Correspondence theory and logical postulates in metaphysics in an application to formal pragmatics

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Overview

Explicit and implicit definitions

- Two types of definition:

  Explicit definition enables replacement of one expression (definiendum) with another (definiens);

  Implicit definition does not enable replacement since instead of definiens it gives a set of postulates which contain occurrences of definiendum.

- The so-called standard theory of definition offers two criteria for good definitions: "the criterion of eliminability (which requires that the defined term be eliminable in favor of previously understood terms) and the criterion of conservativeness (which requires that the definition not only not lead to inconsistency, but not lead to anything —not involving the defined term— that was not obtainable before)".[1]

[1] Nuel Belnap (1993) On Rigorous Definitions. *Philosophical Studies* 72:115–146

Eliminability, conservativeness

Example

Let L be a language and T a theory in L. Let P be a new term, not in the language L. (P can be a singular term, a predicate, or a function symbol). A purported implicit definition of P (relative to T and L) is a set S of sentences in the expanded language $L + \{P\}$. According to Shapiro the following two conditions are "generally held to be individually necessary and jointly sufficient":[2]

Eliminability for each formula $\varphi \in L + \{P\}$ there is a formula $\varphi' \in L$ such that $T \cup S \vDash \varphi \leftrightarrow \varphi'$,

Non-creativity (conservativeness) for each formula $\varphi \in L$, *if* $T \cup S \vDash \varphi$, *then* $T \vDash \varphi$.

[2] Stewart Shapiro (1999) "Implicit definition and abstraction", the talk at University of St. Andrews,
`http://www.st-andrews.ac.uk/arche/papers/hdshapiroimplicitdef.pdf`

Definition of logical terms

- It is obvious that criteria for the implicit definition of logical terms must be different from those for non-logical terms because the eliminability and the conservativeness are defined using the notion of logical consequence. So, it would be circular to apply criteria of eliminability and conservativeness for logical terms.
- Weakening the thesis of Implicit Definition. Let us assume that logical terms are implicitly defined in a partial way by a logical system in which they appear. Also, let us suspend our judgement regarding the (im)possibility of assess the truth of logical principles *in isolation* from a non-logical theory (formulated in a language having the logic to which these principles belong).

Implicit Definition

By the theory of *Implicit Definition* with respect to logic, I shall understand the claim that the meanings of the logical constants are determined by certain logical principles in which they feature: specifically, their meanings are such that the principles come out true.

Thomas Kroedel (2012).
Implicit definition and the application of logic.
Philosophical Studies 158:131–14.

Semantic definitions

- In the semantics part of a logical system one usually finds definitions that apparently are not implicit since they provide the meaning of logical terms.

Example

Definition of necessity operator $\Box$ Let $W \neq \emptyset$, $R \subseteq W \times W$,
$V(p)$: p. letters $\rightarrow \wp W$.
$\langle W, R, V \rangle, w \vDash p$ iff $w \in V(p)$ for propositional letters p;
$\langle W, R, V \rangle, w \vDash \Box\varphi$ iff for all v such that Rwv it is the case that
$\langle W, R, V \rangle, w \vDash \varphi$.

The background of semantic definition

- In fact the meaning has not been provided until it is specified what are the properties of W and R and V, and for this purpose logical principles must be introduced.

Example

Standard semantics of propositional modal logic

- The clause '$w \in V(p)$ for propositional letters' together with classical connective 'iff' presuppose binary valuation, i.e., a certain logic.
- The character of evaluation points $w \in W$ is specified by the necessitation rule RN of inference (if $\vdash \varphi$, then $\vdash \Box\varphi$) and (K) axiom schema $(\Box(\varphi \to \psi) \to (\Box\varphi \to \Box\psi))$: the points are logical points where the laws of logic hold by RN and where consequences of truths of a point are truths of that point by K axiom.
- The properties of relation R are specified by axioms, e.g., reflexivity by (T) $\Box\varphi \to \varphi$, transitivity by (4) $\Box\varphi \to \Box\Box\varphi$, symmetry by (B) $\varphi \to \Box\Diamond\varphi$.

- So, the meaning of the necessity operator is not grasped by the isolated semantic definition for the truth in a model $\langle W.R, V\rangle$ at a point w. Rather, the whole of a logical system is needed to define a logical term.

Going wider

- Is the meaning of the adjective ‘necessary’ understood in the metaphysical sense “fixed” by S5 logical system (propositional logic plus RN, K, T, 4, B)? Not really.
- The thesis: *The meaning of a term cannot be specified in an isolation from other terms.*
- For example, what is the relation between the concept of metaphysical necessity and social obligation?
- Is it so that an obligation ought to be metaphysically possible? If so, in which sense? Interference or subordination?
- In the next page we will consider how the relation between the two concepts is determined by Roman Law Principles *RPL* (interfering concepts) and by Chellas Principle *Ch* (subordination of deontic possibilities to metaphysical).

(RLP, interference) There must be at least one metaphysical possibility among deontic possibilities

- (RLP) $\mathbf{O}p \rightarrow \Diamond p$
- $\forall P\ \mathrm{ST}_x(\mathbf{O}p \rightarrow \Diamond p)$
- $\forall P(\forall y(R_O xy \rightarrow Py) \rightarrow \exists y(R_M xy \wedge Py))$
- minimal valuation $Pu := R_O xu$
- $\forall y(R_O xy \rightarrow R_O xy) \rightarrow \exists y(R_M xy \wedge R_O xy)$
- $\top \rightarrow \exists y(R_M xy \wedge R_O xy)$
- $\forall x \exists y(R_M xy \wedge R_O xy)$

(Ch, subordination) Deontic possibilities lie within the scope of metaphysical possibilities

- (Ch) $\Box p \rightarrow \mathbf{O}p$
- $\forall P\ \mathrm{ST}_x(\Box p \rightarrow \mathbf{O}p)$
- $\forall P(\forall y(R_M xy \rightarrow Py) \rightarrow \forall y(R_O xy \rightarrow Py))$
- minimal valuation $Pu := R_M xu$
- $\forall y(R_M xy \rightarrow R_M xy) \rightarrow \forall y(R_O xy \rightarrow R_M xy)$
- $\top \rightarrow \forall y(R_O xy \rightarrow R_M xy)$
- $\forall x \forall y(R_O xy \rightarrow R_M xy)$

Note that $\{Ch, D\} \vdash RLP$.

Sahlqvist-van Benthem algorithm has been used for computing frame correspondences; a significant number of correspondences can be computed using SQEMA at

`http://www.fmi.uni-sofia.bg/fmi/logic/sqema/new/sqema.jsp`

Implicit definition and inter-categorical relations

- The implicit definition of metaphysical modalities is not completed by making explicit a logical system for $\Box$ and $\Diamond$, which is being used in some theory. Their relations to other modalities constitute their meaning. For example, how do they relate to logical, nomological and historical possibilities? Or, as discussed above, how do they relate to deontic possibilities?
- However, the real challenge does not lie in combining metaphysical modalities with other modalities, but in their interaction with quantifiers.

Williamson's book

- The problem of interaction of metaphysical modalities with quantifiers does not appear in those cases where quantifiers appear within the scope of modalities.

$$\Box\ \forall \qquad \text{(Unproblematic)}$$

  One moves along the paths of accessibility relation and applies first-order logic at evaluation points.

- The real problem appears when quantifiers range over modalities.

$$\forall\ \Box \qquad \text{(Problematic)}$$

- To solve the problem a theoretical option must be chosen. The choice is not theoretically inert but implies a commitment to metaphysical theses.
- An example: the possible truth of *Romeo knows that someone likes him, but he doesn't know who* = $K_r\exists xLxr \wedge \neg\exists xK_rLxr$ shows that $K_r\exists xLxr \rightarrow \exists xK_rLxr$ is not a truth of epistemic logic.

Disputable logic

Indeed, one role for logic is to supply a central structural core to scientific theories, including metaphysical theories, in essence no more above dispute than any other part of those theories.

Timothy Williamson (2013). *Modal Logic as Metaphysics*, [Preface] Oxford University Press.

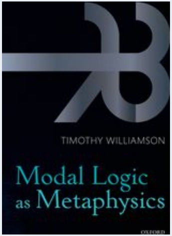
Logic and metaphysics

- The rich, informative and moving exposition, comparison and evaluation of various solutions to the problem of interaction between metaphysical modal operators and quantifiers has been given in Williamson's book.
- For the contemporary philosophy one the crucial questions investigated in the book is the question of relationship between logic and metaphysics.

Maxwell John Cresswell

…the issues raised by the book are among the most important in current work on modal metaphysics, and I very much hope that all metaphysicians of modality make the effort required to come to terms with its many ideas and arguments.

M. J. Cresswell (2014).
Book review *Modal Logic as Metaphysics*,
The Philosophical Quarterly.
doi:10.1093/pq/pqt052



The problem

Are modal axioms metaphysical axioms

For a mixture of technical and philosophical reasons, any such separation of logic and metaphysics became increasingly hard to maintain, especially for principles like the Barcan formula and its converse.

Timothy Williamson (2013).
Modal Logic as Metaphysics, pp.30–31
Oxford University Press.

A semantics of modal quantified logic

- Many semantical systems of quantified modal logic combine first-order structures with relational models of propositional modal logic.
- The resulting combination need not inherit the properties of logics being combined.

A striking instance is incompleteness: often, there is no complete axiomatization for model classes that caused no problems in the propositional case.

Johan van Benthem (2010).
Modal Logic for Open Minds, Chapter 11.
University of Chicago Press.

One among many

- The semantics that will be tentatively presented here is just one among many semantics for quantified modal logic. It ought to be sufficient to ground the thesis that I want to put forward.
- The valuations of predicate letters and individual constants are localized to worlds (evaluation points) and their domains.
- World domains will be allowed to vary so that an object in a world need not exist in a related world, which can also have new objects.
- There are two relations (inter alia) between the domains of related worlds *w* and *v* that are interesting and expressible in modal language:
  1. Increasing (cumulative) relation: $Rwv \rightarrow D_w \subseteq D_v$;
  2. Decreasing ("no object growth"): $Rwv \rightarrow D_v \subseteq D_w$;
- If the relation between domains of related worlds is both increasing and decreasing, then domains are equal across worlds and one can speak about the constant domain.

A model for QML

- The model $M = \langle W, R, D, V \rangle$.
- Domain function D maps a world w to a set of objects D_w.
- Valuation V is a two-place function that interprets a non-logical predicate P^n at a world w as a relation in D_w domain of w, i.e., $V(P^n, w) \subseteq D_w^n$.
- Assignment function a interprets variables and delivers objects.

Definition

$$M, w, a \vDash Pt_1, \ldots, t_n \text{ iff } \langle a(t_1), \ldots, a(t_n) \rangle \in V(P^n, w)$$
$$M, w, a \vDash \neg Pt_1, \ldots, t_n \text{ iff } \langle a(t_1), \ldots, a(t_n) \rangle \in D_w^n - V(P^n, w)$$
$$M, w, a \vDash t_1 \neq t_2 \text{ iff } \langle a(t_1), a(t_2) \rangle \in \bigcup_{v \in W} D_v \times D_w - V(=, w)$$

The unusual addition of special clause for negation of atomic formula secures that the assignment $a[d/x]$ which delivers object d, not in the domain D_w, for variable x, will not satisfy neither Px nor $\neg Px$ at the point w, i.e., if $d \notin D_w$, then $M, w, a[d/x] \nvDash Px$ and $M, w, a[d/x] \nvDash \neg Px$.

Universal modality and universal quantifier

- The problem arises with varying domains since not all the objects available at a domain D_w need not be available in the domain D_v of an accessible world v.
- The assignment that is equal to a except possibly for assigning the object d to variable x is denoted by $a[d/x]$.

Definition

- $M, w, a \vDash \forall x\varphi$ iff for all $d \in D_w$: $\quad M, w, a[d/x] \vDash \varphi$;
- $M, w, a \vDash \Box\varphi$ iff for all v such that Rwv: $\quad M, v, a \vDash \varphi$

Example

ACCESSIBILITY RELATION

$R = \{\langle w, v\rangle, \langle v, w\rangle\}$

WORLD	DOMAIN	VALUATION
w	$D_w = \{a\}$	$V(P, w) = \{a\}$
v	$D_v = \{a, b\}$	$V(P, v) = \{a\}$

FORMULA'S	TRUTH-VALUE AT WORLD	IS
$\forall x \Box Px \rightarrow \Box \forall x Px$	w	**false**
$\forall x \Box Px \rightarrow \Box \forall x Px$	v	**true**
$\Box \forall x Px \rightarrow \forall x \Box Px$	w	**true**
$\Box \forall x Px \rightarrow \forall x \Box Px$	v	**false**

Note that BF fails at w since there is new object b in v, and that CBF fails at v since its object b has been lost in w.

Barcan formula

- Standard translation to first-order language for $\forall x \Box Px \to \Box \forall x Px$, where Exw means 'x is an object in the domain of w'.
    - $\forall x(Exw \to \forall v(Rwv \to Pxv)) \to \forall v(Rwv \to \forall x(Exv \to Pxv))$
    - $\forall v \forall x(Rwv \to (Exw \to Pxv)) \to \forall v(Rwv \to \forall x(Exv \to Pxv))$
    - Let P be the property satisfied at v just by elements of w: $Pxv := Exw$.
    - $\forall v \forall x(Rwv \to (Exw \to Exw)) \to \forall v(Rwv \to \forall x(Exv \to Exw))$
    - $\top \to \forall v(Rwv \to \forall x(Exv \to Exw))$
    - $\forall v(Rwv \to \forall x(Exv \to Exw))$, i.e. $Rwv \to D_v \subseteq D_w$.
- Barcan formula characterizes "inhabited frames" with "no object growth", where the domain of an accessible world is a subset of the world from which it is accessible.

Converse Barcan formula

- Standard translation to first-order language for $\Box\forall xPx \to \forall x \Box Px$, where Exw means 'x is an object in the domain of w'.
  - $\forall v(Rwv \to \forall x(Exv \to Pxv)) \to \forall x(Exw \to \forall v(Rwv \to Pxv))$
  - $\forall x\forall v(Rwv \to (Exv \to Pxv)) \to \forall v\forall x(Exw \to (Rwv \to Pxv))$
  - Let P be the property satisfied at v just by elements of v: $Pxv := Exv$.
  - $\forall x\forall v(Rwv \to (Exv \to Exv)) \to \forall v\forall x(Exw \to (Rwv \to Exv))$
  - $\top \to \forall v\forall x(Exw \to (Rwv \to Exv))$
  - $\forall v(Rwv \to \forall x(Exw \to Exv))$, i.e. $Rwv \to D_w \subseteq D_v$.
- Converse Barcan formula characterizes "inhabited frames" cumulative or increasing domains, where the domain of a world is always a subset of its accessible world.

Necessitism and contingentism

Call the proposition that it is necessary what there is necessitism, and its negation contingentism. In slightly less compressed form, necessitism says that necessarily everything is necessarily something; still more long-windedly: it is necessary that everything is such that it is necessary that something is identical with it. In a slogan: ontology is necessary. Contingentism denies that necessarily everything is necessarily something. In a slogan: ontology is contingent.

Timothy Williamson (2013). *Modal Logic as Metaphysics*, [Preface] Oxford University Press.

- Necessitism thesis: $\forall x \Box \exists y x = y$.
- The thesis holds on cumulative (increasing) inhabited frames.
- Contingentism thesis: $\exists x \Diamond \forall y\, x \neq y$.

Labelled natural deduction

[3]

- Rules for modal operators:

$\Box$Intro	$\Gamma, Rwv \vdash v : \varphi \Rightarrow \Gamma \vdash w : \Box\varphi$	v does not occur in Γ
$\Box$Elim	$\Gamma \vdash Rwv,\ w : \Box\varphi \Rightarrow \Gamma \vdash v : \varphi$	

- Rules for quantifiers:

$\forall$Intro	$\Gamma, w : t \vdash w : \varphi[t/x] \Rightarrow \Gamma \vdash w : \forall x\varphi$	t does not occur in Γ
$\forall$Elim	$\Gamma \vdash\ w : \forall x\varphi, w : t \Rightarrow \Gamma \vdash w : \varphi[t/x]$	

- Relational theory:

$$\begin{array}{rl} \text{T} & \vdash Rww \\ 4 & Rwv, Rvu \vdash Rwu \\ \text{B} & Rwv \vdash Rvw \end{array}$$

- Domain theory:

$$\begin{array}{rl} \text{ID} & Rwv, w : t \vdash v : t \\ \text{DD} & Rwv, v : t \vdash w : t \end{array}$$

[3] Basin, Matthews and Vigano

CBF implies necessitism

1	$w: \Box\forall x\exists y\, x = y \rightarrow \forall x \Box \exists y\, x = y$	
2	Rwv	
3	$v : t$	
4	$v : t = t$	3/ Intro=
5	$v : \exists y\, x = y$	3, 4/ Intro∃
6	$v : \forall x\exists y\, x = y$	3–5/ Intro∀
7	$w : \Box\forall x\exists y\, x = y$	2–6/ Intro□
8	$w : \forall x \Box \exists y\, x = y$	1, 7/ Elim→

Necessitism holds on cumulative domains

1	$w : t$	
2	Rwv	
3	$v : t$	1, 2/ ID
4	$v : t = t$	3/ Intro=
5	$v : \exists y\, x = y$	3, 4/ Intro∃
6	$v : \Box\exists y\, t = y$	3–5/ Intro□
7	$w : \forall x \Box \exists y\, x = y$	1–6/ Intro∀

No object growth

- The thesis that no new objects may appear in metaphysical alternatives has no direct translation. Although validity of BF on an inhabited frame secures "no object growth", the decreasing domain thesis cannot be expressed by an instance of BF: $\forall x \Box \exists y\, x = y \to \Box \forall x \exists y\, x = y$ since it reduces to $\forall x \Box \exists y\, x = y \to \top$ and, finally, to $\top$. The alternative is to claim CBF together with axiom B.
- An extension of the formal language with the reverse modality $\Box^{-1}$ can provide the desired formula.

Definition

$M, w, a \vDash \Box^{-1}\varphi$ iff for all v with $Rvw \quad M, v, a \vDash \varphi$.

- The direct expression of decreasing domain thesis is $\forall x \Box^{-1} \exists y\, x = y$. Is there a literal translation to natural language?

The meaning of modalities

- The philosophical problem of quantified modal logic is that the interaction of two logical terms ($\forall$, $\Box$) cannot be defined in a neutral way. By accepting principles like BF and CBF one commits oneself to metaphysical theses on the nature of possibilities. The same holds for rejections.
- The thesis I'm trying to put forward is that in some cases there is no logico-metaphysical choice. More specifically, the use of imperative mood forces us into rejection of BF and CBF.

Imperatives as requested acts

- There is a long and noteworthy tradition in logic of imperatives where imperatives are treated as a requested acts (von Wright, Lemmon, Belnap, Segerberg, ...). In short, as Belnap put it, *the content of every imperative is agentive*.
- The idea can be easily explained using von Wright's typology of acts:

ACT	IMPERATIVE
$\neg A/A$	Produce A!
$\neg A/\neg A$	Suppress A!
A/A	Maintain A!
$A/\neg A$	Destroy A!

Von Wright's simple semantics of acts

Three points:

1. initial situation,
2. end situation,
3. counter-situation.

Two paths:

1. agency path = initial s. / end s.,
2. nature path (counterfactual path) = initial s. /counter-s.

Produce type of imperatives

- Consider the imperative $!(\underbrace{\forall x \neg Px}_{w} \;/\; \underbrace{\exists x Px}_{v})!$
- There are two ways for the imperative $!(\underbrace{\forall x \neg Px}_{w} \;/\; \underbrace{\exists x Px}_{v})$ to become true:

  1. Rearrangement of the world. In semantic terms, the sufficient condition is that valuation of P changes from $V(P, w) = \varnothing$ at w to $V(P, v) \neq \varnothing$ at v. An example: *Put your head on my shoulder!*.
  2. Object creation. A new object d which is not in the domain at w appears in v: $d \notin D_w$ but $d \in D_v$. The extension of P at v can, and need not remain empty with respect to objects of D_w. An example: *Write a paper!*.

$!(\underbrace{\forall x \neg Px}_{w} \,/\, \underbrace{\exists x Px}_{v})$ is not compatible with BF

- In the special case where Px satisfaction excludes rearrangement the following holds:

w	/	v
$\forall x \neg Px$	/	$\exists x Px$
$\forall x \Box \neg Px$	/	

- Suppose: (i) that Barcan formula ($\forall x \Box Px \rightarrow \Box \forall x Px$) holds, (ii) that imperative alternatives are metaphysically possible (e.g. that the postulate $!(\varphi/\psi) \rightarrow \Diamond\psi$ holds) and (iii) that $\langle w, v\rangle \in R$ (where R is the relation of metaphysical accessibility)! Then the following absurd situation would obtain:

w	/	v
(1) $\forall x \neg Px$	/	(1*) $\exists x Px$
(2) $\forall x \Box \neg Px$	/	
(3) $\Box \forall x \neg Px$, from BF and (2)	/	(3*) $\forall x \neg Px$, from (3) and Rwv
	/	(4*) $\bot$, from (1*) and (3*)

Conclusion

- The use of imperatives presupposes the metaphysics of varying domains. It may be rightfully called the ontology of imperative mood.
- The ontology of imperative mood rejects BF (as proved above) and CBF.
- The implicit definition of metaphysical modality cannot be completed by an unimodal propositional logical system. In order to see what 'necessity' means one must investigate in which way metaphysical modality interacts with all logical terms and elements, like other modalities (e,g. deontic modality), quantifiers or sentential moods (e.g. imperative mood).
- In the context opened by the use of indicatives it is meaningful to dispute whether BF or CBF hold, but not so in the context of imperatives. One either ought to reject BF and CBF or refrain from using imperatives. In short, *ontology of imperatives is contingent*.

Different metaphysics for different types of reasoning

It was stated previously, then, that there are two parts of the soul, the one possessing reason as well as the non-rational part. But now we must divide the part possessing reason in the same manner. Let it be posited that the parts possessing reason are two: one part is that by which we contemplate all those sorts of beings whose principles do not admit of being otherwise, one part that by which we contemplate all those things that do admit of being otherwise. For when it comes to beings that differ in kind from one another, the part of the soul that naturally relates to each is also different in kind, if in fact it is by dint of a certain similarity and kinship that knowledge is available [to the rational parts of the soul]. And let it be said that one of these is 'the scientific;' the other 'the calculative.' For deliberating and calculating are the same thing, and nobody deliberates about things that do not admit of being otherwise. (NE, 1139a)

Aristotle.
Nicomachean Ethics, 1139a.