SPECIMEN DYNAMICUM

1695

While in Italy Leibniz had written an extensive Dynamics (GM., VI, 281–514) which summarized his criticism of Descartes's physical principles and at the same time supplemented what he regarded as an incompleteness in Newton's hypotheses. The manuscript of this work he left at Florence with the Baron Bodenhausen, tutor of the sons of the Duke of Tuscany, with the intention of publishing it after it had undergone the criticism of his friends (see his letter to L'Hospital, January 15, 1696 [GM., II, 305–11]). The present work is a summary of this longer one, made in response to a general demand for his new ideas. Part I was printed in the Acta eruditorum in April, 1695; the second part remained unpublished and was found by Gerhardt among the Hanover manuscripts. The two parts together comprise a mature statement of Leibniz's theory of dynamics.

SPECIMEN DYNAMICUM; FOR THE DISCOVERY OF THE ADMIRABLE LAWS OF NATURE CONCERNING CORPOREAL FORCES, THEIR MUTUAL ACTIONS, AND THEIR REDUCTION TO THEIR CAUSES

[GM., VI, 234–54] Part I

Since we first mentioned a new *science of dynamics*, which was still to be founded, many prominent men in various places have asked for a fuller explanation of its teachings. But as we have not yet found leisure to write a book, we shall here set down some things which may cast some light on it – light which will be returned to us with interest if we succeed in eliciting the opinions of men who combine force of insight with distinction of style. We confess that their judgment will be most welcome and we hope, useful in advancing the perfection of the work.

We have suggested elsewhere that there is something besides extension in corporeal things; indeed, that there is something prior to extension, namely, a natural force everywhere implanted by the Author of nature – a force which does not consist merely in a simple faculty such as that with which the Scholastics seem to have contented themselves but which is provided besides with a striving or effort [conatus seu nisus] which has its full effect unless impeded by a contrary striving.¹ This nisus sometimes appears to the senses, and is in my opinion to be understood on rational grounds, as present everywhere in matter, even where it does not appear to sense. But if we cannot ascribe it to God by some miracle, it is certainly necessary that this force be produced by him within bodies themselves.² Indeed, it must constitute the inmost nature of the body, since it is the character of substance to act, and extension means only the continuation or diffusion of an already presupposed acting and resisting substance. So far is extension itself from comprising substance!

It is beside the point here that all corporeal action arises from motion and that mo-

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tion itself comes only from other motion already existing in the body or impressed upon it from without. For like time, motion taken in an exact sense never exists, because a whole does not exist if it has no coexisting parts. Thus there is nothing real in motion itself except that momentaneous state which must consist of a force striving toward change. Whatever there is in corporeal nature besides the object of geometry, or extension, must be reduced to this force. This reasoning does justice, at last, both to truth and to the teachings of the ancients. Our age has already saved from contempt the corpuscles of Democritus, the ideas of Plato, and the tranquillity of the Stoics which arises from the best possible connection [*nexus*] of things; now we shall reduce the Peripatetic tradition of forms or entelechies, which has rightly seemed enigmatic and scarcely understood by its authors themselves, to intelligible concepts. Thus we believe that this philosophy, accepted for so many centuries, must not be discarded but be explained in a way that makes it consistent within itself (where this is possible) and clarifies and amplifies it with new truths.

This method of study seems to me best suited both for the wisdom of the teacher and for the advancement of the learners; we must guard against being more eager to destroy than to construct, and against being tossed about uncertainly, as if by the wind, among the perpetually changing teachings put forth by certain freethinkers. Then after it has curbed the passion of sects, which is stimulated by the vain lust for novelty, mankind will at length advance with firm steps to ultimate principles in philosophy no less than in mathematics. For if we overlook entirely the harsher things which they say against others, the writings of outstanding men, both ancient and modern, usually contain many true and good things which deserve to be collected and arranged in the public treasury of knowledge. Would that men might choose to do this rather than to waste their time with criticisms that serve only to satisfy their own vanity. Indeed, though fortune has so favored me with the discovery of certain new things of my own that friends often urge me to think only about these, I nevertheless find pleasure in the views of others and appraise each according to its own worth, however this may vary. This may be because I have learned in my widespread activities not to despise anything. But now let us get back on the road.

Active force, which may well be called power, as it is by some, is of two kinds. The first is *primitive* force, which is in all corporeal substance as such, since I believe that a body entirely at rest is contrary to the nature of things. The second is *derivative* force, which is exercised in various ways through a limitation of primitive force resulting from the conflict of bodies with each other. Primitive force, which is nothing but the first entelechy, corresponds to the soul or substantial form, but for this very reason it relates only to general causes which cannot suffice to explain phenomena. Therefore I agree with those who deny that forms are to be used in investigating the specific and special causes of sensible things.³ This I must emphasize to make it clear that in restoring to the forms their proper function of revealing the sources of things to us, I am not trying to return to the word battles of the more popular Scholastics. A knowledge of forms is necessary, meanwhile, for philosophizing rightly, and no one can claim to have grasped the nature of body adequately unless he has paid some attention to such things and has come to understand that the crude concept of a corporeal substance which depends only on sensory imagery and has recently been carelessly introduced by an abuse of the corpuscular philosophy (which is excellent and most true in itself) is imperfect, not to say false.⁴ This can also be shown by considering that such a concept of body does not exclude cessation or rest from matter and cannot provide reasons for the laws of nature which apply to derivative force.

Passive force is likewise of two kinds – primitive and derivative. The *primitive force* of *suffering* or of *resisting* constitutes the very thing which the Scholastics call *materia prima*, if rightly interpreted. It brings it about, namely, that one body is not penetrated by another but opposes an obstacle to it and is at the same time possessed of a kind of laziness, so to speak, or a repugnance to motion, and so does not allow itself to be set in motion without somewhat breaking the force of the body acting upon it. Hence the *derivative force* of *suffering* thereafter shows itself in various ways in *secondary matter*.⁵ But setting aside these general and primary considerations, and having established the fact that every body acts by virtue of its form and suffers or resists by virtue of its matter, we must now proceed to the doctrine of *derivative forces* and *resistances* and discuss the question of how bodies prevail over or resist each other in various ways by their varied impulses. For to these derivative forces apply the laws of action, which are not only known by reason but also verified by sense itself through phenomena.

Here, therefore, we understand by derivative force, or the force by which bodies actually act and are acted upon by each other, only that force which is connected with motion (local motion, that is) and which in turn tends to produce further local motion. For we admit that all other material phenomena can be explained through local motion. Motion is the continuous change of place and thus requires time. But as the moving body has its motion in time, so it has a velocity at every moment of time, a velocity which is the greater in the degree that more space is passed through in less expenditure of time. This velocity along with direction is called *conatus*. *Impetus*. however, consists in the product of the mass $[molis]^6$ of the body by its velocity, and so its quantity is that which Cartesians usually call the quantity of motion, that is, the momentaneous quantity, although speaking more accurately, the quantity of motion, having an existence in time, is an integral of the impetuses (whether equal or unequal) existing in the moving body multiplied by the corresponding intervals of time. In our debate with the Cartesians, however, we have followed their way of speaking. Yet in the scientific use of terms, as we may conveniently distinguish an increase which has already taken place, or one still to come, from one which is now occurring, designating this latter as the increment or element of the increase; so we can distinguish the falling of a body at the present moment from the fall which has already taken place which it increases. So we can also distinguish the present or instantaneous element of motion from the motion extended through time and call it 'motion'. Then what is popularly called motion would be called *quantity* of *motion*. But although we can readily comply with any accepted terminology after its meaning is established, we must be careful about terms until this is done, in order not to be misled by their ambiguity.⁷

Furthermore, just as the calculation of motion carried out through time is integrated from an infinite number of impetuses, so in turn the impetus itself (even though it is a momentaneous thing) arises from a succession of an infinite number of impacts on the same moving body; so it too contains a certain element from which it can arise only through infinite repetitions. Assume a tube AC rotating about a fixed center C with a definite uniform velocity and in the horizontal plane of this page (Figure 29). Assume a ball B moving within the tube without any chain or impediment and hence beginning to move by centrifugal force. It is obvious that the beginning of the conatus of receding from the center (the conatus, namely, by which the ball B tends toward the end of the tube A) is infinitely small with respect to the impetus which it already has from the rotation or that by which the ball B tends from D to D' along with the tube itself, while retaining its distance from the center. But if the centrifugal impulsion proceeding from the rotation is continued for some time, there must arise in the ball, from its own progression, a certain complete centrifugal impetus D'B' comparable to the impetus of rotation DD'. Hence the nisus is obviously twofold, an elementary or infinitely small one which I also call a *solicitation* and one formed by the continuation or repetition of these elementary impulsions, that is, the impetus itself. But I do not mean that these mathematical entities are really found in nature as such but merely that they are means of making accurate calculations of an abstract mental kind.



Fig. 29.

Hence force is also of two kinds: the one elementary, which I also call *dead* force⁸, because motion does not yet exist in it but only a solicitation to motion, such as that of the ball in the tube or a stone in a sling even while it is still held by the string; the other is ordinary force combined with actual motion, which I call *living* force [*vis viva*]. An example of dead force is centrifugal force, and likewise the force of gravity or centripetal force; also the force with which a stretched elastic body begins to restore itself. But in impact, whether this arises from a heavy body which has been falling for some time, or from a bow which has been restoring itself for some time, or from some similar cause, the force. This is what Galileo meant when in an enigmatic way, he called the force of impact infinite as compared with the simple impulsion of gravity. But even though impetus is always combined with living force, the two are nonetheless different, as we shall show below.

Living force in any aggregate of bodies can further be understood in two senses – namely, as total and partial. Partial force in turn is either relative or directive, that is, either proper to the parts themselves or common to all. Respective or proper force is that by which the bodies included in an aggregate can interact upon each other; directive or common force is that by which the aggregate can itself also act externally. I call this 'directive' because the integral force of total direction is conserved in this partial force. Moreover, if it were assumed that the aggregate should suddenly become rigid by the cessation of the motion of the parts relative to each other, this alone would be left. Thus absolute total force is composed of relative and directive force taken together. But this can be understood better from the rules to be treated below.⁹

So far as we know, the ancients had a knowledge of dead force only, and it is this which is commonly called mechanics, which deals with the lever, the pulley, the inclined plane (applicable to the wedge and screw), the equilibrium of liquids, and similar matters concerned only with the primary conatus of bodies in itself, before they take on an impetus through action. Although the laws of dead force can be carried over. in a certain way, to living force, yet great caution is necessary, for it is at this point that those who confused force in general with the quantity resulting from the product of mass by velocity were misled because they saw that dead force is proportional to these factors. As we pointed out long ago, this happens for a special reason, namely, that when for example, different heavy bodies fall, the descent itself or the quantities of space passed through in the descent are, at the very beginning of motion while they remain infinitely small or elementary, proportional to the velocities or to the conatuses of descent. But when some progress has been made and living force has developed, the acquired velocities are no longer proportional to the spaces already passed through in the descent but only to their elements.¹⁰ Yet we have already shown, and will show more fully, that the force must be calculated in terms of these spaces themselves. Though he used another name, and indeed, another concept, Galileo began the treatment of living force and was the first to explain how motion arises from the acceleration of heavy falling bodies. Descartes rightly distinguished between velocity and direction and also saw that in the collision of bodies that state results which least changes the prior conditions. But he did not rightly estimate this minimum change, since he changes either the direction alone or the velocity alone, while the whole change must be determined by the joint effect of both together. He failed to see how this was possible, however, because two such heterogeneous things did not seem to him to be capable of comparison or of simultaneous treatment – he being concerned with modalities rather than with realities in this connection¹¹; not to speak of his other errors in his teachings on this problem.

Honoratius Fabri, Marcus Marci, John Alph. Borelli, Ignatius Baptista Pardies, Claude Deschales, and other most acute men have given us things that are not to be despised in the doctrine of motion, yet they have not avoided these capital errors.¹² So far as I know, Huygens, whose brilliant discoveries have enlightened our age, was also the first to arrive at the pure and transparent truth in this matter, and to free this doctrine from fallacies, by formulating certain rules which were published long ago. Almost the same rules were obtained by Wren, Wallis, and Mariotte, all excellent men in this field, though in differing measure.¹³ But there is no unity of opinion about the causes; hence men who are outstanding in these studies do not always accept the same conclusions. It would seem, indeed, that the true foundations of this science have not yet been revealed. Not everyone has accepted the proposition which seems certain to me - that rebound or reflection results only from elastic force, that is, from the resistance offered by an internal motion. Nor has anyone before me explained the concept of forces itself, a matter which has always disturbed the Cartesians and others who could not understand that the sum of motion or of impetuses, which they take for the quantity of forces, can be different after collision than it was before, because they believed that such a change would change the quantity of forces as well.

While I was still a youth and followed Democritus, and Gassendi and Descartes, his disciples in this matter, in holding that the nature of body consists in inert mass alone, I brought out a small book entitled A Physical Hypothesis, in which I expounded a theory of both abstract and concrete motion. This writing seems to have pleased many distinguished men far more than its mediocrity deserved. There I set up the proposition that assuming this conception of the nature of body to be true, every colliding body must give its conatus to the body receiving the blow or directly opposing it as such. For since it tries to proceed in the moment of impact, and thus to carry the opposing body with it, and (because of the indifference to motion and rest which I then held bodies to have) this conatus must have its full effect upon the opposing body unless it is impeded by a contrary conatus, and indeed, even if it is impeded, since these diverse conatuses must be compounded with each other, it was obvious that no reason could be given why the colliding body should not attain the effect to which it strives, or why the opposing body should not receive the full conatus of the colliding one, so that the motion of the opposing body would be compounded of its own original motion and of that newly received from the external conatus. From this I showed further that if the body is understood in mathematical terms only – magnitude, figure, position, and their change – and conatus is admitted only at the moment of impact itself, no use being made of metaphysical notions such as active power in form, or of passive power and resistance to motion in matter, if therefore it is necessary to determine the outcome of the collision solely by the geometric composition of conatuses, as I have explained; then it must follow that the conatus of even the smallest colliding body must be transmitted to even the largest receiving body, and thus that the largest body at rest will be carried away by a colliding body, no matter how small, without any retardation of its motion, since such a notion of matter involves no resistance to motion but rather indifference to it. Thus it would be no more difficult to move a large body than a small one, and hence there would be action without reaction, and no estimation of power would be possible, since anything could be accomplished by anything.¹⁴ Since this and many other matters of the same kind are contrary to the order of things, and in conflict with the principles of a true metaphysics, I believed at that time (and indeed, rightly) that in this construction of the system the most wise Author of things had particularly avoided these things which would have followed per se from the bare laws of motion derived from pure geometry.

Later, however, after I had examined everything more thoroughly, I saw wherein the systematic explanation of things consists and discovered that my earlier hypothesis about the definition of a body was incomplete. In this very fact, along with other arguments, I found a proof that something more than magnitude and impenetrability must be assumed in body, from which an interpretation of forces may arise. By adding the metaphysical laws of this factor to the laws of extension, there arise those rules of motion which I should call systematic – namely, that all change occurs gradually, that

every action involves a reaction, that no new force is produced without diminishing the earlier force, so that a body which carries another with it is retarded by the body carried away, and that there is neither more nor less power in the effect than in the cause. Since this law is not derived from the concept of mass, it must follow from something else which is in bodies, namely, from force itself, which always preserves the same quantity even though it is used by different bodies. I concluded, therefore, that besides purely mathematical principles subject to the imagination, there must be admitted certain metaphysical principles perceptible only by the mind and that a certain higher and so to speak, formal principle must be added to that of material mass, since all the truths about corporeal things cannot be derived from logical and geometrical axioms alone, namely, those of great and small, whole and part, figure and situation, but that there must be added those of cause and effect, action and passion, in order to give a reasonable account of the order of things. Whether we call this principle form, entelechy, or force does not matter provided that we remember that it can be explained intelligibly only through the concept of forces.

I cannot agree with certain prominent men today, however, who see the inadequacy of the popular concept of matter, but call in God *ex machina* and remove all force of action from things themselves, as is done in a work on the Mosaic Philosophy, as Fludd called it. For although I should agree that they have shown clearly that there can be no distinct influx of one created substance into another, if the matter is taken in metaphysical rigor, and I also admit freely that all things arise by a continuous creation from God, yet I think that there is no natural truth in things for which we must find the reason in the divine action or will but that God has always put into things themselves some properties by which all their predicates can be explained. Certainly God has created not only bodies but also souls, to which the primitive entelechies correspond. But I shall demonstrate these matters elsewhere by following out their proper reasons more thoroughly.

Meanwhile, even though I hold that an active principle which is superior to material concepts and so to speak, vital exists everywhere in bodies, I do not agree with Henry More and other men distinguished for piety and spirit, who make use of some Archeus – I know not what – or hylarchic principle, even to explain phenomena; as if there are some things in nature which cannot be explained mechanically and as if those who undertake a mechanical explanation aim to deny incorporeal beings, with a suspicion of impiety – or as if it were necessary to appoint intelligences for the revolving starry orbs as Aristotle did, or to say that the elements are driven upward and downward by their own form, a teaching that is certainly as naïve as it is fruitless.¹⁵

With these things, I say, I do not agree, and this philosophy pleases me no more than the theology of those who believed so firmly that Jupiter thunders and snows that they even branded those who sought after the specific causes of these things with the crime of atheism. In my judgment the best answer, which satisfies piety and science alike, is to acknowledge that all phenomena are indeed to be explained by mechanical efficient causes but that these mechanical laws are themselves to be derived in general from higher reasons and that we thus use a higher efficient cause only to establish the general and remote principles. Once this is established, we need not admit entelechies any more than we admit superfluous faculties or inexplicable sympathies, as long as we are dealing only with the immediate and particular efficient causes of natural things. For the first and most universal efficient cause must not enter into special problems, aside from our viewing the ends to which the Divine Wisdom adhered in ordering things in such a way, lest we neglect any opportunity to sing the most beautiful hymns in his praise.

In fact, as I have shown by the remarkable example of the principles of optics, the celebrated Molyneux having warmly approved my interpretation in his Dioptrics¹⁶, final causes may be introduced with great fruitfulness even into the special problems of physics, not merely to increase our admiration for the most beautiful works of the supreme Author, but also to help us make predictions by means of them which would not be as apparent, except perhaps hypothetically, through the use of efficient causes. Philosophers have in the past perhaps not sufficiently observed this advantage of final causes. It must be maintained in general that all existent facts can be explained in two ways - through a kingdom of power or efficient causes and through a kingdom of wisdom or *final causes*; that God regulates bodies as machines in an architectural manner according to laws of magnitude or of mathematics but does so for the benefit of souls and that he rules over souls, on the other hand, which are capable of wisdom, as over citizens and members of the same society with himself, in the manner of a prince or indeed of a father, ruling to his own glory according to the laws of goodness or of morality. Thus these two kingdoms everywhere permeate each other, yet their laws are never confused and never disturbed, so that the maximum in the kingdom of power, and the best in the kingdom of wisdom, take place together. But here we have undertaken to set up the general rules for effective forces, which we can then use in explaining special efficient causes.

Next I arrived at a true estimation of forces and at exactly the same one, moreover, by widely different ways. One was a priori, based on the most simple consideration of space, time, and action; this I shall explain elsewhere. The other was a posteriori, by calculating force by the effect it produces in expending itself. For by effect I mean here not any effect whatever but that for which force is expended or consumed and which may therefore be called *violent*. The force which a heavy body exercises in moving along a perfectly horizontal plane is not of this kind, because however far such an effect is prolonged, it always retains the same force, and though we use the same principle in calculating this effect also, which we may call harmless, we now exclude it from consideration. Moreover, I choose that particular form of violent force which is most capable of homogeneity or of division into similar and equal parts, such as is found in the ascent of a body endowed with gravity. For the elevation of a heavy body to 2 or 3 feet is exactly double or triple the elevation of the same weight to 1 foot. Hence the elevation of a body twice as heavy to 3 feet is exactly six times as great as the elevation of a simple body for 1 foot, assuming of course, for the sake of exposition, that heavy bodies have the same weight at different heights, for though this is not true in fact, the error will be imperceptible. Homogeneity is not so easily established in elastic bodies. Therefore, since I wanted to compare bodies with different masses and different velocities, I saw of course that if body A is a simple unit and body B twice as large, but the velocity is equal in the two cases, the force of the former must be a simple unit and that of the latter double, since whatever is assumed to occur once in the former must be taken to occur exactly twice in the latter. For in B the matter is twice the equal and equivalent of A, and nothing more. But if the bodies Aand C are equal, but the velocity of A is simple and that of C twice as large, I saw that the properties of A would not be exactly doubled in C, since, though the velocity

is doubled, the body is not also. I saw that an error had been committed here by those who think that force is itself doubled merely by the doubling of a property. I have already observed and explained long ago that we do not yet have the true *art of calculating* in spite of the many 'Elements of Universal Mathematics' that have been written and that this art consists in arriving at length at homogeneous things, that is, at an accurate and complete duplication of things as well as their properties. No better or more illuminating example of this method can be given than the one shown in this argument.¹⁷

In order to obtain these results, therefore, I considered whether these two bodies, equal in magnitude but different in velocity, could produce any effects that were both equipollent with their causes and homogeneous with each other. In this way things which cannot easily be compared directly can at least be compared accurately through their effects. I assumed that an effect must be equal to its cause if the entire force is expended or consumed in producing it; the length of time taken to produce the effect does not matter here. Assume therefore that bodies A and C are equally heavy and that their force is converted into an ascent, as happens if they are understood to be at the ends of vertical pendulums PA and EC at the moment at which they receive the given velocities, A's velocity being simple and C's twice as large (Figure 30). But the



demonstrations of Galileo and others have established the fact that if body A, with a velocity of 1, ascends to a maximum height A_2H of 1 foot over the horizontal, the body C, with a velocity of 2, will ascend to a height C_2R of 4 feet. Hence it follows that a heavy body whose velocity is double that of another has a force four times that of the other, since the expenditure of all its force can accomplish exactly four times as much. In lifting 1 pound (that is, itself) 4 feet, it lifts 1 pound 1 foot exactly four times. In the same way we can conclude in general that the forces of equal bodies are proportional to the squares of their velocities and that the forces of bodies in general are proportional, compositely, to their simple masses and the squares of their velocities.

I have confirmed the same thing by reducing the contrary opinion, which is popularly accepted, especially among the Cartesians, to an absurdity, namely, to perpetual motion. According to this opinion, forces are believed to be proportional to the pro-

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ducts of their masses and velocities. I have also sometimes used this method to give an a posteriori definition of two states unequal in force and at the same time to provide a sure criterion for distinguishing the greater from the smaller. If by substituting either for the other, we can get a perpetual mechanical motion or an effect greater than its cause, then these forces are not in the least equipollent, but that which was substituted for the other was more powerful, since it brought something greater to pass. I assume it to be certain, however, that nature never substitutes for forces something unequal to them but that the whole effect is always equal to the full cause. Thus we, in turn, can safely substitute things which are equal to these forces in our reckoning, freely assuming that this is just as if we actually carried out the substitution itself, without any fear¹⁸ of perpetual mechanical motion. If it were true, as men are commonly persuaded, that a heavy body A of magnitude 2 (for this is now our assumption) and velocity 1, and heavy body C with magnitude 1 and velocity 2, are equipollent to each other we should be able to substitute one for the other with impunity -a thing which is not true. For let us assume that A, with magnitude 2, has acquired a velocity of 1 in its descent A_2A_1 from the height A_2H , which is 1 foot¹⁹; and then let us substitute for it, as it exists on the level of A_1 , the weight C (which they claim is equipollent to it) with magnitude 1 and velocity 2, which ascends to C_2 , a height of 4 feet. Thus, merely by the descent of a 2-pound weight A from the height of 1 foot A_2H , we have, by substituting its supposed equipollential, brought about a rise of 1 pound to 4 feet, which is double the former effect. Therefore we have gained this much force or achieved a perpetual mechanical motion, which is absurd. Whether we can actually accomplish this substitution through the laws of motion is irrevelant, for equipollents can safely be substituted for each other in the mind. But I have also thought out various methods by which we can actually carry out, as nearly as we wish, the transfer of the whole force of the body A to the body C, previously at rest, so that C alone is placed in motion while A is brought to rest. The result should be that to the double weight of velocity 1 there would succeed a 1-pound weight with velocity 2, if these are equipollent, and as we have shown, an absurdity arises from this. These considerations are not worthless, nor are they merely verbal, for they have important applications in the comparison of machines and motions. For if enough force is received, from water power, animals, or some other cause, to keep a heavy body of 100 pounds in constant motion, so that it can complete a horizontal circle 30 feet in diameter in a fourth of a minute, and someone claims that a weight twice as large put in its place would complete half the circle in the same time, and with less expenditure of power, and claims that this means a profit to you, you may know that you are being deceived and are losing half of the force. But having now put these errors to flight, we will propound the true and genuinely admirable laws of nature a little more distinctly in the second part of this study.20

Part II

The fact that the nature of body, and indeed of substance in general, is not well enough understood has resulted, as we have already suggested, in outstanding philosophers of our time locating the notion of body in extension alone and being driven therefore to take refuge in God to explain the union between soul and body and even the communication between bodies themselves. For it must be admitted that it is impossible for mere extension, which involves only geometric concepts, to be capable of action and

passion. So only one possibility seemed to remain for them - that when man thinks and starts to move his arm, God, as if by an original agreement, moves his arm for him; and conversely, that when there is a motion in the blood and animal spirits, God excites a perception in the soul. But such views are foreign to the true method of philosophizing and should have shown their authors that they were depending on a false principle and had not set up a correct concept of substance, since such consequences followed from it. We show, therefore, that there is in every substance a force of action and that if it is created substance, there is also a force of suffering. We show too that the concept of extension is not complete in itself but requires a relation to something which is extended and whose diffusion or continuous repetition it implies, and therefore that it presupposes also a bodily substance which involves the power to act and resist, and which exists everywhere as corporeal mass, the diffusion of which is contained in extension. Sometime we shall use this view to throw new light on the union of body and soul.²¹ But now we must show that there follow from it wonderful and most useful practical theorems which apply to dynamics, the science which deals particularly with the laws of corporeal forces.

First of all, we must recognize that force is something absolutely real even in created substances but that space, time, and motion have something akin to a mental construction [de ente rationis] and are not true and real per se but only insofar as they involve the divine attributes of immensity, eternity, and activity or the force of created substances. Hence it follows at once that there is no vacuum in space and time; that motion apart from force (or insofar as it involves only a consideration of the geometric concepts of magnitude, figure, and their variations) is in fact nothing but change of situation; and thus that motion insofar as it is phenomenal consists in a mere relationship. Descartes, too, acknowledged this when he defined it as translation from the position of one body to the position of another. But he forgot his definition when he deduced its consequences and set up rules of motion as if motion were something real and absolute. Therefore, we must hold that if any number of bodies are in motion, we cannot determine from the phenomena which of them are in absolute determinate motion or rest; rest can be attributed to any one of them you may choose, and yet the same phenomena will be produced. It follows therefore (Descartes did not notice this) that the equivalence of hypotheses is not changed by the impact of bodies upon each other and that such rules of motion must be set up that the relative nature of motion is saved, that is, so that the phenomena resulting from the collision provide no basis for determining where there was rest or determinate absolute motion before the collision. Thus the rule of Descartes, which claims that a body at rest can in no way be driven from its place by a smaller body, hardly squares with the truth. He has other rules of this kind, too, than which nothing is further from the truth. It also follows from the relative nature of motion that the action of bodies upon each other or their force of percussion is the same, provided they approach each other at the same velocity. That is to say, if the given phenomena appear the same, whatever may be the true hypothesis or however we may ascribe motion or rest to them, the same result will be produced in the unknown or the resulting phenomena, even with respect to the action of bodies upon each other. This conforms to our experience; we will feel the same pain whether our hand strikes a stone which is at rest, suspended from a thread, if you will, or the stone strikes our hand at rest with the same velocity. Meanwhile we speak as the situation demands, in whatever way provides the more fitting and simpler explanation of the phenomena, just as we make use of the motion of a *primum mobile* in the study of spheres and must use the Copernican hypothesis in planetary theory. Thus we already cause those violent arguments which have been carried on with so much energy, even by theologians, to disappear completely.²² For even though force is something real and absolute, motion belongs to the class of relative phenomena, and truth is found not so much in phenomena as in their causes.

From our concepts of body and of forces there arises also this principle – that whatever happens in substances must be understood to happen spontaneously and in an orderly way. With this is connected the principle that no change occurs through a leap. If this is established, it follows also that there can be no atoms. That the force of this conclusion may be grasped, let us assume that bodies A and B collide, that A_1 comes to A_2 and B_1 to B_2 , and that, colliding at A_2B_2 , they are deflected from A_2 to A_3 and from B_2 to B_3 (Figure 31). Assuming then, that there are atoms, that is, bodies of



maximum hardness and therefore inflexible, change would obviously occur through a leap or in a moment, for the direct motion becomes retrograde at the very moment of collision, unless we assume that the bodies rest instantaneously after the collision, that is, that they lose their force – a thing which, besides being absurd on other grounds, would still contain a change through a leap, namely, an instantaneous change from motion to rest without passing through intermediate degrees. We must thus recognize that if bodies A and B collide and come from A_1 and B_1 to the place of collision A_2B_2 , they are there gradually compressed like two inflated balls, and approach each other more and more as the pressure is continuously increased; but that the motion is weakened by this very fact and the force of the conatus carried over into the elasticity of the bodies, so that they then come entirely to rest (Figure 32). Then as



the elasticity of the bodies restores itself, they rebound from each other in a retrograde motion beginning from rest and increasing continuously, at last regaining the same velocity with which they had approached each other, but in the opposite direction so that they regress and return to the positions A_3 and B_3 , which coincide with A_1 and

 B_1 if the bodies are assumed equal and with equal velocities. From this it is now clear that no change occurs through a leap but only by a gradually diminished progression finally reduced to rest, after which regression begins. Just so, one figure is not made from another (an oval from a circle, for instance) except by innumerable intermediate figures, and nothing passes from one place to another or from one time to another except by passing through all the intermediate places or times. Thus rest, and much less a motion in the opposite direction, cannot come from motion except through all the intervening degrees of motion. This is of such great importance in nature that I wonder that it has been so little noticed. There follows also from these matters the view which Descartes attacked in his letters and which some great men are even now unwilling to admit – that all rebound arises from elasticity, and a reason is given for many brilliant experiments which show that a body is bent before it is propelled; Mariotte has shown this most beautifully. Finally, there follows also that most admirable principle of all – that there is no body, however small, which has no elasticity and is not thus permeated by a still subtler fluid; and thus that there are no elementary bodies, nor any most fluid matter, nor any solid globes of some second element, I know not what; but that analysis proceeds to the infinite.

It is also in agreement with this *law of continuity*, which excludes a leap from change, that the case of rest can be considered as a special case of motion, namely, the case of a disappearing or minimal motion, and that the case of equality can be held for a case of disappearing inequality. The consequence is that the laws of motion must be set up in such a way that particular rules are not necessary for equal and resting bodies, but that these arise from the rules for unequal and moving bodies as such. Or if we wish to formulate particular rules for rest and equality, we must be careful not to set up such rules as do not agree with a hypothesis which considers rest as the limit of motion, and equality as the smallest inequality. Otherwise we shall violate the harmony of things, and our rules will not agree with each other. I first published this new device for testing our own rules and those of others in the Nouvelles de la république des *lettres* for July, 1687, Article 8, and called it a general principle of order arising from the concept of the infinite and the continuous, adding to this the axiom that as the data are ordered, the unknowns are also ordered [datis ordinatis etiam quaesita sunt ordinata]. I expressed the matter universally in this way – if in a given series one value approaches another value continuously, and at length disappears into it, the results dependent on these values in the unknown series must also necessarily approach each other continuously and at length end in each other. So in geometry, for example, the case of an ellipse continuously approaches that of a parabola as one focus remains fixed and the other is moved farther and farther away, until the ellipse goes over into a parabola when the focus is removed infinitely. Therefore all the rules for the ellipse must of necessity be verified in the parabola (understood as an ellipse whose second focus is at an infinite distance). Hence rays striking a parabola in parallel lines can be conceived as coming from the other focus or as tending toward it. Therefore, since the case in which the body A strikes B in motion can be varied continuously in the same way, so that as the motion of A remains constant, the motion of B can be assumed to be greater or smaller and at length to disappear into rest and then into increasing motion in the contrary direction, I maintain that the result of the collision when both are in motion, whether it be the result in A or in B, must continuously approach the result of the collision in the case when B is at rest, and must finally merge with it. So the case of

rest in the given series as well as its results in the unknown series is the limit of the cases of directed motion, or the common limit of linear or continuous motion, and so, as it were, a special case of both. When I examined the Cartesian rules of motion by means of this touchstone, which I carried over from geometry into physics, it turned out that a kind of hiatus or leap was revealed which is contrary to the nature of things, for when the quantities involved were expressed graphically, the motion of B before collision in all its cases being taken for the abscissas, and the motion of B after collision as the unknown, for the ordinates, and a line was drawn through the ordinates according to their values by Descartes's rules, this line proved not to be one continuum but something with amazing gaps, with leaps of an absurd and unintelligible kind.²³ On that occasion I had also observed that the rules of the Reverend Father Malebranche did not meet this test in all respects, and after weighing the matter again with his usual candor, that distinguished man admitted that this led him to change his rules, and he brought out a small book to this effect. Yet it must be admitted that he had not yet sufficiently mastered the use of this new device and has left things which even now do not yet fit together completely.²⁴

From what has been said it also follows, remarkably, that every passion of a body is spontaneous or arises from an internal force, though upon an external occasion. But I mean by this the passion proper to it, which arises from percussion, or which remains the same whatever hypothesis may be chosen or to whatever body we may ascribe rest or motion. For since the percussion is the same regardless of what body the true motion belongs to, it follows that the effect of percussion will be equally distributed between both, and thus that both act equally in the collision, so that half of the effect comes from the action of one, the other half from the action of the other. And since half of the effect or passion is also in one and half in the other, it suffices to derive the passion which is in one from the action which is in it, so that we need no influence of one upon the other²⁵; even though the action of one provides an occasion for the other to produce a change within itself. Certainly when A and B collide, the resistance of the bodies combined with elasticity causes them to be compressed through the percussion, and the compression is equal in both, whatever may be the hypothesis about their original motion. Experiments show this, too, if we let two inflated balls collide, whether both are in motion or one is at rest, and even if the one at rest is suspended from a string so that it can swing back with ease, for if the velocity of approach or relative velocity is always the same, the compression or elastic tension will be the same and will be equal in both. Then the balls A and B will restore themselves by force of the active elasticity compressed within them, repel each other, and burst apart as if driven by a bow, each being driven back from the other with equal force, and thus receding, not by force of the other, but by its own force. But what is true of inflated balls must be understood of every body insofar as it suffers in percussion. Repercussion and repulsion, namely, arise from elasticity within the body itself, or from the motion of an ethereal fluid matter which permeates it, and so from an internal force existing within it. But, as I have said, I mean the proper motion, belonging to the bodies, separate from the common motion, or motion which can be ascribed to their common center of gravity; hence their proper motion is to be thought of (in a hypothetical way) as if they were carried in a ship which has a motion common to their center of gravity, while they themselves move in such a way that the phenomena can be saved, both with regard to the composite motion common to the ship or to their center and that proper

to themselves.²⁶ It is also understood from what has been said that there is never an action of bodies without reaction and that both are equal to each other and in contrary directions.

Also, since only force and the effort arising from it at any moment exist (for as we have explained above, motion never truly exists), and every effort tends in a straight line, it follows that all motion is in straight lines, or compounded of straight lines. Hence it not only follows that whatever moves in a curve strives always to proceed in a straight line tangent to it, but there also arises here, the true notion of firmness, which one would hardly expect. For if we assume that some one of those bodies which we call firm (although nothing is in fact absolutely firm or fluid but has a certain degree of firmness and fluidity, being called firm by us only out of a predominant regard for our senses) if we assume one of these bodies to rotate about its center, its parts will strive to fly off on a tangent; indeed, they really begin to fly off. But because this separation from each other disturbs the motion of the body surrounding them, they are thus repelled or crowded into each other again, as if there were a magnetic force in the center which attracts them, or as if there were a centripetal force in the parts themselves. The result is a rotation compounded of the rectilinear effort along the tangent and this centripetal impulse together. So all curvilinear motion arises as a continuous composition of rectilinear efforts with centripetal ones, and at the same time we understand that this crowding together by the surrounding bodies is the cause of all firmness. Otherwise it would be impossible for all curvilinear motion to be composed of mere rectilinear motions. This gives us another unexpected argument against atoms. Nothing more foreign to nature can be conceived, moreover, than to seek firmness in rest, for there is never any true rest in bodies, and nothing but rest can arise from rest. But though A and B may be at rest in relation to each other, if not actually, at least relatively (accurately speaking, however, this never occurs, for no body ever preserves exactly the same distance from another for any length of time. however small), and though whatever once rests will always be at rest unless a new cause is added yet it does not follow that, because B resists a striking body, it will also resist that which separates it from others, so that A would at once follow when the resistance of B is overcome or B is itself propelled. But if true attraction, which is not found in nature, were explained from a primitive firmness, or through rest or something similar, this would certainly follow. Firmness is therefore not to be explained except as made by the crowding together by the surrounding matter. For pressure alone does not adequately explain the problem, as if only the separation of B from A is prevented; it must be understood that they do in fact separate from each other but are again driven together by the surrounding matter, so that this conservation of their union is produced by the composition of two motions. Thus those who conceive of certain slabs or imperceptible layers in bodies, like two slabs of polished marble which fit together exactly, which it is difficult to separate because of the resistance of the surrounding matter, and who explain the firmness of two sensible bodies in this way, may indeed often be speaking the truth; but since they presuppose some firmness in the slabs themselves, they have given no ultimate explanation of firmness. From these considerations it can be understood why I cannot support some of the philosophical opinions of certain great mathematicians on this matter, who admit empty space and seem not to shrink from the theory of attraction but also hold motion to be an absolute thing and claim to prove this from rotation and the centrifugal force arising from it. But since rotation arises only from a composition of rectilinear motions, it follows that

For references see p. 450

if the equipollence of hypotheses is saved in rectilinear motions, however they are assumed, it will also be saved in curvilinear motions.²⁷

It can also be understood from what has been said that the motion common to a system of bodies does not change their actions among themselves, because the relative velocity with which they approach each other and so the force of collision with which they approach each other are not changed. There follow from this the outstanding experiments which Gassendi reported in his letters about a motion imparted by a moving body which is itself being translated; he did this to answer those who thought they could infer that the earth is at rest from the motion of projectiles. It is certain, however, that if people are being transported in a large ship (assumed to be closed, or at least so constructed that the passengers cannot observe external things), and the ship moves at a great velocity, yet smoothly and without acceleration, they will have no principle by which to discern whether the ship is at rest or in motion (on the basis, that is, of what is happening within the ship), even if they play ball or carry out other motions. This must be noted in support of those who believe in the Copernican theory, which they do not rightly understand. According to them, bodies projected from the earth into the air are caught up by the air which is turning with the earth, and so follow the motion of the earth, and likewise fall back to earth as if this were at rest. This view is rightly to be judged inadequate, since the most learned men who use the Copernican hypothesis think rather that whatever is on the surface of the earth moves with the earth, and if it is shot by a bow or catapult, it carries with it the impetus impressed on it by the rotation of the earth, together with the impetus impressed by its projection. Hence, since its twofold motion is in part common with the earth, in part peculiar to its projection, it is not surprising that this common motion changes nothing. Meanwhile it must not be concealed that if projectiles can be driven so far, or the ship be conceived as so large and moving with so great a velocity that before the descent the earth or the ship will describe an arc perceptibly different from a straight line, a difference would be perceived, because then the motion of the earth or ship, being circular, would not remain common with the motion impressed on the missile by the ship or by the rotation of the earth, which was rectilinear. In the striving of heavy bodies toward a center, moreover, an external action is added which can produce a diversity of phenomena, no less than if there were a compass in the enclosed ship which pointed to the pole and which would certainly indicate a variation in the ship's direction. But whenever the equipollence of hypotheses is involved, every factor contributing to the phenomena must be included. It is also understood from these matters that the composition of motions or the resolution of one motion into two or any number whatever can safely be used, even though, according to Wallis, one brilliant man has raised plausible doubts. For the matter certainly deserves to be proved and cannot be assumed to be known in itself, as many have done.

REFERENCES

¹ See No. 45, which had appeared in the same journal the preceding year.

² The criticism here implied of the occasionalists and Malebranche becomes explicit in the *New System* (No. 47) and even more so in *On Nature Itself* (No. 53). The continuous miracle imputed to the occasionalists was involved in the denial of any proper activity according to law within created beings themselves.

³ Primitive force thus belongs to the realm of metaphysics, not of mechanical science. But it

inheres in the individual monads and the laws of their functional dependence, which would be revealed if physical analysis and synthesis could be completed.

⁴ On the nature of the corpuscular philosophy see p. 349, note 14. It is abused by any theory which regards mechanism as ultimate or applicable to substance itself. The "crude notion of a corporeal substance" advocated by Descartes thus exceeds the limits of a sound corpuscular philosophy by reducing substance to extension.

⁵ Materia prima is here defined in relation to the quality of resistance and inertia in the phenomenal body. Contrast No. 39, and p. 366, note 4, where it is related to the monad or individual substance. This dual function of *materia prima* appears throughout the subsequent discussion of matter. Cf. Introduction, Secs. III and VII. Secondary matter, on the other hand, includes the physical property of inertia and resistance in the composite body involved in physical transactions.

⁶ On Leibniz's concept of mass see p. 103, note 8, and p. 329, note 21. Mass is not equivalent to matter but is a quantitative measure of inertia or *materia prima* as experienced in *materia secunda*. But compare the distinction between *moles* and *massa* on p. 508, note 12.

⁷ See p. 301, note 2. Leibniz now defines momentary motion and aggregate motion in the sense in which he used quantity of motion and quantity of force in 1686. Every body has a velocity at a particular moment of time: v = (ds/dt). The product of the mass by this velocity is here called the quantity of momentary motion, or merely 'motion': mv = (mds/dt), while the quantity of motion over a period of time would be the integral:

$$m \int_0^{\infty} ds/dt \, dt = ms$$
.

But, since distance is proportional to v^2 , this is Leibniz's own quantity of force as defined in No. 34.

Leibniz	Modern	Formula
Conatus, momen- tary velocity	Velocity (vecto- rial)	v = ds/dt
Impetus, momen- tary motion (quantity of motion for Descartes)	Momentum	mv
Quantity of motion for Leibniz	•••••	$m\int_0^t v dt$
Dead force (sollicitation)	Force of accel- eration (cf. po- tential energy)	a=dv/dt ma
Living force	Vis viva (cf. ki- netic energy)	
(i) In a single body(ii) In a compound system(a) Absolute or		$m\int_0^t vdt = m\int_0^t ds/dt \ dt = ms \ \text{or} \ mv^2$
(a) Absolute of total force		$\sum mv^2$
direction		Σ mv

The following table presents the mathematical equivalents of Leibniz's concepts:

⁸ I. e., force of acceleration.

⁹ Not only the total living force of a material system is conserved, therefore, but also (1) the internal relative force of its members with regard to each other and (2) the total directive progress of the system. The result of (1) is that the center of gravity of the system is not changed by the motions of its component members. The principle of the conservation of direction of

progress differs from Descartes's principle of the conservation of quantity of motion in considering the algebraic, not merely the arithmetic, sum of the progressive motion (*mv*) of its members; thus motion in opposite directions would carry opposite numerical signs (cf. p. 44, note 25). In this connection see H. Poincaré, 'Note sur les principes de la mécanique dans Descartes et dans Leibnitz', in Émile Boutroux's edition of *La Monadologie*, Paris 1881, pp. 225-31.

¹⁰ That is, to the derivative of the distance, ds/dt, or v. See p. 301, note 3.

¹¹ See No. 42, Part II, on Arts. 40–44. Among Descartes's reifications of modal or conceptual differences which Leibniz had criticized are the distinction between rest and motion, the reduction of direction to the same and contrary motions, and now, the absolute separation of direction and speed. Descartes's distinction had saved his interactionism. Leibniz has just asserted that both force and direction of motion are conserved in a material system, and this implies that mind can effect neither; parallelism is thus inevitable.

¹² See p. 302, note 4. Johannes Marcus Marci von Kronland, *De proportione motus* (1639); Ignace Gaston Pardies, *Discours du mouvement locale* (1670).

¹³ On Huygens see p. 301, note 2, and p. 302, note 8. It was the laws of motion of Huygens and Wren, formulated in 1668 in response to an invitation of the Royal Society, which occasioned Leibniz's own first efforts to develop laws of motion in 1669 (see Kabitz, *Phil. des jungen Leibniz*, pp. 65–68, 135–48). Mariotte had further developed some of Wren's work.
¹⁴ See p. 329, note 23.

¹⁵ Henry More's *Opera omnia* reached Leibniz soon after publication in 1679, but his influence is probably negligible beyond a few figures of speech which Leibniz appropriated from him. His argument that force and motion are due to a hylarchic principle which imparts an 'essential spissitude' to the bodies in which it inheres is developed in the *Metaphysical Enchiridion*, chap. xiii, scholium. Cf. p. 204, note 20; p. 328, note 14; and p. 508, note 2.

¹⁶ William Molyneux's *Dioptrica nova* appeared in two volumes (1692 and 1709). His reference was to the 'Unicum opticae, catoptricae, et dioptricae principium', which Leibniz published in the *Acta eruditorum* in 1682 (see No. 50).

¹⁷ Though the following argument has already been given three times (Nos. 34, 35, and 42), it is here developed with an emphasis upon methodology, which is a special case of the general method of analysis and synthesis. In No. 34 Leibniz attributed his method of analysis and the substitution of equivalents to Huygens' analysis of the compound pendulum (see p. 302, note 8).

¹⁸ Reading metu (Dut.) for motu (G.).

¹⁹ Reading unus for minus (G.).

²⁰ Dut. adds "in the month of May". The second part was not published, however. The dynamic interpretation which Leibniz gives of his principles in this second part should be compared with their a priori logical exposition in the 'First Truths' earlier (No. 30).

²¹ The New System was published this same year (No. 47).

²² See the last letter in No. 43, and p. 420, note 12, for Leibniz's attempt to have his reconciliation of the two views accepted at Rome.

²³ Leibniz's graph is on p. 412, note 34.

²⁴ See No. 37. Malebranche undertook to correct his laws of motion as presented in the 'Recherche' in his *Traité des lois de la communication des mouvements* (1692).

²⁵ The passive force involved here corresponds to relative or proper force as defined in Part I, not to directive or total force. The instance of the ship (the prototype of popular expositions of relativity?) makes this clearer.

²⁶ To 'save phenomena' means not merely to avoid theories which contradict them but to provide analyses (here causal) that explain them.

²⁷ This criticism is directed at both Newton and Huygens, who had held that circular motion is an argument for absolute motion (see No. 43, II, III).