Social dynamics of common knowledge

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Normative contexts for reconstruction of collective intentionality

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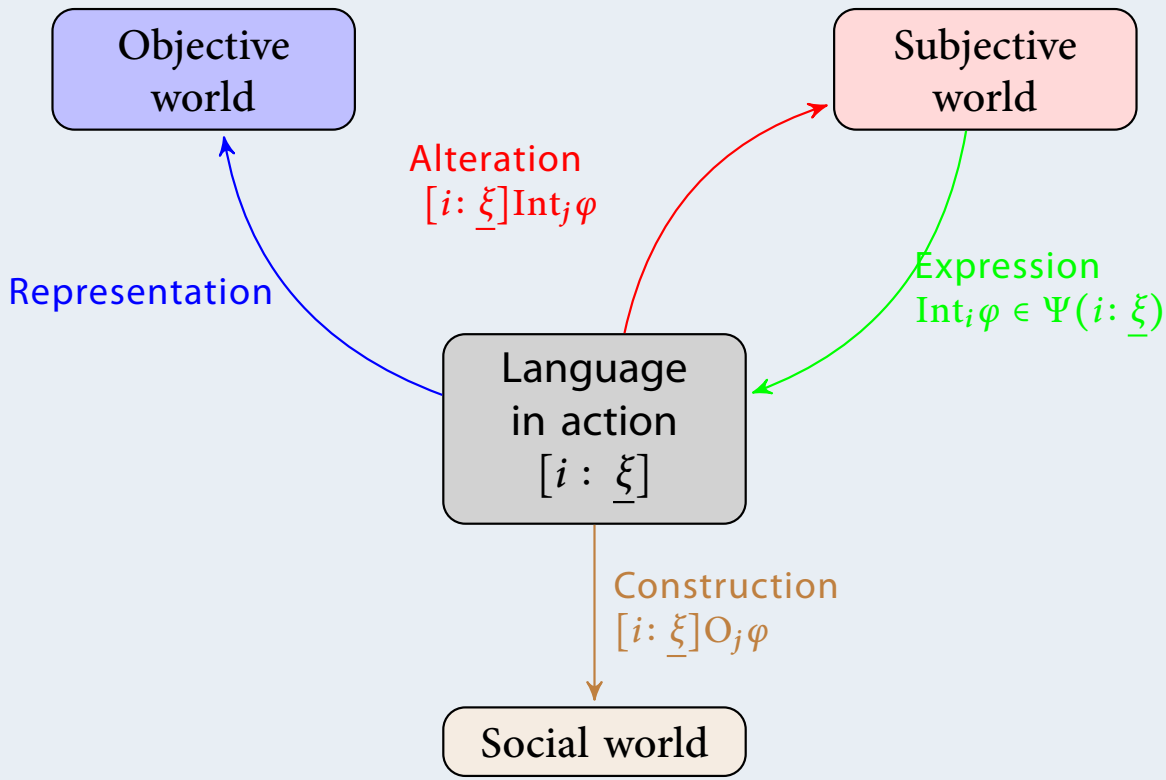
Overview

Locution as expression and action

Habermas ontology and formal pragmatics



Example

φ stands for "positive properties create an ultrafilter" (Abbreviation)

$$\mathrm{B}_{\mathrm{kurt}}\varphi \in \Psi(\mathrm{kurt}\colon \underline{.\varphi}) \qquad \text{(Expression)}$$

$$[\mathrm{kurt}\colon \underline{.\varphi}]\mathrm{F}_{\mathrm{kurt}}\mathrm{kurt}\colon \underline{.\neg\varphi} \qquad \text{(Construction)}$$

$$[\mathrm{kurt}\colon \underline{.\varphi}]\mathrm{O}_{\mathrm{howard}}\mathrm{B}_{\mathrm{howard}}\varphi \qquad \text{(Construction)}$$

$$[\mathrm{kurt}\colon \underline{.\varphi}]\mathrm{B}_{\mathrm{howard}}\varphi \qquad \text{(Fulfilment)}$$

Principal norm of communication

Definition

Let G be a communication group and $i \in G$.

$$\text{If } \mathrm{Int}_i\varphi \in \Psi(i\colon \underline{\xi}) \text{, then } [i\colon \underline{\xi}]\mathrm{O}_j\mathrm{Int}_j\varphi \text{ for any } j \in G. \tag{1}$$

Two special cases

Principal communication norm applied to the sender: $i = j$

If $\mathrm{Int}_i\varphi \in \Psi(i\colon \underline{\xi})$, then $[i\colon \underline{\xi}]\mathrm{O}_i\mathrm{Int}_i\varphi$ for any $j \in G$. (Norm of sincerity)

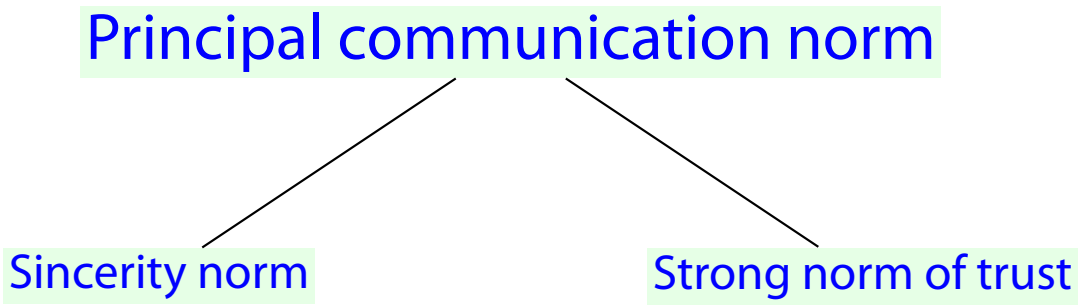
Principal communication norm as applied to the sender herself yields the *norm of sincerity*: after the sender's locution that expresses an intentional state the sender is obliged to have that state.

Principal communication norm applied to a sender-receiver pair: $i \neq j$

If $\mathrm{Int}_i\varphi \in \Psi(i\colon \underline{\xi})$ and $j \neq i$, then $[i\colon \underline{\xi}]\mathrm{O}_j\mathrm{Int}_j\varphi$. (Strong norm of trust)

General communication norm as applied to a sender-receiver pair yields the *strong norm of trust*: after the sender's utterance of a locution that expresses an intentional state having certain type and content, the receiver is obliged to have the state of the same type and content.

An overview


Principal communication norm
Sincerity norm
Strong norm of trust

The first postulate

Definition (General norm observance belief)

The observance of the principal communication norm is a general (form of) belief in a group iff

- actors have veridical beliefs about (all instances of) the principal communication norm
- actors believe that the norm is always observed by everyone in the group

Postulate (1)

Communication group has general belief that principal communication norm is observed.

Getting rid of deontic operator

If everybody in the communication group correctly recognizes the language norm and believes that it is observed by everyone in the group, then everybody in the group believes that language in use can alter intentional states.

Proposition

Communication group has general alteration belief.

Proof.

Let B_G denote "for all $i \in G$, B_i". General alteration belief follows from language norm and norm observance belief in normal doxastic logic.

$$\text{If } [i\colon \underline{\xi}]\mathrm{O}_j\varphi, \text{ then } \mathrm{B}_G[i\colon \underline{\xi}]\mathrm{O}_j\varphi \qquad \text{(Language norm belief)}$$

$$\mathrm{B}_G([i\colon \underline{\xi}]\mathrm{O}_j\varphi \to [i\colon \underline{\xi}]\varphi) \qquad \text{(Norm observance belief)}$$

$$\mathrm{B}_G[i\colon \underline{\xi}]\varphi \qquad \text{(Alteration belief)}$$

□

Provisional semantics

- Denote by M, w point w in relational structure M.
- Associate with each locution operator $[i : \underline{\xi}]$ an operation that transforms structure M into structure $M|i: \underline{\xi}$. In this way the effects of language use are modelled as changes of relation structures.

Provisional definitions

- $M, w \vDash [i: \underline{\xi}]\varphi$ iff $M|i: \underline{\xi}, w \vDash \varphi$.
- $M, w \vDash \mathrm{B}_i\varphi$ iff $M, v \vDash \varphi$ for all v such that $R_{\mathrm{B}_i} wv$.

Axiom (Logical axiom on operators switch)

$\mathrm{B}_i[i: \underline{\xi}]\varphi \leftrightarrow [i: \underline{\xi}]\mathrm{B}_i\varphi$

Operator switch axiom

Proposition

Logical axiom

$$\mathrm{B}_j[i\colon \underline{\xi}]\varphi \leftrightarrow [i\colon \underline{\xi}]\mathrm{B}_j\varphi \qquad \text{(Operator switch)}$$

is sound with respect to the provisional semantics.

Proof.

- $M, w \vDash \mathrm{B}_j[i\colon \underline{\xi}]\varphi$ iff
- $M, v \vDash [i\colon \underline{\xi}]\varphi$ for all v such that $R_{\mathrm{B}_j} wv$ iff
- $M|i\colon \underline{\xi}, v \vDash \varphi$ for all v such that $R_{\mathrm{B}_j} wv$.

- $M, w \vDash [i\colon \underline{\xi}]\mathrm{B}_j\varphi$ iff
- $M|i\colon \underline{\xi}, w \vDash \mathrm{B}_j\varphi$ iff
- $M|i\colon \underline{\xi}, v \vDash \varphi$ for all v such that $R_{\mathrm{B}_j} wv$.

□

The phenomenon of social ontology: making it the case by believing it to be the case

Proposition

If everyone in a communication group is rational and believes that the general communication norm is being observed, then the general communication norm is being observed.

Proof.

Let $[i\colon \underline{\xi}]\mathrm{O}_j\mathrm{Int}_j\varphi$. Suppose that j is rational and violates the norm. After the locution $i\colon \underline{\xi}$ has been performed actor j does not have required intentional state, $\neg\mathrm{Int}_j\varphi$. Assuming the introspective accessibility of intentional states, the actor j is aware of her own violation, $\mathrm{B}_j\neg\mathrm{Int}_j\varphi$. On the other hand, actor j believes that she has the required state since she believes that the norm is observed by her, $\mathrm{B}_j\mathrm{Int}_j\varphi$. Since j is transparency rational she does not have inconsistent beliefs. Therefore, *the communication norm generally believed to be observed is in fact observed.* □

Collective intentionality as social reflection

Proposition (1)

If there is a general norm observance belief, then social reflexive belief of the first order is produced by language use.

Proof.

1. Let $\mathrm{Int}_i\varphi \in \Psi(i\colon \underline{\xi})$.
2. $\mathrm{B}_G[i\colon \underline{\xi}]\mathrm{Int}_j\varphi$ for any $j \in G$, by the general alteration belief.
3. $[i\colon \underline{\xi}]\mathrm{B}_G\mathrm{Int}_j\varphi$ for any $j \in G$, by the operator switch axiom.

□

Weak trust

Definition (Weak norm of trust)

If $\mathrm{Int}_i\varphi \in \Psi(i\colon \underline{\xi})$ and $j \neq i$, then $[i\colon \underline{\xi}]\mathrm{O}_j\mathrm{B}_j\mathrm{Int}_i\varphi$. (Norm)

Corollary 1

The general communication norm observance belief satisfies the weak norm of trust.

If $\mathrm{Int}_i\varphi \in \Psi(i\colon \underline{\xi})$ and $j \neq i$, then $[i\colon \underline{\xi}]\mathrm{B}_j\mathrm{Int}_i\varphi$. (Norm observance)

Proof.

Use Proposition 1. □

The second postulate: Social reflective beliefs

Postulate (2 : Socially reflexive belief about general communication norm observance belief)

Everybody in communication group believes that norm observance belief is a general belief in the communication group.

The postulate on social reflection is the conjunction of (2) and (4). It implies (4) socially reflective alteration belief.

$$\text{If } [i\colon \underline{\xi}]\mathrm{O}_j\varphi, \text{ then } \mathrm{B}_G\mathrm{B}_G[i\colon \underline{\xi}]\mathrm{O}_j\varphi \tag{2}$$

$$\mathrm{B}_G\mathrm{B}_G([i\colon \underline{\xi}]\mathrm{O}_j\varphi \to [i\colon \underline{\xi}]\varphi) \tag{3}$$

$$\mathrm{B}_G\mathrm{B}_G[i\colon \underline{\xi}]\varphi \tag{4}$$

Transitivity

- If $[i\colon \underline{\xi}]\mathrm{O}_j\varphi$ (for any $i, j \in G$) is a language norm, then postulate (1) implies general alteration belief, $\mathrm{B}_G[i\colon \underline{\xi}]\varphi$ and postulate (2) implies socially reflective alteration belief, $\mathrm{B}_G\mathrm{B}_G[i\colon \underline{\xi}]\varphi$. Since conjunction implies conditional, the previous fact implies:

$$\mathrm{B}_G[i\colon \underline{\xi}]\varphi \rightarrow \mathrm{B}_G\mathrm{B}_G[i\colon \underline{\xi}]\varphi \tag{5}$$

- Relying on logical equivalence of any order of locution operator and doxastic operator, the transitivity principle (6) follows from (5):

$$[i\colon \underline{\xi}]\mathrm{B}_G\varphi \rightarrow [i\colon \underline{\xi}]\mathrm{B}_G\mathrm{B}_G\varphi \tag{6}$$

Social transparency

The transitivity principle shows that language generated beliefs are transparent. Within the scope of a locution operator the social introspection variant of modal axiom schema (4) holds:

$$[s\colon \underline{\xi}]\ \mathrm{B}_i\varphi \rightarrow [s\colon \underline{\xi}]\ \mathrm{B}_j\mathrm{B}_i\varphi \quad \text{for all } i, j \in G \qquad \text{(Social introspection)}$$

In technical terms, this means that doxastic accessibility relation is transitive over accessibility relations for different actors. For example, if $R_B(i)$ and $R_B(j)$ are accessibility relations modelling beliefs of actors i and j, then the principle of social introspection will hold just in case that the transitive closure of their union equals their union.

Production of collective intentionality: language in use produces socially transparent intentional states

Iterated application of social reflection axiom schema (4) allows for construction of finite sequences of doxastic modal operators of arbitrary length and composition, $\mathrm{B}_{x_1} \ldots \mathrm{B}_{x_n}$ where $x_i \in G$.
If one takes that transparency (introspective accessibility) is a necessary condition of intentionality phenomena, then collective intentionality includes social reflection, i.e., mutual beliefs about the intentional states of the others. The use of language if governed by general communication norm (implying sincerity and strong trust) and supported by general belief that it is being observed and generally believed as being observed can explain the production of collective intentionality: 1. after the locution the type and content of expressed intentional states become shared states within the group (because belief in norm being observed in one's own case implies norm being observed, otherwise the actor would be irrational), 2. shared intentional states become the content of the common belief (thanks to Postulate 2).

If $[i\colon \underline{\xi}]\mathrm{O}_j\varphi$, then $[i\colon \underline{\xi}]\mathrm{B}_G\varphi \rightarrow [i\colon \underline{\xi}]\mathrm{B}_G\mathrm{B}_G\varphi$ (Socially transparent beliefs)

Related research: logic in society

- In *dynamic epistemic logic* and *dynamic preference logic* by Johan van Benthem, Hans van Ditmarsch, Wiebe van der Hoek, Barteld Kooi, Fenrong Liu, and others, as well as in dynamic command logic by Tomoyuki Yamada, there is a common presupposition in modelling: the locution effects are exactly those that would obtain in the presence of generally believed general belief about communication norm being observed.
- The modelling does not deal with the possible clash between non-introspective intentional states of an actor before and after the locution. It should be noted that only intentional states of individual and social introspection get revised as an effect of a locution. For example, let $[i\colon \underline{\xi}]\mathrm{B}_j\varphi$ and let $\mathrm{B}_i\neg\mathrm{B}_j\varphi$ and $\mathrm{B}_j\neg\mathrm{B}_j\varphi$ be the case before the locution, then after the locution revision takes place and the former beliefs must contracted and replaced by $\mathrm{B}_i\mathrm{B}_j\varphi$ and $\mathrm{B}_j\mathrm{B}_j\varphi$.

Related research: society in logic

- It was in the field of imperative logic that a need for adding new elements in the model arouse. Liu and Ju have introduced the elements of social hierarchy. Not every imperative creates the obligation to do on the side of the receiver, but only those that are issued at a higher level in a relevant social hierarchy. So, they restrict the power of creating *obligations-to-do*

$$[i\colon !\, j \; \mathit{stit}\; \varphi]\mathrm{O}_j \; j \; \mathit{stit}\; \varphi$$

  to hierarchical social relations

$$j < i \rightarrow [i\colon !\, j \; \mathit{stit}\; \varphi]\mathrm{O}_j \; j \; \mathit{stit}\; \varphi$$

- This insight can be generalized and used for extending dynamic model along the following lines:
  1. Social relations determine the effects of language in use.
  2. Changes in social relation might be required for the resolution of communicative incoherence.

Diversity of normative contexts for language generated collective intentionality

- The generally believed general belief in the principal communication norm observance is just a special case of normative context for construction of collective intentionality. There are other normative contexts as well. It may well be the case, as it is argued here, that the changes in normative context are conditions of possibility of creating collective intentionality thus making the dynamics of social relations a necessary part of the dynamics of collective intentionality.

Communicative authority

Definition

Actor i is an epistemic authority for actor j iff actor i's assertion that it is the case that φ produces the obligation of actor j to believe that it is the case that φ.

$$\mathrm{EA}(i,j) \text{ iff } [i\colon \cdot\underline{\varphi}]\mathbf{O}_j \mathrm{B}_j \varphi \qquad \text{(Epistemic authority relation)}$$

Definition

Actor i is a bouletic authority for actor j iff actor i's request that j see to it that it is the case that φ produces the obligation of actor j to see to it that it is the case that that φ.

$$\mathrm{BA}(i,j) \text{ iff } [i\colon \underline{!\, j \mathit{\ stit\ } \varphi}]\mathbf{O}_j\, j \mathit{\ stit\ } \varphi \qquad \text{(Bouletic authority relation)}$$

Definition

Actor i is a communicative authority for actor j iff actor i's locution produces the obligation of actor j to have the intentional state expressed by the locution.

$$\mathrm{CA}(i,j) \text{ iff } \mathrm{Int}_i\varphi \in \Psi(i{:}\ \underline{\xi}) \text{ implies } [i{:}\ \underline{\xi}]\mathbf{O}_j\mathrm{Int}_j\varphi$$

(Authority relation generalized)

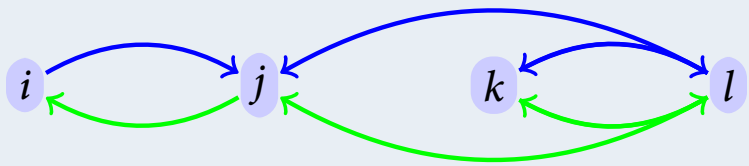
- The relation of communicative authority is just a restriction of the binding force of the general communication norm to pairs of actors.
- If the general communication norm binds every pair of actors, this type of distribution of communicative authority will be hereafter called α-type of communicative authority configuration.

Communication authority configurations

- There is a multitude of configurations of communicative authority distributions (v. Example below).
- Let $EA \cup BA$ stand for *communicative authority*, and let $EA \cap BA$ stand for *strong communicative authority*.

Example

$x\rightarrow$ stands for *... is an epistemic authority for ...*, $x\rightarrow$ stands for *... is a bouletic authority for ...*.
1. Actor i is an epistemic but not a bouletic authority for j. Actor j is a bouletic but not an epistemic authority for i.
2. Actors j and k are communicative equals devoid of authority.
3. Actors k and l are communicative equals with mutual strong communicative authorities.
4. l is a strong communicative authority for j.



Is communicative authority relation reflexive?

Proposition

If the norm of sincerity holds, then everybody is one's own authority.

Proof.

1. Suppose $\mathrm{Int}_i\varphi \in \Psi(i\colon \underline{\xi})$.
2. $[i\colon \underline{\xi}]\mathrm{O}_i\mathrm{Int}_i\varphi$, by the norm of sincerity.

□

- We want to keep reflexivity because any meaningful language use requires that logically rooted linguistic commitments are satisfied.
- In its purely linguistic variant the sincerity norm equals the so-called "principle of undeniability of sincerity conditions": if $\mathrm{Int}_i\varphi \in \Psi(i\colon \underline{\xi})$, then $[i\colon \underline{\xi}]\mathrm{F}_i\, i\colon .\underline{\neg\mathrm{Int}_i\varphi}$, by the norm of sincerity.

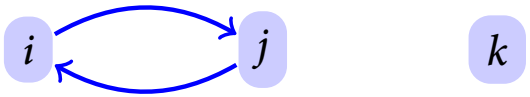
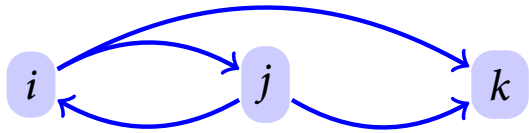
The division

- There are two equality types of communicative authority distribution: (α-type) where communicative authority relation is universal within a group G, $CA = G \times G$ (i.e. sincerity norm and strong trust norm hold); and (β-type) where communicative authority relation is reduced to identity relation, $CA = Id$.
- The third type (γ-type) comprises all transitive and reflexive relations that are neither universal nor socially empty (i.e., where only sincerity norm holds). A communicative authority relation R belongs to γ-type iff R is transitive and reflexive, $R \neq G \times G$, $R \neq \varnothing$.

Equality with authority (α-type) $\forall i \forall j CA(i,j)$
Equality without authority (β-type) $\forall i \forall j \ (i \neq j \rightarrow \neg CA(i,j))$
Non-equality (γ-type) $\exists i \exists j (i \neq j \wedge \neg CA(i,j)) \wedge \exists i \exists j (i \neq j \wedge CA(i,j))$

Communicative hierarchy

- Communicative inequality (γ-type) comes in a variety of subtypes.

Some γ-subtypes	
Not connected subtype $\exists i \exists j(\neg CA(i,j) \wedge \neg CA(j,i))$	Not symmetric subtype $\exists i \exists j(CA(i,j) \wedge \neg EA(j,i))$
Examples	
Not connected but symmetric	Connected but not symmetric
	

- Let us call "hierarchical" those γ-subtypes that are connected but not symmetrical.

Communicative incoherence

- A logical phenomenon similar to attitude inconsistency at individual level can be found at the collective level. Let it be called *communicative incoherence*: the term 'incoherence' points to the fact that there is a disagreement within the group, the adjective 'communicative' shows that the disagreement can be discovered only in communication. Unlike individual states which are introspectively accessible, collective intentionality is transparent only for those intentional states that are produced by language in use in the context of postulates 1 and 2, i.e., in the context of generally believed general belief that the general communication norm is being observed.

Communicative incoherence

Definition

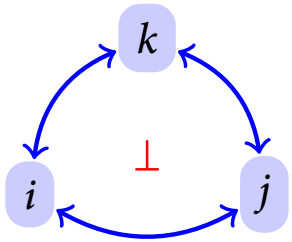
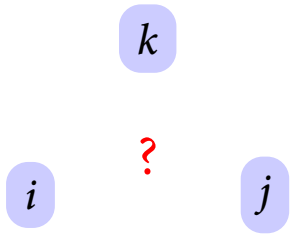
Discourse $i : \underline{\xi_1} \quad \ldots \quad j : \underline{\xi_n}$ is *communicatively incoherent* iff it makes the same intentional state both permitted and forbidden for the same actor, i.e. iff $[i : \underline{\xi_1}] \ldots [j : \underline{\xi_n}](\mathrm{P}_i\varphi \wedge \mathrm{F}_i\varphi)$.[1]

[1] Equivalently, if it makes the same locution both permitted and forbidden for the same actor.

Resolving communicative incoherence

- The communicative incoherence is a state of disequilibrium and it initiates an informational process aimed to resolve the imbalance. For example, scientific knowledge is subordinated to the requirement of *intersubjective testability* (H. Feigl) and the discovery of the communicative incoherence shows that the requirement has not been met and, therefore, corrective actions must be taken.
- The resolution of communicative incoherence can have one of the two forms:
  1. the *conservative form* retains the existing communicative authority relations and assigns the obligation to revise their own intentional states to individual actors,
  2. the *non-conservative form* transforms communicative authority relations.

Non-conservative form

General communication norm α-type  Communicative incoherence	$\Rightarrow$ $\Rightarrow$	Argumentation β-type  Subdetermined process of reaching understanding

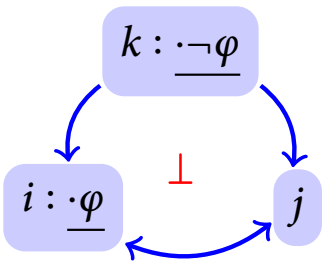
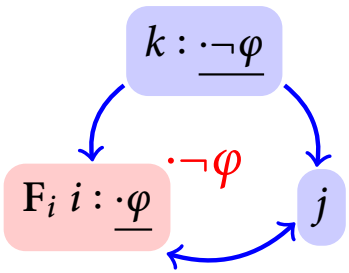
Transition from α- to β-type of communicative authority distribution

As soon as [validity claims][2] are problematized and made the object of a justified controversy, interlocutors switch (in however rudimentary a fashion) from communicative action to another form of communication, namely, a practice of argumentation, willing to convince one another of their views as well as to learn from one another.

Jürgen Habermas (2003).
Truth and Justification, str. 77, MIT Press.

[2] Truth, normative rightness, sincerity.

Conservative form of communicative incoherence resolution

γ-type N	$\Rightarrow$	γ-type N
		
Communicative incoherence	$\Rightarrow$	Determined resolution

- An actor's refusal to revise her intentional states (or at least to perform only those locutions expressing the revised intentional states) might end in excommunication.

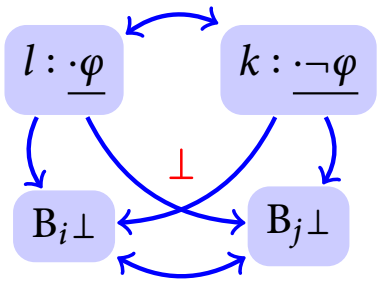
Non-compliance with conservative type of resolution of communicative incoherence

Samuel Hirszenberg (1907) *Excommunicated Spinoza.*

Obstacles to conservative resolution

- The appeal to higher authority cannot provide the solution for certain subtypes of γ-configurations.

Unsolvable disagreement
Conflicting authorities k and l



Conclusions

- The non-conservative resolution of communicative incoherence includes the change in communicative authority relations. Thus, social dynamics is a condition of possibility of logical dynamics in reconstruction of collective intentionality.
- The transition from α-type to β-type configuration is a rational socio-logical revision form. The failure of communication as "informational exchange" is replaced with argumentation, which if successful, restores the collective intentionality and enables the restoration of α-type.
- On the other hand, the conservative resolution of communicative incoherence has many obstacles and cannot be considered adequate for the construction of collective intentionality in the community of scientists and philosophers.
- The communicative authority configuration of the community of scientists and philosophers is not of stable γ-type, but a transformative structure of α–β–α–… transitions.

Prototype $\mathcal{L}_{\text{effect}}$ dynamic modal language for communication theory

Definition (The prototype language $\mathcal{L}_{\text{effect}}$)

$$\mathcal{L}_{\text{world}} \quad p \text{ is a sentence of propositional logic}$$

$$\mathcal{L}_{\text{reality}} \quad \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Diamond\varphi \mid \mathrm{D}_{\mathsf{i}}\varphi \mid \mathrm{B}_{\mathsf{i}}\varphi \mid \mathsf{i} \text{ stit } \varphi \mid \mathrm{O}_{\mathsf{i}}\varphi \mid \chi$$

$$\mathcal{L}_{\text{utterance}} \quad \xi ::= !\mathsf{i} \text{ stit } \varphi \mid \cdot\varphi \mid \cdot\varphi \rightarrow !\mathsf{i} \text{ stit } \varphi$$

$$\mathcal{L}_{\text{locution}} \quad \chi ::= \mathsf{i} : \underline{\xi}$$

$$\mathcal{L}_{\text{effect}} \quad \epsilon ::= \varphi \mid [\chi]\epsilon \mid \neg\epsilon \mid (\epsilon \wedge \epsilon) \mid \circledast_{\mathsf{i}}\epsilon \mid \ulcorner \circledast_{\mathsf{i}}\varphi \urcorner \in \Psi(\mathsf{i} : \underline{\xi})$$

Cf.

Žarnić, B. (2013)
Logical roots of linguistic commitment.
In *Theory of Imperatives from Different Points of View, vol. II*. Eds. A. Brożek, J. Jadacki, and B. Žarnić. Warsaw: Wydawnictwo Naukowe Semper.