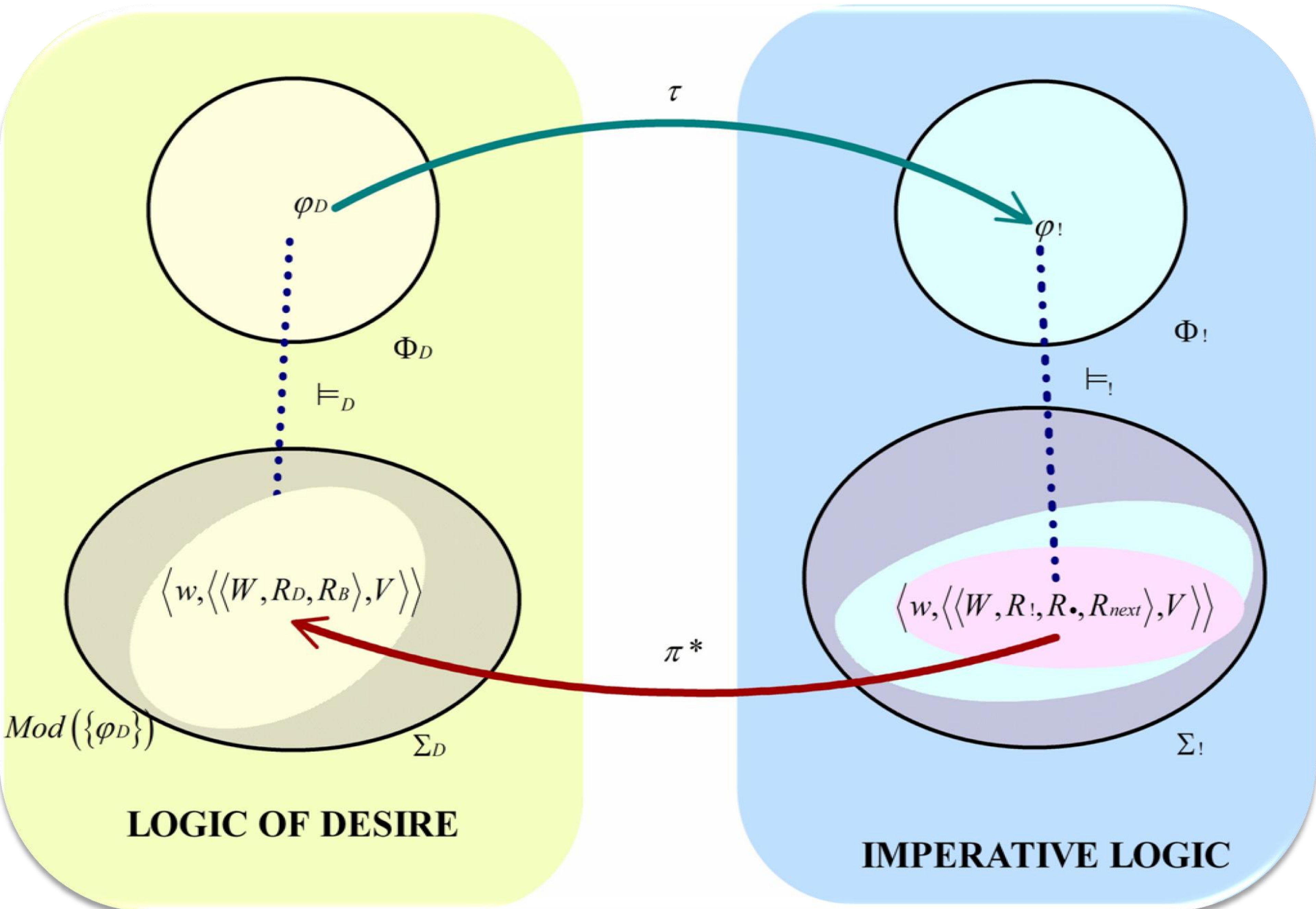
- Robert Matthews has developed an elaborate approach along this Davidson-Suppes lines in his book *The Measure of Mind* (OUP, 2007), trying to investigate the relation of "real" psychological states and their representational space.
- I consider Matthews approach unsatisfactory on several accounts.
    - The main objection to his application of measurement theoretic semantics is the reduction of representational space to relations between assertoric sentences.

Working hypothesis

- In my opinion the measurement theoretical approach should relate the logic of the language of intentionality with the logic of speech acts (or, rather, logic of "mood designated sentences").

A sketch

- In a sketch, the source (measured system) is a system of intentionality $L_{INT} = \langle \Phi_{INT}, \Vdash_{INT}, \nVdash_{INT} \rangle$, $\Vdash_{INT} \subseteq \wp\Phi_{INT} \times \Phi_{INT}$ and $\nVdash_{INT} = \wp\Phi_{INT} \times \Phi_{INT} - \Vdash_{INT}$.
- The target system, the representational space is a logic of speech acts $L_{SA} = \langle \Phi_{SA}, \Vdash_{SA}, \nVdash_{SA} \rangle$.
- The role of homorphic function is played by translation function τ and we have to prove that semantic relations (sequitur and non sequitur relations) are preserved by τ.
- For this variant of representation theorem we may relay on our theorem using parsimonious projection and translation constant given that logic of system of intentionality has strong negation.


τ
φD
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⟨w,⟨⟨W, RD, RB⟩,V⟩⟩
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π*
Mod({φD})
ΣD
Σ!
LOGIC OF DESIRE
IMPERATIVE LOGIC