Disjunctive facts and superposition of states

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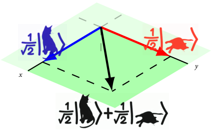

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Overview

1. Motto
2. Methodological introduction: the Socratic method
3. Confrontation of Tractatus postulates with the superposition theory
   - Maudlin's instructive presentation of quantum logic
4. Estimated costs of TLP revisions

Principle of tolerance

Carnap on logic(s)

Principle of Tolerance: It is not our business to set up prohibitions, but to arrive at conventions. …In logic, there are no morals. Everyone is at liberty to build up his own logic, i.e. his own form of language, as he wishes. All that is required of him is that, if he wishes to discuss it, he must state his methods clearly, and give syntactical rules instead of philosophical arguments.[a]

Carnap, Rudolf [1931]
Logical Syntax of Language.
(2001) London: Routledge.

[a] p.52

Φ&Φ

- There are methods of science and there are methods of philosophy.
- Let us call 'Socratic method' the method of building a theory through dialogue, the method in which the at least two interlocutors are needed, one of which is a philosopher and the other is an expert in the field that is relevant for the philosophical question under investigation.
- It is well known that many questions are of common interest both for scientists and philosophers. For example, the theories of time and space are presupposed by any empirical theory and yet it is only physics that has made explicit the theory of time and space. So, if a philosopher wants to investigate the questions of time and space using the Socratic method, then a physicist is needed for the dialogue to take place.

External consistency

- *A theory ought to be consistent!* is the first requirement to which theory building is subordinated. *The consistency of a theory ought to be proved!* is a closely related requirement and it shows that logical research is necessary part of any theory building.
- Less known is the requirement *A theory ought to be externally consistent* or *The union of theories ought to be consistent*.
- When a philosopher confronts her or his theory with a scientific theory and examines the external consistency of the former, then a variant of Socratic method is at work and a dialogue between philosophy and science takes place.

A variant of the Socratic method

- Suppose a philosophical theory is confronted with a scientific theory and external inconsistency has been discovered. Then a revision must take place. Typically, the revision will involve a contraction in which some old postulates will be abandoned, and an expansion in which some new one will be introduced.
- Like in any revision, the learning happens and probably to the benefit of both sides. The gain for philosophy is that it will acquire a new theory, superior to the old one with respect to external consistency. The gain for the science is that its implications will become explicit and thus the implicit general knowledge, active in public communication and technical application of science and in science education, will be revised and improved.

World and language

- Wittgenstein's theory exposed in *Tractatus logico-philosophicus* is beautiful in its apparent simplicity and its closeness to the common-sense. Let me summarize the main postulates:

Ontology	The world is that which is the case. The world divides into facts. A fact is an existent state of affairs.
Epistemology	A simple sentence is true if it is a picture of (if it represents) a fact.
Logic	The meaning of a sentence is defined by its truth-conditions. A complex sentence is a truth-function of its constituent parts.

Confrontation

- Let us abbreviate by TPL the aforementioned philosophical postulates.
- Let us abbreviate by SPT (superposition theory) the physical theory (possibly under certain interpretation) that admits the existence of superposition of states.
- The method will be: 1. merging the two theories, TPL∪SPT, 2. showing that TPL is externally inconsistent with respect to SPT, TPL∪SPT$\models \perp$, 3. evaluating the costs of the possible ways to revise TPL.
- Superposition theory will be extracted in two ways. First, the text analysis will be used. Second, the postulate on existence of superposition of states will be introduced relying on the most instructive Maudlin's presentation of quantum logic (2005) since SPT is clearly stated in quantum logic (Birkhoff-von Neumann, 1936)

Main references

- Birkhoff, G., and von Neumann, J. The logic of quantum mechanics. *The Annals of Mathematics 37* (1936), 823–843.
- Maudlin, T. (2005) The tale of quantum logic. *Hilary Putnam*, Y. Ben-Menahem (ed.), Cambridge University Press.
- Allday, J. (2009) *Quantum Reality : Theory and Philosophy*. Taylor & Francis

From J. Allday *Quantum Reality*

The mathematical representation of an initial quantum state uses a symbol such as $|\varphi\rangle$, and an expansion (summed list) over a series of final quantum states as follows:

$$|\varphi\rangle = a_1 |A\rangle + a_2 |B\rangle + a_3 |C\rangle + \dots$$

The amplitudes are a collection of complex numbers related to the probability that the initial state $|\varphi\rangle$ will change into *one* of $|n\rangle$ final states as the result of a measurement.

Quantum states can be formed by combining other states in quantum SUPERPOSITION. These combinations can be formed from states that would, classically, be impossible simultaneously.

◊

The textbook quote shows: 1. The author presupposes the existence of the *representation relation* between the language (of mathematical formulas) and reality (of quantum states). 2. The author admits possibility of a specific *type of combination* of simpler states into a complex state. 3. The author admits the possibility of *coexistence* of “states that would, classically, be impossible simultaneously”.

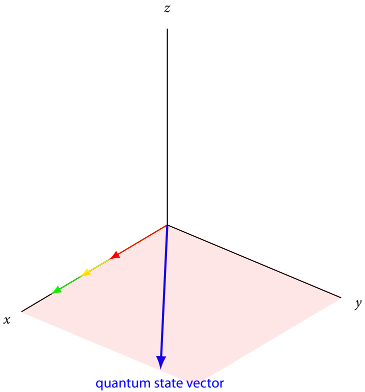
Symbols

- We will proceed to Maudlin's instructive exposition of the semantics of quantum logic in Euclidean space, which I will call 'geometrical interpretation'.
- The following symbols will be used:
  - x, y, z for axes in Euclidean space, $x - y$ for the plane passing through axes x and y.
  - $\vee$ for the disjunction of the first-order logic ('or'),
  - $\sqcup$ for the disjunction of the quantum logic ('join', 'squarecup'),
  - $\wedge$ both the FOL conjunction and $\sqcap$ for the quantum conjunction,
  - $X, Y, \ldots$ for the sentences (i.e. formulas) of mathematical language used in quantum theory.
  - $[\![X]\!]$ for geometrical interpretation of the sentence X. In geometric interpretation $[\![X]\!]$ will stand for some part of the Euclidean space.

Propositions interpreted geometrically

In quantum theory, the wave-function of a system is represented by a ray in a high-dimensional complex vector space called Hilbert space. But in order to illustrate quantum logic, we can stick to plain old Euclidean space — even a three-dimensional Euclidean space. So think of the quantum state of a system as represented by a vector in Euclidean space. Propositions in quantum logic are represented by subspaces of the Hilbert space — in our Euclidean model, you can think of a proposition as a straight line or a plane through the origin. The entire space is also a proposition — the tautological proposition, as we shall see.

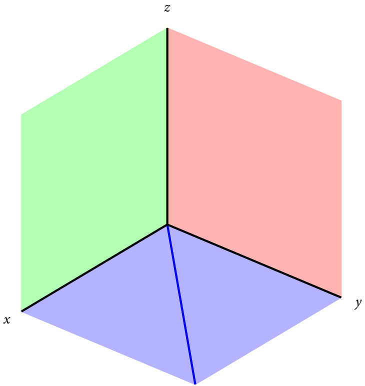
Tim Maudlin
(2005) The tale of quantum logic.
Hilary Putnam, Y. Ben-Menahem (ed.), Cambridge University Press.



In 3D geometrical interpretation proposition is a subspace: a line, a plane, or the entire space. Each subspace is a set of objects called vectors which assign a 'probability amplitude' to some state, each line has an infinite number of vectors with different probability amplitudes for the same state. Blue vector represents reality.

The entire space is also a proposition — the tautological proposition, as we shall see. A proposition in quantum logic is true of a system just in case the vector that represents the system lies in the subspace that represents the proposition. So in our little model, the x-, y-, and z-axes all represent propositions, as do the $x - y$, $y - z$, and $x - z$ planes. Similarly, the line $x = y, z = 0$ represents a proposition, and so on.

Maudlin, T. (2005) The tale of quantum logic. *Hilary Putnam*, Y. Ben-Menahem (ed.), Cambridge University Press.



The line $x = y, z = 0$.

Now think of the vector that lies in the $x - y$ plane and bisects the right angle between the positive x and y axes. According to our truth conditions, if that vector represents a system, then the proposition associated with the $x - y$ plane is true (since the vector lies in that plane) and the proposition associated with the line $x = y, z = 0$ is true (since the vector lies on that line), but the propositions associated with the x- and y-axes are not true. Let's call the vector just described "V", the proposition associated with the x-axis "X" and the proposition associated with the y-axis "Y". Let us also call the proposition associated with the $x - y$ plane "$X - Y$", and so on.

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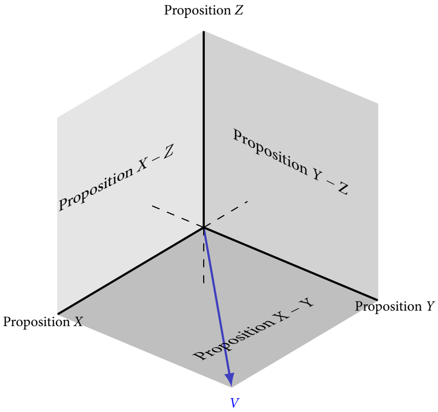

Proposition $X - Y$ is true. Propositions X, Y, Z, $X - Z$, $Y - Z$ are false. Proposition $X - Y - Z$ is a tautology and, therefore, true.

The meet is, in fact, just like the connective "$\cap$": the meet of two propositions (subspaces) is just the intersection of those subspaces. Thus, the meet of the proposition $X - Y$ and the proposition $X - Z$ is just the proposition X, since the x-axis is the intersection of the $x - y$ plane and the $x - z$ plane. In symbols, $X - Y \sqcap X - Z = X$. The meet of $X - Y$ and X is just X. And the meet of $X - Y$ with the line $x = y, z = 0$ is just that line, since it lies in the $x - y$ plane. The meet of X and Y is the point at the origin, which is the logically false proposition $\perp$, since no vector can lie within it (so it cannot be true).

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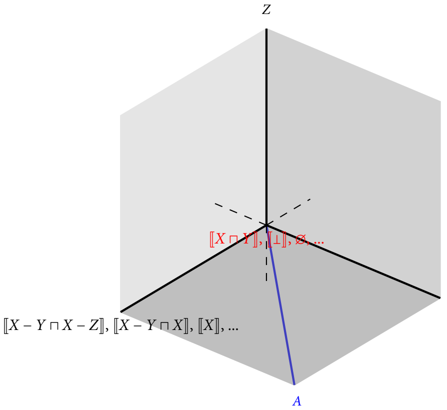


Meet of $[\![X - Y]\!]$ and $[\![A]\!]$ is $[\![A]\!]$.
Meet of $[\![X]\!]$ and $[\![Y]\!]$ is $\varnothing = [\![\perp]\!]$.

Truth

- Classical definitions of logical connectives as functions $\text{connective}^n : \{\text{true}, \text{false}\}^n \to \{\text{true}, \text{false}\}$:
  - $X \wedge Y$ is true iff X is true and Y is true.
  - $X \vee Y$ is true iff X is true or Y is true.
  - $\neg X$ is true iff X is not true.
- In order to obtain the semantic definitions for connectives of the quantum logic the following is to be done:
  1. Replace '[...] is true' with 'the vector that represents the system lies within the subspace that represents [...]'!
  2. Associate each quantum *n*-ary connective with a function from *n* subspaces to a subspace of the high-dimensional complex vector space.
- Ad 2. Subspaces can be partially ordered by inclusion since the inclusion is a reflexive, transitive and antisymmetric relation. Since any non-empty finite set of subspaces has the 'lowest upper bound' (supremum) and the 'greatest lower bound' (infimum), the partial ordering forms a lattice.

From the seminal paper

- The algebraic interpretation in Birkhoff–von Neumann seminal paper:
  - 1. *Implication [is a] partial ordering.*
    "…the experimental propositions concerning a system $\mathfrak{S}$ correspond to a family of subsets of its phase-space Σ, in such a way that "x implies y" (x and y being any two experimental propositions) means that the subset of Σ corresponding to x is contained set-theoretically in the subset corresponding to y." (X implies Y iff $[\![X]\!] \subseteq [\![Y]\!]$)
  - 2. *Implication structure forms a lattice.*
    "In any calculus of propositions, it is natural to imagine that there is a weakest proposition implying, and a strongest proposition implied by a given pair of propositions. In fact, investigations of partially ordered systems from different angles all indicate that the first property which they are likely to possess, is the existence of greatest lower bounds and least upper bounds to subsets of their elements."

Birkhoff, G., and von Neumann, J.
The logic of quantum mechanics.
The Annals of Mathematics 37 (1936), 823–843.

The easy case: conjunction and meet

- Classical conjunction: $X \wedge Y$ is true iff X is true and Y is true.
- Quantum conjunction: The vector that represents the system lies within the subspace that represents the proposition $X \sqcap Y$ iff the vector that represents the system lies within the intersection of the subspace that represents the proposition X and the subspace that represents the proposition Y.
- The quantum conjunction is interpreted as the intersection of subspaces and, consequently, the logical behaviour of the classical and the quantum conjunction is analogous.

The hard case: disjunction and join

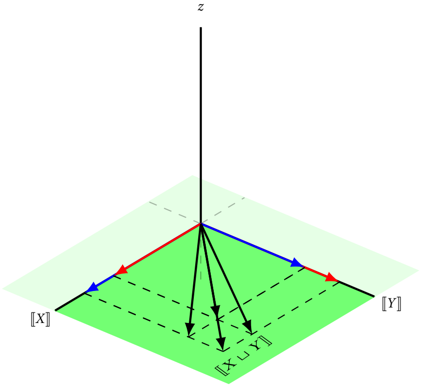
- Classical disjunction: $X \vee Y$ is true iff X is true or Y is true.
- Quantum disjunction: The vector that represents the system lies within the subspace that represents the proposition $X \sqcup Y$ iff the vector that represents the system lies within the subspace that is spanned by the subspace that represents the proposition X and the subspace that represents the proposition Y.
- The logical behaviour of the classical and the quantum disjunction is *n*ot analogous.
  - Quantum disjunction is not just the union of the subspaces associated with the disjuncts but their span. Therefore, it is possible that $X \sqcup Y$ holds while neither X nor Y holds.
  - This fact makes quantum disjunction similar to classical proof-theoretical notion of disjunction: provability of a disjunction does not imply provability of at least one disjunct. On the other hand, quantum disjunction is not to similar to the intuitionistic proof-theoretical notion of disjunction or to the dialogue logic disjunction. In intuitionistic logic provability of a disjunction demands provability of at least one disjunct. In dialogue logic the disjunction stands for the right of the opponent of the disjunction to seek from the proponent the justification for one disjunct chosen by the proponent.

Things are not so simple for the join of two propositions. The join of two propositions is represented by the subspace that is spanned by the subspaces associated with the propositions being joined. This is not the same as the union of the subspaces, since it will include vectors that lie in neither of the subspaces being joined. But the idea is still quite intuitive. Consider all of the vectors that can be made by adding a vector from subspace A to a vector from subspace B, by normal vector addition. This new set of vectors constitutes the subspace spanned by A and B. So the x-y plane is spanned by the x-axis and the y-axis, since every vector in the x-y plane can be written as the sum of a vector on the x-axis and a vector on the y axis. Or, in symbols, $X \sqcup Y = X - Y$.

Tim Maudlin

(2005) The tale of quantum logic.

Hilary Putnam, Y. Ben-Menahem (ed.), Cambridge University Press.



$$[\![X \sqcup Y]\!] = \{x + y \mid x \in [\![X]\!] \wedge y \in [\![Y]\!]\}$$

The semantics of the join is not the same as classical disjunction: therein lies the main difference between classical and quantum logic. Of course, if A is true, then so is $A \sqcup B$ for arbitrary B: if the vector which represents the system lies in the subspace associated with A, then it lies in any subspace spanned by that subspace and another. But in the opposite direction, the semantics are not like disjunction. In particular, $A \sqcup B$ can be true even though neither A nor B is true. Suppose, for example, the vector which represents the system lies in the $x-y$ plane but, like V, is parallel neither to the x-nor the y-axis. Then $X \sqcup Y$ is true even though neither X nor Y is.

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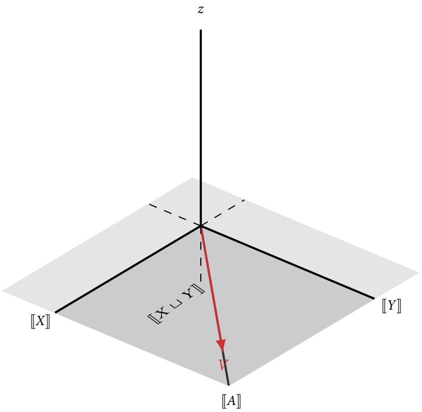


$[\![X \sqcup Y]\!] = [\![X - Y]\!]$
$[\![X \sqcup Y]\!] - ([\![X]\!] \cup [\![Y]\!]) \neq \varnothing$
$V \in [\![X \sqcup Y]\!]$
$V \notin [\![X]\!] \cup [\![Y]\!]$
Counterexample for distributivity:
$V \in [\![A \sqcap (X \sqcup Y)]\!]$ since $[\![A \sqcap (X \sqcup Y)]\!] = [\![A]\!]$ but
$V \notin [\![(A \sqcap X) \sqcup (A \sqcap Y)]\!]$ since
$[\![(A \sqcap X) \sqcup (A \sqcap Y)]\!] = \varnothing$.

Superposition

- Suppose that propositions X and Y are mutually exclusive in empirical sense: X is observed if and only if Y is not observed.
- Suppose that it makes sense to claim that the proposition $X \sqcup Y$ has a truth-value prior to observation and that it is true.
- According to the semantic definition, the vector V that represents the system lies in the subspace $[\![X \sqcup Y]\!]$.
- Assume that $V \notin [\![X]\!] \cup [\![Y]\!]$.
- Then $X \sqcup Y$ is true although neither X nor Y is true.
- Since the vector V represents the system, there must be some state of affairs σ that is represented by the vector.
- In the state of affairs σ neither X nor Y is true since $V \notin [\![X]\!] \cup [\![Y]\!]$. However, if X is not true, then Y must be true, and if Y is not true, then X must be true. So σ makes contradictions true, both $X \wedge \neg X$ and $Y \wedge \neg Y$ hold in σ. The state of affairs σ is impossible and, therefore, it does not exist.
- However, σ is represented by V and, therefore, must exist.

- There are different ways of avoiding paradox:
    - Abandon quantum LOGIC.
    - Revise ONTOLOGY by allowing 'disjunctive states of affairs'.
    - Revise EPISTEMOLOGY by abandoning realism.
    - Revise LOGIC by admitting connectives which are not truth-functional.
- Our thesis: if we keep quantum logic and opt for realism regarding the possibility of representing (the possibility of truth) of the sentence describing a non-observable states of affairs, then we are forced to revise our ontology.

Superposition as a truth-maker for quantum disjunction

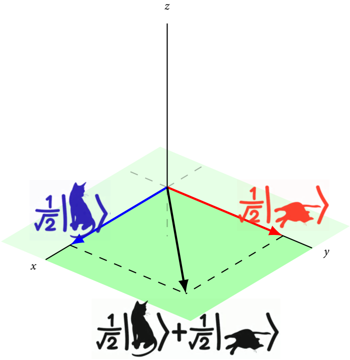
- For the antirealist the quantum disjunction can be regarded as an 'inference ticket' allowing the deduction that if $X \sqcup Y$ holds, then the measurement will show that either X or Y holds.
- In addition to this, realism requires a truth-maker for the quantum disjunction.
- A disjunctive state of affairs or disjunctive fact can play the role of a truth-maker for the quantum disjunction.
- Not everybody accepts disjunctive type of composition of states of affairs or superposition. The most famous (purported) counterexample is given by Schrd̈inger's thought experiment about cat that is both alive and dead.

Schrödinger's counterexample

One can even set up quite ridiculous cases. A cat is penned up in a steel chamber, along with the following device (which must be secured against direct interference by the cat): in a Geiger counter, there is a tiny bit of radioactive substance, so small, that perhaps in the course of the hour one of the atoms decays, but also, with equal probability, perhaps none; if it happens, the counter tube discharges and through a relay releases a hammer that shatters a small flask of hydrocyanic acid. If one has left this entire system to itself for an hour, one would say that the cat still lives if meanwhile no atom has decayed. The ψ-function of the entire system would express this by having in it the living and dead cat (pardon the expression) mixed or smeared out in equal parts.

It is typical of these cases that an indeterminacy originally restricted to the atomic domain becomes transformed into macroscopic indeterminacy, which can then be resolved by direct observation. *That prevents us from so naively accepting as valid a "blurred model" for representing reality.* In itself, it would not embody anything unclear or contradictory. There is a difference between a shaky or out-of-focus photograph and a snapshot of clouds and fog banks.

Schrödinger, E.
The present situation in quantum mechanics [1935].
Proceedings of the American Philosophical Society 124, 5 (1980), 323–338



$$[\![\text{Alive} \sqcup \text{Dead}]\!] = \{x + y \mid x \in [\![\text{Alive}]\!] \wedge y \in [\![\text{Dead}]\!]\}$$

The result

- If SPT is accepted then the either the contractions ought marked red ought to be performed over TPL or the meaning of the term 'fact' must be changed to accommodate disjunctive facts or one must abandon realistic standpoint:

Ontology	The world is that which is the case.The world divides into facts. A fact is an existent state of affairs. [1]
Epistemology	A simple sentence is true if it is a picture of (if it represents) a fact.
Logic	The meaning of a sentence is defined by its truth-conditions. A complex sentence is a truth-function of its constituent parts.

[1]The world can be divided into parts only if there is only one type of composition, one way of putting the facts together. States in the superposition cannot be said to properly exist, rather they, so to speak, "subexist".

Cost analysis

Consequences of SPT	Possible contractions in TLP	Estimated cost with respect to general knowledge
Not all sentential connectives are truth-functional.	Revise logic.	Low cost since it has been already done (e.g. modal logic).
Mutually exclusive states can co-exist in the disjunctive mode of combination of states of affairs.	Revise ontology.	High. Many opponents.
Picture relation (representation relation) holds between complex sentences and reality.	Revise epistemology.	Low.
Picture relation (representation relation) does not hold between complex sentences and reality.	Revise epistemology and adopt anti-realism.	Medium.

Thank you! Hvala!