Dialetheic deontic logic and language-created reality

Berislav Žarnić
Faculty of Humanities and Social Sciences, University of Split

Speaking of the Ineffable, East and West
Logic Conference

Center for Logic, Methodology and Decision Theory, University of Rijeka
Croatian Logic Association

Rijeka, 12th June 2015

Overview

Dialetheic deontic logic

- In 1987. Priest presented the system of dialetheic deontic logic, a logic with three values and free from "explosion principle".

  Graham Priest (2006).
  In contradiction : a study of the transconsistent. — Expanded edition.
  (First edition published 1987 by Martinus Nijhoff Publishers). Oxford: Oxford University Press.

- The deontic operator $\mathbf{O}$ is receives two interpretations in a given possible world w: it has an extension $\omega^+(w) \subseteq \mathcal{L}$ and anti-extension $\omega^-(w) \subseteq \mathcal{L}$ which are exhaustive $\omega^+(w) \cup \omega^-(w) = \mathcal{L}$ but not in general exclusive, i.e., it is allowed in the system that $\omega^+(w) \cap \omega^-(w) \neq \varnothing$.
- The extension of $\mathbf{O}$ is closed under entailment, i.e., if α entails β and $\alpha \in \omega^+(w)$, the $\beta \in \omega^+(w)$, and closed under adjunction, if $\alpha \in \omega^+(w)$ and $\beta \in \omega^+(w)$, the $(\alpha \wedge \beta) \in \omega^+(w)$. It is allowed for the extension to be inconsistent, i.e., $\alpha \in \omega^+(w)$ and $\neg\alpha \in \omega^+(w)$. Since the DDL logic is free from "explosion principle" the fact $\alpha \in \omega^+(w)$ and $\neg\alpha \in \omega^+(w)$ does not imply $\omega^+(w) = \mathcal{L}$.

Dialetheic deontic logic

Interpretation

Interpretation is 6-tuple $\langle G, W, R, \omega^+, \omega^-, v\rangle$ where

1. P is a set of propositional letters, F is the set of formulas,
2. $W \neq \varnothing$,
3. $R \subseteq W \times W$,
4. $G \in W$,
5. $v : W \times P \to \{\{1\}, \{0\}, \{1, 0\}\}$ where v extends to F by definitions of dialetheic propositional logic,[1]
6. $\omega^+ : W \to \wp F$, $\omega^- : W \to \wp F$ where $\omega^+(w) \cup \omega^-(w) = F$ for all $w \in W$.

Graham Priest (2006).
In contradiction : a study of the transconsistent. — Expanded edition.
(First edition published 1987 by Martinus Nijhoff Publishers). Oxford: Oxford University Press.

[1] Alternatively (Part IV) interpretation ρ is a relation between formulas and values 1 and 0, i.e., $\rho \subseteq F \times \{1, 0\}$.

Dialetheic deontic logic

- The informal reading for O is as follows:

  ...if the world were such that all extant obligations were duly fulfilled, then it would be the case that α.

- The truth conditions are:
  - $1 \in v_w(\mathrm{O}\gamma)$ iff $\gamma \in \omega^+(w)$
  - $0 \in v_w(\mathrm{O}\gamma)$ iff $\gamma \in \omega^-(w)$
- Operators P and F have standard definitions, $\neg\mathrm{O}\neg$ and $\mathrm{O}\neg$.

Dialetheic deontic logic

- Priest (1987) approach is similar to the approach of other philosophers who think about the logic of normativity in terms of properties of norm-sets. Examples:
  - Alchourrón and Bulygin (1981) treat norm-sets (normative systems) as deductively closed sets,
  - Broome (2007) treats norm-sets as closed under equivalence.
- There are no obstacles in treating norm-sets as simple sets consisting just of sentences that correspond to contents of norms and leave to question of their logical properties aside. This rout will be taken here.

The place for contradictions

- Contradiction is a relation between sentences which can be defined in different ways.

  Syntax In syntactic definition the two sentences p and q are contradictory in classical logic if each proves and is provable from the negation of the other, $\{p\} \vdash \neg q$ and $\{\neg q\} \vdash p$.

  Semantics In semantic definition the two sentences p and q are contradictory in classical logic if their interpretations are exclusive, $\mathrm{Int}(p) \cap \mathrm{Int}(q) = \varnothing$, and exhaustive, $\mathrm{Int}(p) \cup \mathrm{Int}(q) = \mathrm{Int}(\top)$.

  Pragmatics In pragmatic definition the two sentences p and q are contradictory in classical logic if the discourse $p\ q$ fails, $[\mathrm{i} :\cdot \underline{p}][\mathrm{j} :\cdot \underline{q}]\bot$, while the discourse $p\ \neg q$ is redundant, $[\mathrm{i} :\cdot \underline{p}]\varphi \to [\mathrm{i} :\cdot \underline{p}][\mathrm{j} :\cdot \underline{q}]\varphi$.

  - There are different but equivalent ways to define the pragmatic effects: either by describing the receiver's intentional state or the state of collective intentionality or the state of sender's linguistic commitments and so on.

Contradiction and destruction

- Contradiction creates destruction according to the principle *ex contradictione quodlibet*. In classical logic the presence of contradictory sentences in the premises destroys the proof by making every sentence provable, their presence in the theory destroys its descriptive power by making no interpretation possible, their presence in the discourse destroys communication by making it impossible to reach understanding.
- According to Tarski's (1930) theory of consequence relation ($Cn : \wp\mathcal{L} \to \wp\mathcal{L}$) there must exist a sentence whose consequence is the whole of the language ($\mathcal{L}$). Tarski's Axiom 5 states this property: *there is a sentence x such that $Cn(\{x\}) = \mathcal{L}$.*
- Is it rational to abandon the whole theory once a contradiction has been discovered? Is it rational to end communication when the disagreement of attitudes has been revealed?

Reason for revision

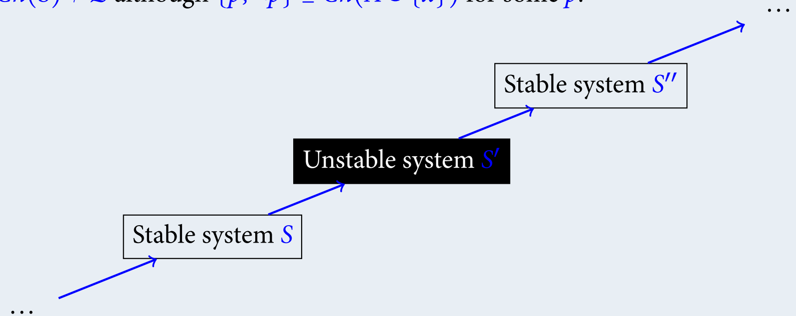
- The thesis I am defending is that the discovery of contradiction is not a reason for destruction (of a theory or a communicative exchange), but a reason for reconstruction either of a systems and its logic.
- A good example for reason for reconstruction is given in AGM concept of revision. Revision is complex and underdetermined theoretical change occurring when a new sentence x cannot be consistently added to a theory A. Revision takes two steps: 1. contraction, which is an under-determined change of A to a contracted theory A^* to which x can be consistently added, and 2. expansion of A^* with x.

Alchourrón, Carlos, Peter Gärdenfors, and David Makinson. 1985.
On the Logic of Theory Change: Partial Meet Contraction and Revision Functions.
Journal of Symbolic Logic 50:510–530.

- So, the fact that the theory growth leads to contradiction, i.e., $\{p, \neg p\} \subseteq Cn(A \cup \{x\})$ for some p, is not a reason to destroy it, but to revise it.
- The classical logic was not intended to provide a model of theoretical or normative dynamics: contradiction leads to destruction. Nevertheless, as the example of AGM theory shows, classical logic can be used in building the model of theoretical change: contradiction leads to reconstruction.

Instability phase

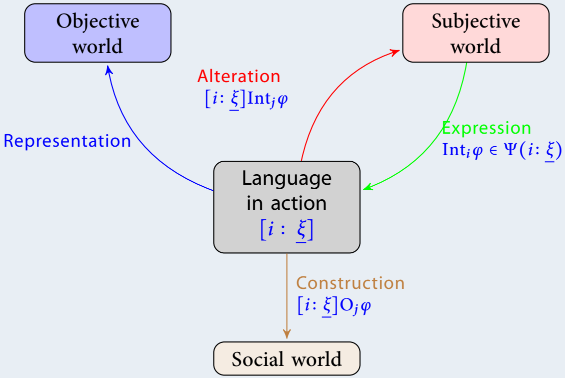
The state of a system is unstable if it contains a contradiction. According to classical logic there is no unstable state of a theoretical or a normative system since contradiction destroys the system. It is an empirical fact that there are theories and normative systems that contain a contradiction but do not "explode" but retain some kind of consequence that is not universal, i.e. $Cn(S) \neq \mathcal{L}$ although $\{p, \neg p\} \subseteq Cn(A \cup \{x\})$ for some p.



A theoretical or a normative system passing through unstable state towards a stable state (or a state with lesser degree of instability).

Are there contradictions in reality?

Habermas ontology and formal pragmatics



One can argue that there are contradictions in the reality: —in objective world quantum theory (in realistic interpretation) admits coexistence of “orthogonal states”, —in subjective world examples of “cognitive dissonances”, cases of contradictory beliefs and desires abound, —in social world normative conflict has been well described in literature and philosophy. Nevertheless, the existence of contradiction as an ontological phenomenon does not imply the destruction of the phenomenon.

Language-created reality

- How contradiction as a relation between sentences can become a contradiction in the social world?
- Suppose that a lawmaker l proclaims a sequence $\frac{n+1}{2}$ of conditional imperatives $l : \underline{(\cdot\varphi_1 \rightarrow !\varphi_2) \ldots (\cdot\varphi_n \rightarrow !\varphi_{n+1})}$ thus creating the set of norms $\mathcal{N} = \{(\varphi_1 \rightarrow \varphi_2), \ldots, (\varphi_n \rightarrow \varphi_{n+1})\}$ for the group $G = \{r_1, \ldots, r_k\}$.

Example

Imagine that the normative system $\mathcal{M}$ is created by the single command 'Do not see to it that something is the case if you desire it not to be the case' directed to a single actor i.[2] The content of the command is: 'If actor i desires that $\neg p$, then it is not the case that i sees to it that p is the case', or symbolically: (2). The content describes what conformation with the norm looks like and so it belongs to the single command norm-set $\mathcal{M}$, or symbolically: (3).[3]

$$\cdot\, \mathrm{D}_{\mathsf{i}}\neg p \rightarrow !\neg \mathsf{i}{:}\mathrm{stit}\, p \tag{1}$$

$$\mathrm{D}_{\mathsf{i}}\neg p \rightarrow \neg \mathsf{i}{:}\mathrm{stit}\, p \tag{2}$$

$$\ulcorner \mathrm{D}_{\mathsf{i}}\neg p \rightarrow \neg \mathsf{i}{:}\mathrm{stit}\, p \urcorner \in \mathcal{M} \tag{3}$$

[2] This normative system can be interpreted as founded on Nietzsche's maxim "Be thyself!".

[3] 'Quine quotes' are used for forming the name of an expression. Their use can be omitted if there is no possibility of confusion, but in cases where the same formula is both used and mentioned, Quine quotes will be used.

Norm-set membership and deontic value

The connection between the language of deontic logic and the language of norm-sets is established by the following equivalence:

$$\varphi \in \mathcal{N} \text{ } \textit{iff} \text{ } \mathrm{O}\varphi$$

Obligations

Social inconsistency

Suppose that $\mathcal{N}$ implies the falsehood in a certain situation w, i.e., $\bot \in Cn(\mathcal{N} \cup \{\varphi\})$ where φ is a non-agentive sentence that is true in w.
Opposite acts are acts resulting in a mutually exclusive state of affairs. For example, $p \to \mathsf{i}\ \mathsf{stit}\ r$ and $q \to \mathsf{j}\ \mathsf{stit} \neg r$ are agentive sentences about opposite acts and they cannot simultaneously be true in a situation where $p \wedge q$ holds:

$$\bot \in Cn(\{p \to \mathsf{i}\ \mathsf{stit}\ r, q \to \mathsf{j}\ \mathsf{stit} \neg r, p \wedge q\}$$

The norm-set $\mathcal{N} \supseteq \{p \to \mathsf{i}\ \mathsf{stit}\ r, q \to \mathsf{j}\ \mathsf{stit} \neg r\}$ is *socially inconsistent* in a situation where $p \wedge q$ holds.
Does social inconsistency imply destruction of $\mathcal{N}$-part of the social world?

Absurdity of normative explosion

Example

Let $\{p \to \mathsf{i}\ \mathsf{stit}\ r, q \to \mathsf{j}\ \mathsf{stit} \neg r\} \subseteq \mathcal{N}$. Suppose that:

1. $p \wedge q$ is true in situation w;
2. actor i sees to it that r, $\mathsf{i}\ \mathsf{stit}\ r$;
3. actor j does not see to it that $\neg r$, $\neg \mathsf{j}\ \mathsf{stit}\ \neg r$.

Has the norm-set $\mathcal{N}$ been complied with by the act of the first actor (i) and violated by the act of the second (j), or both of them have done and have not done what was required of them since the norm-set has been destroyed in the situation by the presence of jointly unsatisfiable requirements?
On the other hand, $\mathcal{N}$ might be inconsistent with respect to $p \wedge q$-type of situations and consistent with respect to all other types of situations. What is the range of "explosion" of $\mathcal{N}$ with respect to $p \wedge q$-type of situations? Does this fact destroy the norm-set in general, with respect to any situation?

Consistency of a norm-set

- The notion of consistency of a norm-set is not the same notion as consistency of a theory. Theory as a set of sentence is semantically consistent if there is an interpretation that makes true every sentence in the set.
- The consistency of a norm-set cannot be defined in the similar way as the previous example shows.

Definition

Let be $c(\mathcal{N}, w)$ be the function that picks consequents of those norm-conditionals the antecedents of which are true in w, i.e., $c(\mathcal{N}, w) = \{y \mid x \rightarrow y \in \mathcal{N} \text{ and } w \vDash x \text{ for some } x\}$. Norm set $\mathcal{N}$ is strongly consistent if for all w, $c(\mathcal{N}, w)$ is consistent.

Two approaches

Option 1: "all or nothing" approach

If a norm set is not *strongly consistent*, then any act is obligatory in any situation.

Option 2: flexible approach

	$\bot \notin Cn(c(\mathcal{N}, w))$ $\mathcal{N}$ is in a stable state at w	$\bot \in Cn(c(\mathcal{N}, v))$ $\mathcal{N}$ is in an unstable state at v
Logic in use by the norm-subject and norm-applier	classical	non-clasical

In theoretical Option 2, which seems to be the more preferable option with respect to the economy of time, the fact of inconsistency of a norm-set with respect to a certain situation is a reason for actors to engage in revision of the norm-set or its underlying logic.

Social roles and metanormativity

Von Wright's conjecture on the meta-normative character of axioms of deontic logic

[. . .] classic deontic logic, on the descriptive interpretation of its formulas, pictures a gapless and contradiction-free system of norms. A factual normative order *may* have these properties, and it may be thought desirable that it *should* have them. But can it be a *truth of logic* that a normative order has ("must have") these "perfection"-properties?

Georg Henrik von Wright (1999). Deontic logic: a personal view. *Ratio Juris* 12: 26–38.

In a norm governed social interaction there are three actor roles: the norm-giver, the norm-subject and the norm-applier role; and there are three types of norm related actions: norm-promulgation, norm-regulated action, norm-based judgement. In this kind of interaction, a norm-giver by norm-promulgation regulates the actions of a norm-subject whose observance of the norms is judged by a norm-applier. For each role there are specific meta-norms that regulate the relation of the role and norm-set.

Example (Norm-subject)

A specific type of desirability appears in the metanormative thesis (OU):

$$O(Op \to p)$$

This thesis which can be plausibly interpreted as *Conformation to norms is desirable*, *Duty must be done*, *Norms ought to be realized*, and so on. B. Chellas approves the use of the second order obligation within the thesis: "Note that OU is a theorem of deontic S5 ... The schema expresses the thesis that it ought to be the case that whatever ought to be the case be the case. It is a much discussed principle in deontic logic, because it is one of the few plausible cases of a theorem of the form OA in which A is non-trivial..." (Chellas, 1980:193). It is not plausible, however, to interpret the thesis as a claim about a desirable property of a norm-set since the claim *It is desirable that norms require only what is the case* results in a kind of normative collapse. Rather, the thesis can be understood as an observance principle since it shows that a norm is that which ought to be observed. Unlike the norm-giver, the norm-subject has no obligations with respect to the properties of norm-sets, and unlike the norm-subject, the norm-giver has no obligations with respect to the observance of norms.

Roles in norm governed interaction:	Their second-order obligations:	
Norm-giver g	ought to create norm-sets with perfection properties.	$O_g(O_s p \rightarrow P_s p)$
Norm-subject s	ought to observe norms.	$O_s(O_s p \rightarrow p)$
Norm-applier (Judge) j	ought to apply norms.	$O_j(O_s(p \rightarrow q) \rightarrow (O_s p \rightarrow O_s q))$

Rational reactions according to flexible approach to consistency maintenance

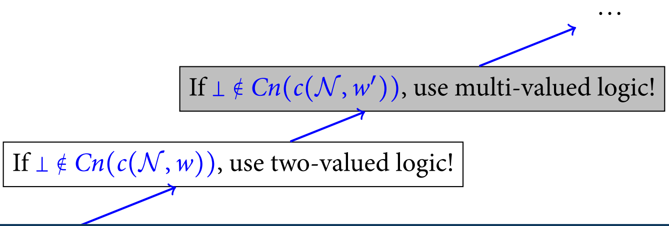
ROLES in norm governed interaction:	The reasons they have in a situation w at which the norm system $\mathcal{N}$ is inconsistent, i.e., $\bot \in Cn(c(\mathcal{N}, w))$:
NORM-GIVER	has a reason to revise $\mathcal{N}$ so to restore consistency
NORM-SUBJECT	has a reason to change the logic of normative reasoning (from classical to non-classical multi-valued)
NORM-APPLIER (JUDGE)	has a reason to change the logic of normative reasoning (from classical to non-classical multi-valued)

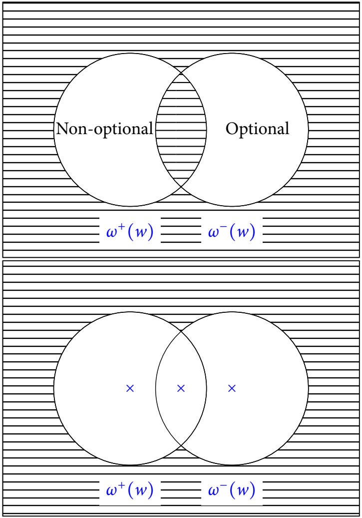
Position of dialetheic deontic logic

- The question is not *Which logic is the proper logic of normativity*, but *Is social reality created by a less than perfect use of language in norm promulgation*. If the answer to the second question is affirmative, then the first question can be dismissed as a meaningless one. The presupposition of the first question is the thesis *There is exactly one logic of normativity* which is not proved. On the other hand, if there is a deontic logic that can both "non-explosively" deal with situations at which a norm-set is inconsistent and deal "classically" with situations at which it is consistent, then it is a matter of terminology whether one should take them to be the two logics (the one dealing with situations of normative inconsistency and the other dealing with situations of normative consistency) or just one logic (whose special case is the two-valued logic).

Conclusion

- From the meta-normative standpoint it is the thesis on the existence of the two logics of normativity that ought to be adopted. The norm-giver is subordinated to the requirement of creating consistent norm-sets, sets that are strongly consistent. The logic of sets having perfection property of being consistent does not admit contradictions (such as $\mathrm{O}p \wedge \mathrm{P}\neg p$) and does not lie on the same level with paraconsistent logic. The latter is the logic of suboptimal social world while the former shows the value that ought to be achieved.
- The logics of normativity may vary with the state of the system or it might be just one logic, like DDL, with classical logic as its sublogic, but the logic of metanormativity is different and provides no place for contradictions.




Non-optional
Optional
$\omega^+(w)$
$\omega^-(w)$
$\omega^+(w)$
$\omega^-(w)$