Odabrane teme dinamične logike

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Semantika propozicijske logike

U semantici polazimo od tumačenja rečenica da bismo odredili njihova logička svojstva i odnose.
Npr. kada definiramo neovisnost rečenice R od teorije T u propozicijskoj logici, onda izričemo tvrdnju koja ide preko svih tumačenja i kažemo postoji tumačenje h u kojemu su istinite i sve rečenice iz T i rečenica $\neg R$, ali postoji također tumačenje h' u kojemu su istinite i sve rečenice iz T i rečenica R, tj. $T \not\models R$ i $T \not\models \neg R$.

Jedan svijet i odnos jezika i svijeta

- Filozofija zasnovna na logici prvoga reda, logici koja logički dio znanstvenog rječnika nalazi samo u veznicima, predikatu identiteta i dvama kvantifikatorima, izložena je u Wittgensteinovom djelu *Tractatus logico-philosophicus* (1921.).

Tractatus teorija jezika

Svijet		Prirodna znanost
$\Downarrow$		$\Uparrow$
Činjenica		Istinit iskaz
$\Downarrow$		$\Uparrow$
Stanje stvari		Iskaz
$\Downarrow$		$\Uparrow$
Odnos predmeta		Predikati i imena
▲	ODNOS SLIKE	▲

$\Downarrow$ je put razlaganja.
$\Uparrow$ je put tvorbe.

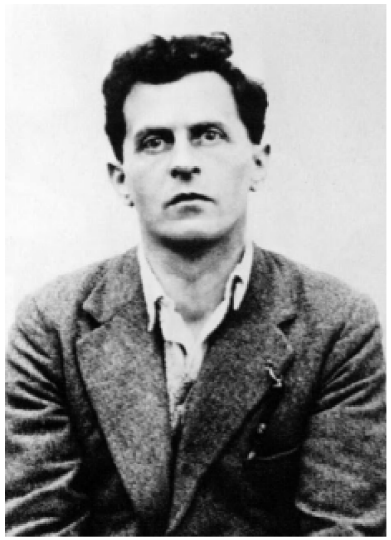
Je li jedan svijet dostatan?

- Nazovimo modalnim iskazima iskaze kojima je oblik: ‘nužno je da . . . ’, ‘obavezno je da . . . ’, ‘i vjeruje da . . . ’, ‘i želi da . . . ’, ‘uvijek će biti slučaj da . . . ’ i slične.
- Prema kriteriju iz *Tractatus*-u, modalni iskazi nisu iskazi jer nisu istinitosnofunkcionalni.
- U semantičku teoriju “jednog svijeta” ne mogu se uklopiti modalni iskaz jer istinitosna vrijednost koju njihovi “elementarni iskazi” imaju u *Svijetu* ne determinira istinitosnu vrijednost modalnog složenog iskaza.

Primjer

Istinitost tvrdnje o onome što treba biti, što je obavezno da bude, ne ovisi o onome što jest slučaj u *Svijet*u..
Neka **O** stoji za ‘Obavezno je da’.
I $p \wedge \mathbf{O}p$ i $\neg p \wedge \mathbf{O}p$ su zadovoljive.

Tractatus logico-philosophicus



5 Iskazi su istinitosne funkcije elementarnih iskaza. . . . **6.42** Stoga [. . .] etičkih iskaza ne može biti.
. . .
7 O čemu se ne može govoriti, o tome treba šutjeti..

Logička terminologija

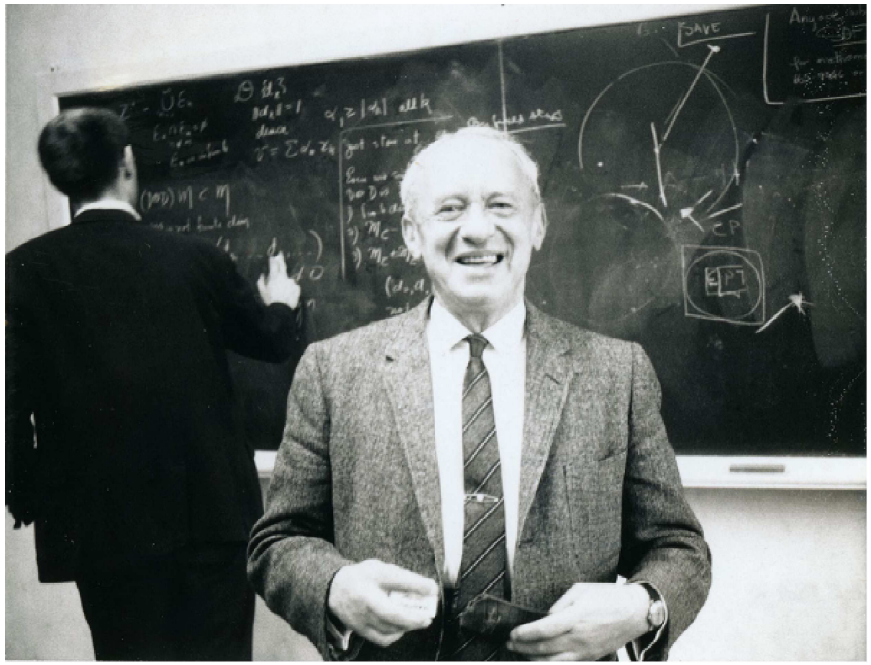
Zajednički dio rječnika

"Logika" [...] je ime discipline koja analizira značenje onih pojmova koji su zajednički svim znanostima te koja ustanovljava opće zakone koji upravljaju tim pojmovima.

Alfred Tarski.
Uvod u logiku i metodologiju deduktivnih znanosti, 1. izd. 1941.

Primjedba

Određenje logike kakvo daje Tarski, preusko je s današnjeg stajališta: on govori o logici prvog reda. Logika analizira značenje i onih pojmova koji nisu zajednički svim znanostima, npr. deontična logika. ◊



Alfred Tarski
(Varšava, 1901.–Berkeley, 1983.)

Deontična logika kao modalna logika

Kako je okrivena deontična logika

Jednoga dana dok sam se šetao obalom rijeke Cam—u to sam vrijeme živio u Cambridgeu (Engleska)—pogodila me je misao da se modalni atributi "moguće", "nemoguće" i "nužno" odnose među sobom jednako onako kako se međusobno odnose kvantifikatori "neki", "nijedan" i "svi". Uskoro sam otkrio da se formalna analogika između kvantifikatora i modalnih pojmova proteže iznad obrasca uzajmne definiblnosti . . . [Prije dovršenja knjige *An Essay in Modal Logic* 1951.] došao sam do sljedećeg neočekivanog zapažanja—ovoga puta tijekom rasprava s prijateljima—naime da normativni pojmovi o dopuštenju, zabrani i obvezi podliježu istom obrascu uzajamne povezanosti koja postoji između kvantifikatora i osnovnih modalnosti.

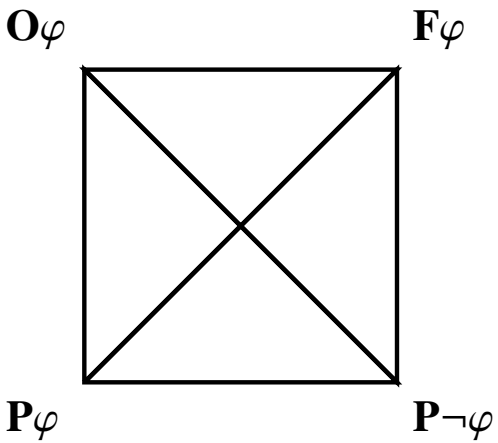
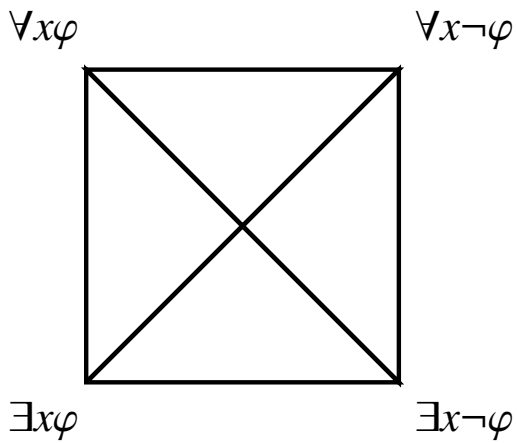
Georg Henrik von Wright.
Deontic logic: a personal view.
Ratio Juris, 12:26–38, 1999.

Ludwig Wittgenstein i Georg Henrik von Wright,
(fotografija iz 1950. snimljena u Von Wrightovom vrtu)

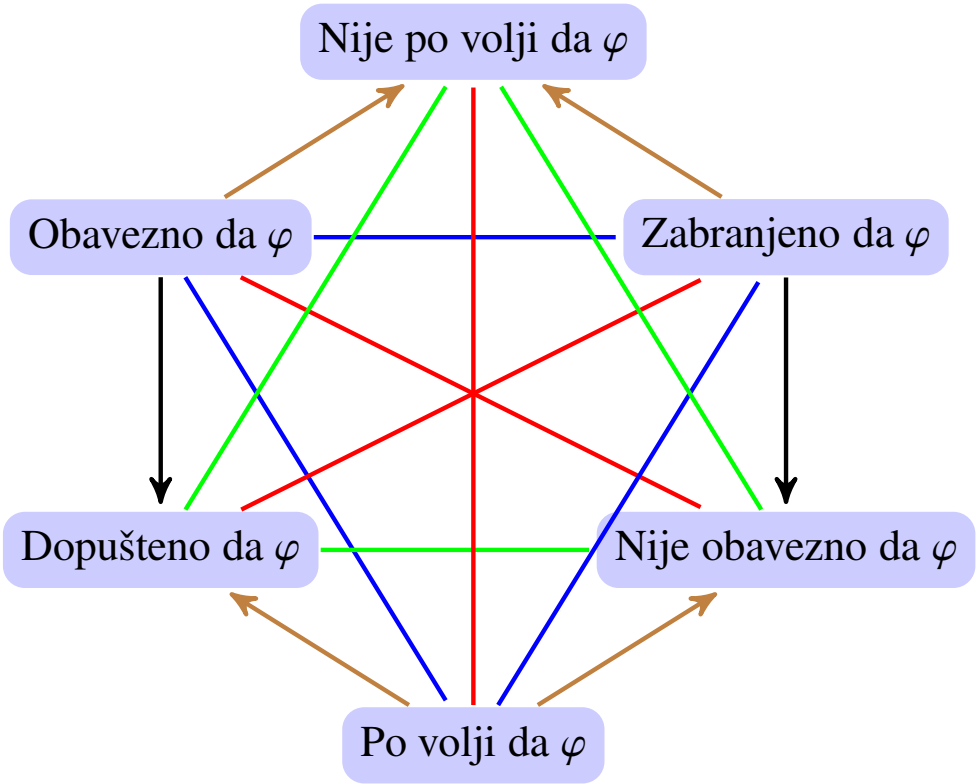
Analogija kvantifikacije i modaliteta

Dualnost; logički kvadrat

Kvantifikatori	Aletični modaliteti	Deontični modaliteti
$\forall x\varphi \quad (\neg\exists x\neg\varphi)$ Sve je ...	$\Box\varphi \quad (\neg\Diamond\neg\varphi)$ Nužno je...	$\mathbf{O}\varphi \quad (\neg\mathbf{P}\neg\varphi)$ Obvezno je ...
$\exists x\varphi \quad (\neg\forall x\neg\varphi)$ Nešto je ...	$\Diamond\varphi \quad (\neg\Box\neg\varphi)$ Moguće je ...	$\mathbf{P}\varphi \quad (\neg\mathbf{O}\neg\varphi)$ Dopušteno je ...
$\forall x\neg\varphi \quad (\neg\exists x\varphi)$ Nijedan nije ...	$\Box\neg\varphi \quad (\neg\Diamond\varphi)$ Nemoguće je ...	$\mathbf{F}\varphi \quad (\mathbf{O}\neg\varphi$, tj. $\neg\mathbf{P}\varphi)$ Zabranjeno je ...

Modalna logika	Logika prvog reda
	

Odnosi modaliteta

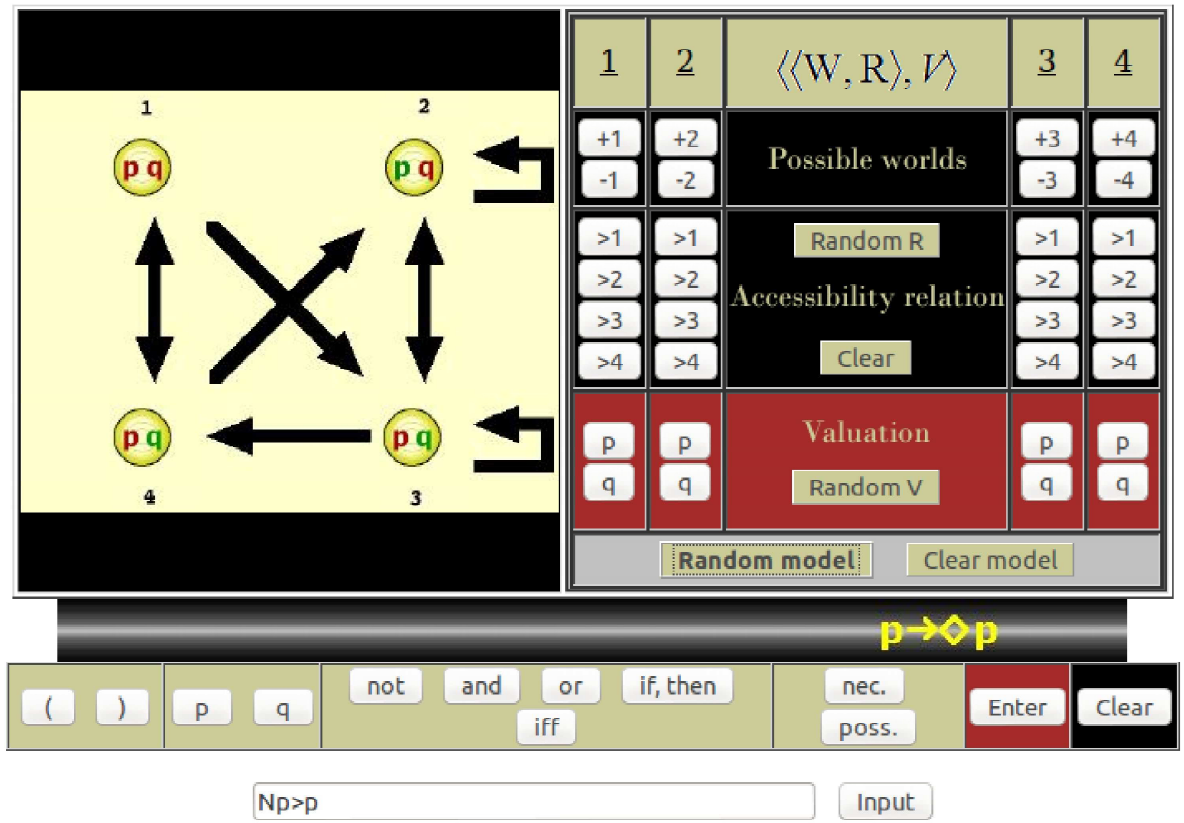


Obavezno $\mathbf{O}\varphi \leftrightarrow \neg\mathbf{P}\neg\varphi \leftrightarrow \mathbf{F}\neg\varphi$

Zabranjeno $\mathbf{F}\varphi \leftrightarrow \neg\mathbf{P}\varphi \leftrightarrow \mathbf{O}\neg\varphi$

Implikacije prikazane crnom strelicom, $\mathbf{O}\varphi \rightarrow \mathbf{P}\varphi$ i $\mathbf{F}\varphi \rightarrow \mathbf{P}\neg\varphi$, istovrijedne su.

Modalni kalkulator:
`http://www.ffst.hr/~logika/implog/calculators/modal/modal.html`
Instructions:
`http://www.ffst.hr/~logika/implog/doku.php?id=program:possible_worlds`



Aksiomi i pravila deontične logike

- Standardna deontična logika KD je normalna logika, a to znači da u njoj vrijede:
  - K aksiom(ski oblik):
$$\mathbf{O}(\varphi \to \psi) \to (\mathbf{O}\varphi \to \mathbf{O}\psi)$$
  - RN pravilo necesitacije:
$$\text{Ako } \vdash \varphi, \text{ onda } \vdash \mathbf{O}\varphi.$$
- Jedini specifični aksiom(ski oblik) deontične logike jest:
  - D aksiom(ski oblik):
$$\mathbf{O}\varphi \to \mathbf{P}\varphi$$
- Aksiom K možemo shvatiti kao tvrdnju "logičke posljedice obveza i same su obveze".
- Pravilo RN možemo shvatiti kao tvrdnju "dopuštenja su logički moguća" (obratom nad $\Box\varphi \to \mathbf{O}\varphi$).
- Aksiom D odgovara tvrdnji "dopušteno je ispuniti obvezu" ili "što se mora to se smije".

Rimsko pravo

Definicija logičke nužnosti: $\vdash \varphi$ akko $\Box\varphi$.

Lema

$\vdash \mathbf{P}\varphi \rightarrow \Diamond\varphi$

Stavak (Ultra posse nemo obligatur.)

$\vdash \mathbf{O}\varphi \rightarrow \Diamond\varphi$ *(ili, drukčije iskazano,* $\vdash \neg\Diamond\varphi \rightarrow \neg\mathbf{O}\varphi$*).*

Dokaz.

1. Pretpostavimo $\vdash \mathbf{O}\varphi$.
2. $\vdash \mathbf{P}\varphi$, iz (1) zbog D aksioma.
3. $\vdash \Diamond\varphi$, iz (2) po lemi.

□

Neočekivani rezultati

- Na pozitivnoj strani, uvođenje relacijske semantike za modalnu logiku ("semantika mogućih svjetova", istodobno i neovisno otkriće Stiga Kangera i Saula Kripkea) donijelo je neočekivane uvide. Pokazalo se da analiza značenja modalnih pojmova određivanjem za njih važećih aksioma nije poduhvat u kojemu se implicitnim definicijama unutar postojećeg jezika fiksira značenje pojmova, nego nastanak novog jezika ili, radije, otkriće svojevrsnosti logike jezika intencionalnosti, u kojem opisujemo čovjeka kao osobu, kao nositelja doživljaja, činitelja radnji i podređenika zahtjevima.
- Jezik propozicijske modalne logike razlikuje se u kategoriji od jezika propozicijske logike, potonji nema izražajnu moć koja bi omogućila razlikovanje struktura. Jezik propozicijske modalne logike izražajnu moć takve vrste ima: on može razlučiti konačne strukture do granice bisimilarnosti. Pri tome je slabiji od jezika logike prvog reda kojoj granica razlučivosti leži u izmorfizmu konačnih struktura.
- Jezik propozicijske modalne logike omogućio je projekt izgradnje univerzalnog simbolizma kao znanstvenog jezika zahvati jezik znanosti o čovjeku.

Izražajna snaga jezika propozicijske modalne loggike

Jezik propozicijske modalne logike izražajniji je od jezika propozicijske logike: on, za razliku od prvospomenutoga, može razlikovati strukture. Pitanje je koliko blizu takav jezik može doći do svog predmeta opisa.

Definicija

Bisimulacija E jest odnos između struktura $\mathfrak{M} = \langle W, R, V\rangle$ i $\mathfrak{M}' = \langle W', R', V'\rangle$ takav da

$$wEw'$$

akko:

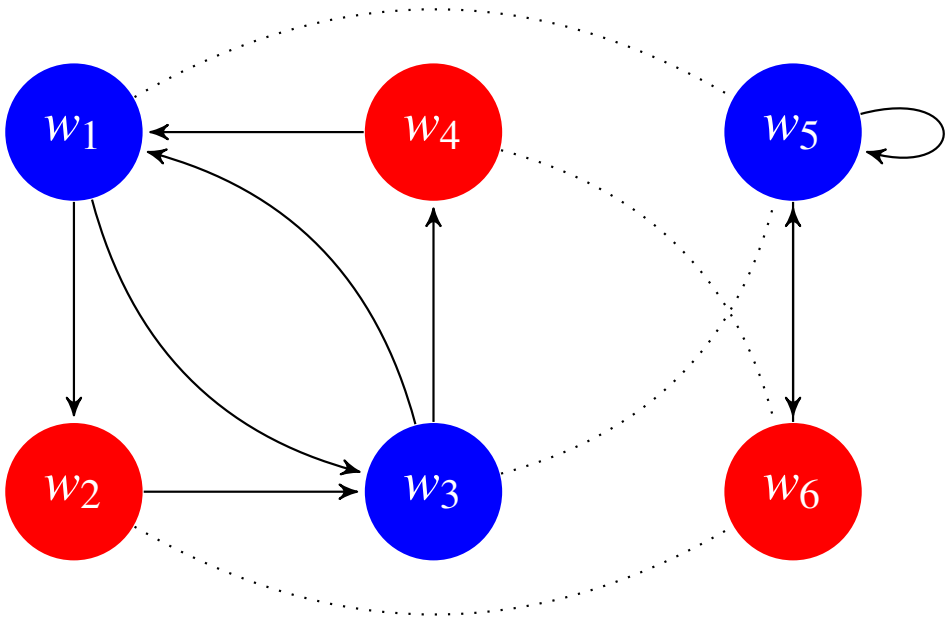
atomarna harmonija $w \in V(p) \leftrightarrow w' \in V'(p)$ za svako propozicijsko slovo p,

"naprijed" $Rwv \rightarrow \exists v'(vEv' \wedge R'w'v')$,

"natrag" $R'w'v' \rightarrow \exists v(vEv' \wedge Rwv)$.

Strukturalna sličnost

Bisimulacija jest neki odnos slike, ali nije mnogo jak.
U metafori: —jezik logike prvog reda pruža pogled odzgo, “ptičju perspektivu”;
—jezik propozicijske modalne logike pruža pogled odozdo, “žablju perspektivu”.



$\mathfrak{M}$ $\mathfrak{N}$

Odnosi R i R' prikazani su strelicama. Točkasta crta stoji za odnos bisimulacije E. Točke w i w' jednake su boje ako im se vrednovanja poklapaju (i.e., $w \in V(p)$ akko $w' \in V'(p)$ za svako slovo p). Slika pokazuje kako “žablja perspektiva” (za razliku od “ptičje”) ne može razlikovati “simetrične” točke od “refleksivnih”.

Semantika modalne logike

U modalnoj se logici tumačenje rečenice provodi pozivanjem na strukturu sačinjenu od različitih tumačenja. Način kako su tumačenja međusobno povezana pokazuje "što znači" neki modalni operator. Npr. deontični operator obligacije O "znači" serijalnu povezanost tumačenja. U usporedbi: značenje veznika fiksiramo prirodom tumačenja, značenje modalnih operatora—prirodom veze među tumačenjima.

Dinamični pristup

Sljedeći korak u razvoju semantičke teorije nastaje kada promatramo nizove sačinjene od struktura tumačenja. U ovakvom tumačenju približavamo se predteorijskom shvaćanju jezika gdje razumijevanje opisujemo nekim glagolom promjene. Npr. netko nas je obavijestio, razuvjerio, utješio, nagovorio ili nadahnuo svojim riječima. Prijelaz s jedne vrste tumačenja na sljedeću nije raskid s prethodnim nego njegov razvoj.

	Tumačenje
Propozicijska logika	vrednovanje
Modalna logika	struktura vrednovanja
Dinamična logika	promjene struktura vrednovanja

U dinamičnoj logici cijela rečenica jest jedan logički element. Njezino značenje sastoji se u promjenama na strukturi tumačenja. Jednostavne promjena su obnavljanje i provjera: pomak prema najbližoj strukturi tumačenja koja zadovoljova rečenicu i ispitivanje može li se takva provjera provesti. Na ovaj su način pomirena načela kompozicionalnosti (značenje cjeline ovisi o značenju dijelova) i načelo kontekstualnosti (značenje dijela ovisi o cjelini). Prvo je načelo ostvareno jer nakon pomaka strukture tumačenja dolazimo do oblika tumačenja gdje se iz tumačenja dijelova rečenice gradi njezino tumačenje. Drugo načelo ostvaruje se tako što je polazište semantičkog pomaka promjenljivo i nove rečenice tumače se na onoj strukturi tumačenja do koje su dovele prethodne rečenice: one time tvore kontekst u kojemu se odvija tumačenje nove rečenice.

Ovakav način razmišljanja otvara horizont u kojemu se mogu prepoznati složeni fenomeni u svijetu intencionalnosti. Rečenice se razumijevaju u procesu tumačenja koji može uključivati različite radnje kao što su odbacivanje pojedinih tumačenja, izmjenu relacija među tumačenjima, dodavanje tumačenja, uvjetovanje takvih radnji uspješnim provjerama i slično. Prema tome, razumijevanje rečenice jedan je program, jedan niz radnji koje se provode nad polaznim tumačenjima. Zbog toga dinamična semantika nije rivalski projekt u odnos na statičnu nego njezina akomodacija. Uzmimo kao primjer AGM teoriju o dinamici teorije: ekspanzija i kontrakcija osnove su promjene, a revizija—složena (kontrakcija radi omogućavanja i provođenja ekspanzije). Ako dopuštenja shvatimo kao ukinuća zabrana onda razumijevanje iskaza dopuštenja "znači" reviziju vlastitog voljnog stanja i deontičnog obrasca.

Imamo li razloga odlučiti se za treću vrstu tumačenja? Zar nam modalna logika nije dovoljna da bismo prepoznali odnose značenja u jeziku? Je li doista nužno uvesti lik tumača u semantičku teoriju i odustati od omiljene teorijske slike jezika na kojoj nema nikoga za koga bi jezik značio nešto?
Odgovor na svako od ovih pitanja je potvrdan. Pojmovni okvir sačinjen od pojmova lišenih tumača ne predstavlja krajnji doseg teorijske optike: vidokrug se širi i novi fenomeni postaju vidljivi kada se teorijski aparat obogati semantičkim pojmovima koji uključuju tumače, one jedine za koje značenjski fenomeni jesu fenomeni.

Obratit ćemo pažnju na dva značenjska fenomena koji postaju vidljivi i shvatljivi nakon što se promijeni pojmovni okvir. Prvi je fenomen retraktivnih rečenica, rečenica koje za sadržaj imaju ukidanje onoga što je ranije izrečeno. Takve rečenice nisu ekstremni slučajevi u kojima se pokazuje komunikacijski krah, radije, one su legitiman potez. Npr. u deontičnoj logici, ako se dopuštenje izjednači s ukidanjem zabrana, tvrdokorni problem distributivnosti permisije nad disjukcijom $(\mathrm{P}(p \vee q) \rightarrow (\mathrm{P}p \wedge \mathrm{P}q))$ zadobiva prirodno rješenje. Drugi fenomen o kojem će biti riječi, jest fenomen slabog slijeda, nemonotonog i netranzitivnoga slijeda. Takve vrste slijeda promatraju se ili kao teorijske mogućnosti ili kao povezane s posebnom prirodom koje imaju rečenice koje govore o pravilnostima koje dopuštaju iznimke. Međutim dinamični pojmovni okvir otvara nam pogled na jednu stvarnu i od davnina prepoznatu i imenovanu (prima facie slijed) varijantu netarskijevskog slijeda.

Rečenice, govorni činovi i komunikacija

- Semantika dinamične logike je komunikacijska semantika jer uzmimamo u obzir "ono što rečenice čine".
  1. Slušatelj j nalazi se u nekom (intencionalnom, umnom) stanju σ.
  2. Govornik i izriče rečenicu φ.
  3. Slušatelj j obnavlja (*update*) ili razvija (*upgrade*) svoje stanju σ i prelazi u stanje σ'
- Osnovnu ideju lako zahvaćamo jednostvnim iskaznim oblikom:

$$\sigma[\varphi] = \sigma'$$

Dynamic semantics: beginnings and the Dutch school

- The beginnings of dynamic approach in philosophical logic can be traced back to two papers of David Lewis from 1979:

  
  David Lewis. 1979. A problem about permission.
  In *Essays in Honour of Jaakko Hintikka*, eds. E. Saarinen et al., Dordrecht: Reidel, pp. 163175.

  
  David Lewis. 1979. Scorekeeping in a language game.
  Journal of Philosophical Logic 8:339–359.
- Now there is a number of semantical theories that can and have been classified under the heading ‘dynamic semantics’ (e.g. discourse representation theory of Hans Kamp).
- We will discuss a variant of a family of dynamic semantical system that have been developed over the last three decades by a number of authors associated with the Institute for Logic, Language, and Computation, at University of Amsterdam: Johan van Benthem, Frank Veltman, Jeroen Groenendijk, Martin Stokhof, Jan van Eijck, Paul Dekker, an a large number of other researchers that have worked or studied there, or had been inspired by the approach.
- A rich repository of resources can be find at the Institute’s site:
  `http://www.illc.uva.nl/`

Blurring the semanitic/pragmatic distinction

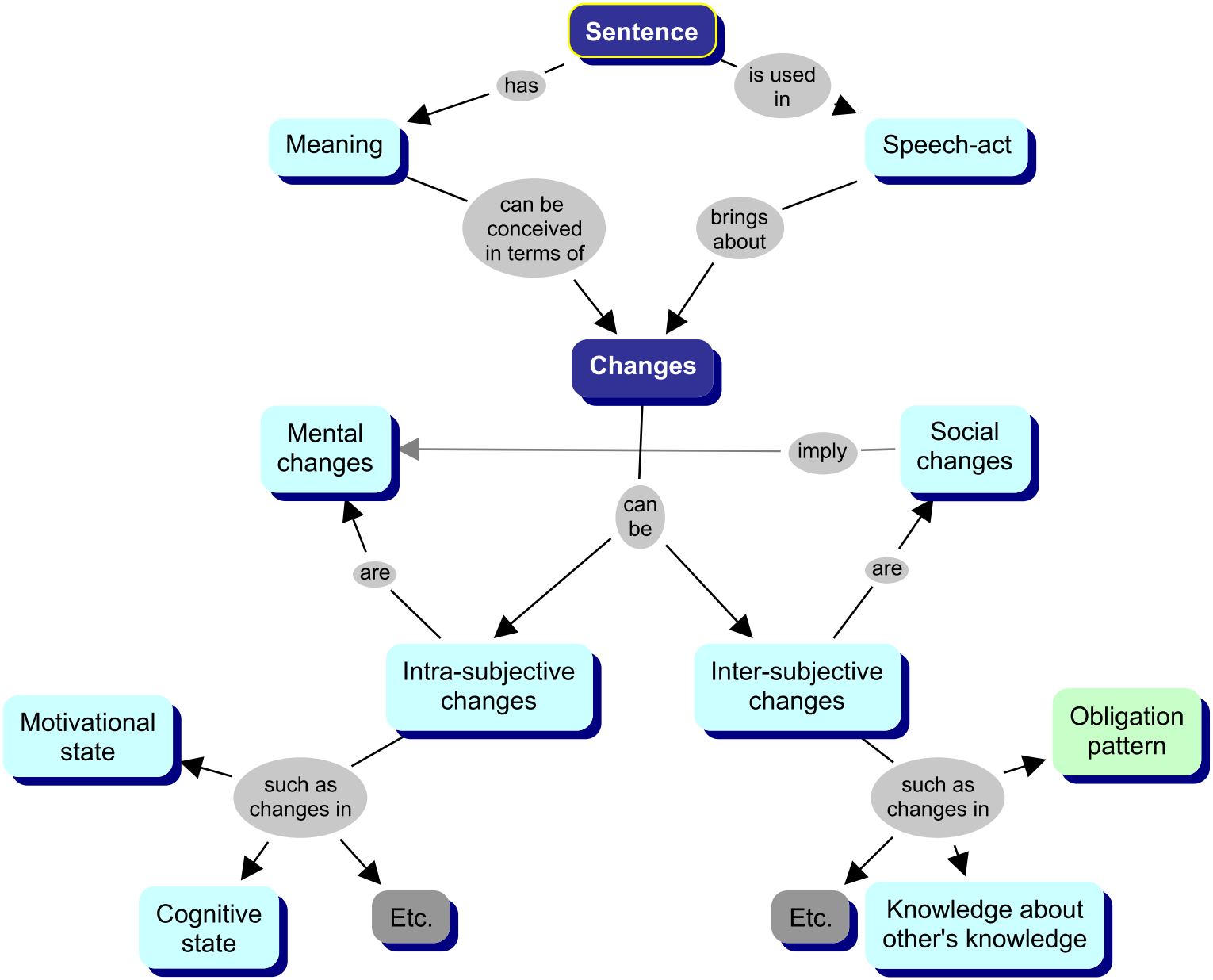
- Let us take a look at an early formulation of the theory:

The dynamic view on meaning

. . . the meaning of a sentence does not lie in its truth conditions, but rather in the way it changes (the representation of) the information of the interpreter. The utterance of a sentence brings us from a certain state of information to another one. The meaning of a sentence lies in the way it brings about such a transition.

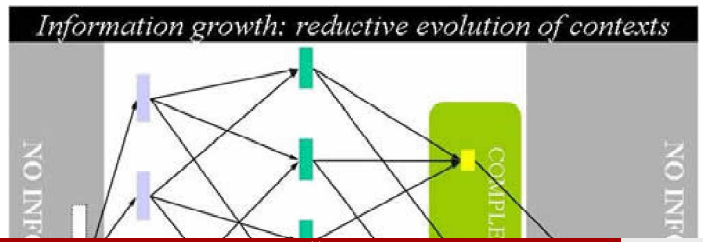
Jeroen Groenendijk and Martin Stokhof. 1991. Dynamic predicate logic. *Linguistics and Philosophy* 14:39–100.

- By the equation 'meaning=change-potential' the pragmatics and semantics have become blended together, and the speech acts rather then sentences have become the objects of logical analysis.


Sentence
has
is used in
Meaning
Speech-act
can be conceived in terms of
brings about
Changes
Mental changes
imply
Social changes
can be
are
are
Intra-subjective changes
Inter-subjective changes
Motivational state
such as changes in
Obligation pattern
such as changes in
Cognitive state
Etc.
Etc.
Knowledge about other's knowledge

NAJJEDNOSTAVNIJA *update* SEMANTIKA

- Osnovne ideje: informacijsko stanje je skup vrednovanja, vrednovanje je skup propozicijskih slova ("jedan Tractatus-svijet"), rečenice su funkcije (odnosi vrste n—1) koje uzimaju neki skup vrednovanja i isporučuju neki skup vrednovanja.
- Informacija kao "mjera (ne)znanja":
- Skup svih vrednovanja $W = \wp A$ s obzirom na skup A propozicijskih slova. Zapis $\wp A$ označava skup svih podskupova od A, tj. $\wp A = \{w \mid w \subseteq A\}$.
- Informacijsko stanje $\sigma \subseteq W$ je skup skupova slova, tj. skup vrednovanja ("skup mogućih Tractatus-svjetova"). Ovdje vrijedi "manje je više": što je manji broj vrednovanja w u stanju σ, to je σ bogatija u informacijskome smislu.
- Granični slučajevi:
  Minimalno info-stanje ako $\sigma = W$ (tj. $|\sigma| = |W|$ gdje $|\ |$ označava brojnost nekog skupa), onda σ ne sadržava nijednu informaciju;
  Maksimalno info-stanje ako $|\sigma| = 1$, σ onda punu informaciju s obzirom na A;
  Apsurdno info-stanje ako $\sigma = \emptyset$ (tj. $|\sigma| = 0$), onda σ predstavlja krah procesa pribavljanja informacija.



Update functions

- Truth in a valuation: for $w \in W$, $p \in A$, $\varphi, \psi \in \mathcal{L}_A$
  $w \models p$ iff $p \in w$,
  $w \models \neg\varphi$ iff $w \not\models \varphi$,
  $w \models (\varphi \wedge \psi)$ iff $w \models \varphi$ and $w \models \psi$.
- Update function [1]: $\cdot[\cdot] : \wp W \times \mathcal{L}_A \to \wp W$.
- Update-sentences φ^+ act upon info-states delivering info-state in which they are accepted:
- $\sigma[\varphi^+] = \{w \in \sigma \mid w \models \phi\}$,
- $\sigma[\varphi^+] = \sigma[\varphi^+][\varphi^+]$.
- Use calculator!

[1] Combined infix and postfix notation is used.

Adding and testing

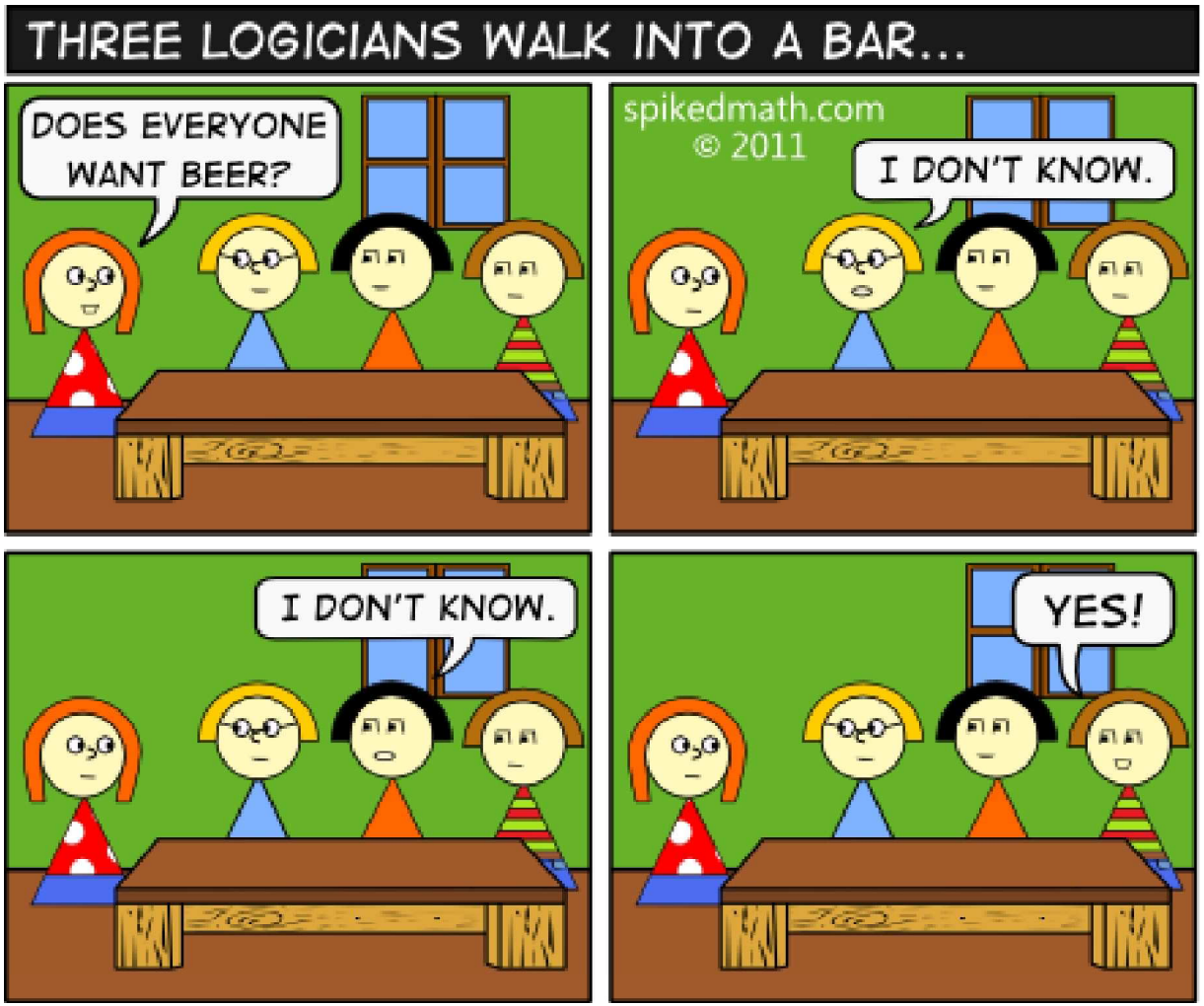
- Relative consistency testing (can an information contained in φ be added in a context σ without causing informational breakdown)
  - Acceptability testing:

$$\sigma\left[\varphi^{?^{consistency}}\right] = \begin{cases} \sigma & \text{if } \sigma[\varphi] \neq \emptyset, \\ \emptyset & \text{otherwise.} \end{cases}$$

- Relative validity testing (will the context σ be changed by adding information contained in φ)
  - Acceptance testing:

$$\sigma\left[\varphi^{?^{validity}}\right] = \begin{cases} \sigma & \text{if } \sigma[\varphi] = \sigma, \\ \emptyset & \text{otherwise.} \end{cases}$$

Mental processes



Mental states

Exercise

Let us analyse the mental process described in the cartoon above! We will identify the belief state of each actor with the set of situation that she considers possible. Let $p(q, r)$ stand for 'the first actor (the second actor, the third actor) wants beer'. A possible situation can identified with a valuation. In the beginning every actor knows only her own desires and is ignorant of desires of others, and therefore there are exactly four valuations that each of them considers possible.
An actor's ignorant answer to the waiter's question shows that she wants beer (for if she did not want beer her answer to the question 'Does everybody want beer' would be 'No') and that answer gives information to the other actors who update their belief state accordingly. Let us use calculator at
`http://www.ffst.hr/~logika/implog/calculators/update/update.html`
and reconstruct the dynamics of the belief change of the last actor!

Pojam slijeda kao sadržavanja informacija

Information

Whenever i **L**-implies j, i asserts all that is asserted by j, and possibly more. In other words, the information carried by i includes the information carried by j as a (perhaps improper) part. Using '$In(\ldots)$' as an abbreviation for the presystematic concept 'the information carried by ...,' we can now state the requirement in the following way:
R3-1. $In(i)$ includes $In(j)$ iff i **L**-implies j.
By this requirement we have committed ourselves to treat information as a set or class of something. This stands in good agreement with common ways of expression, as for example, "The information supplied by this statement is more inclusive than (or is identical with, or overlaps) that supplied by the other statement."

Rudolf Carnap and Yehoshua Bar-Hillel. *An Outline of a Theory of Semantic Information*. Technical Report no. 247. Research Laboratory of Electronics, Massachusetts Institute of Technology, 1952.

A MORE RECENT EXAMPLE

Sentences adding information

The information containment conception: P implies c if and only if the information of c is contained in the information of P. In this sense, if P implies c, then it would be redundant to assert c in a context where the propositions in P have already been asserted; i.e., no information would be added by asserting c.

Jose M. Saguillo. Logical Consequence Revisited. *The Bulletin of Symbolic Logic* (1997) 3: 216-241

Adding information

- Two notions “adding information” and “information as a set or class of something” show
  1. that sentences can do something, namely they can “add information”, and
  2. that semantic relations occur at the level of sets, since “information [is] a set or class of something”.
- Putting these two together we get that sentences act on sets.
- Two notions of “information containment” are relevant:
  1. Conclusion adds no information to **any** context that includes all information contained in premises.
  2. Conclusion adds no information to the context that includes **only** the information contained in premises.[This notion corresponds to “ignorant-update-to test” and a variant of it will be introduced later as *prima facie* consequence.]

Classical consequence as a special case

In dynamic semantics the notion of consequence can be generalized. The use of 'therefore φ' is justified in context σ if φ produces no change in σ: $\sigma[\varphi] = \sigma$. Then special cases of relations between text and sentence can be distinguished: is the text order irrelevant (tt) or not (ut), is the relation "localized" (0-ut), etc.

test-to-test $p_0; ...; p_n \models_{tt} q$ iff for all contexts σ:

$$\sigma[p_1] = ... = \sigma[p_n] = \sigma \rightarrow \sigma[q] = \sigma$$

update-to-test $p_0; ...; p_n \models_{ut} q$ iff for all contexts σ:

$$\sigma[p_1]...[p_n] = \sigma[p_1]...[p_n][q]$$

ignorant-update-to-test $p_0; ...; p_n \models_{0-ut} q$ iff for the empty context (carrying no information) 0:

$$0[p_1]...[p_n] = 0[p_1]...[p_n][q]$$

Johan van Benthem. 1996. *Exploring Logical Dynamics*
Stanford: Center for the Study of Language and Information

'Therefore': sentence operator or metalinguistic predicate?

- If one thinks about the semantics as something to do with the actions performed on "sets of something", then one is not obliged to treat natural language expressions 'therefore' and 'might' as a metalinguistic predicate. In the dynamic semantics perspective 'therefore' and 'might' can be understood as "modalities of introspection:" reflections on one own state of mind.

Primjer

Denote by $\mathcal{L}_0$ the language in which some logical constants occur. Then we need a meta language $\mathcal{L}_1$ to state that a sentence $p \in \mathcal{L}_0$ is a consequence of a set of sentences $\Gamma \subseteq \mathcal{L}_0$ since operator 'therefore' does not belong to the language $\mathcal{L}_0$.

- It may seem odd that by saying 'p therefore q'
  - either (i) the speaker mentions sentences p and q (using their names) but does not use them,
  - or (ii) the speaker simultaneously uses and mentions p and q since she is asserting p and q (using the sentences) as well as (mentioning them while) asserting the existence of consequence relation between 'p' and 'q'.

Lowering

- One may choose whether to treat `might` and `therefore` as logical operators or as metalogical predicates.

metalogical predicate	*logical operator*
$\texttt{therefore}(\Gamma, p)$ i.e. $\Gamma \models p$	$\texttt{therefore}\, p \in \mathcal{L}_0$
$\texttt{might}(\Gamma, p)$ i.e. $\Gamma \cup \{p\} \not\models \bot$	$\texttt{might}\, p \in \mathcal{L}_0$

- The "logical operator option" is taken when we interpret some natural language sentences as "test functions" both for stating relative consistency and for stating relative validity (context validity), i.e

$$sentence\_function(context) = \begin{cases} context \text{ if the condition is met,} \\ failure \text{ otherwise.} \end{cases}$$

  - Advantages:
    - Adverb 'therefore' is treated unambiguously (instead of signifying different relations in the contexts with different logics).
    - Gain in sensitivity to different phenomena of "information containment".
    - The drawback is that correctness of the use of 'therefore' need not imply existence of a consequence relation.
  - The advantage of might-operator is that the statements on consistency become part of the object language.
- 'might' is logical operator, 'therefore' is metalogical predicate is the position

Tarskian consequence

Axiom 1. $|S| \leq \aleph_0$.
Axiom 2. If $X \subseteq S$, then $X \subseteq Cn(X) \subseteq S$.
Axiom 3. If $X \subseteq S$, then $Cn(Cn(X)) = Cn(X)$.
Axiom 4. If $X \subseteq S$, then $Cn(X) = \bigcup_{Y \subseteq X \text{ and } |Y| < \aleph_0} Cn(Y)$.
Axiom 5. There exists a sentence $x \in S$ such that $Cn(\{x\}) = S$.

Alfred Tarski. 1928. On some fundamental concepts of metamathematics.
In Alfred Tarski. *Logic, semantics, metamathematics : papers from 1923 to 1938* (trans. by J.H.Woodger), Clarendon Press, Oxford, 1956

For the countable languages S (axiom 1) it holds that: (i) consequences of sentences remain within the same language and premises are their own consequences (reflexivity, axiom 2), (ii) consequences of consequences of some set of sentences are already consequences of that set (transitivity), (iii) the consequences of a set of sentences X do not exceed their finite subsets consequences, which in return are retained in their superset X consequences (compactness and monotonicity, axiom 4), (iv) there is in the language at least one sentence such that their consequences include all and only sentences of that language (existence of "*falsum*", "absurdity", "explosive sentence", "informational breakdown" etc., axiom 5).

Enriched language

Primjer

In the language enriched with "modalities of introspection" the structural properties of classical (i.e. Tarskian) consequence relation do not hold.

irreflexivity might $\neg p$; $p \not\models_{ut}$ might $\neg p$,

non-monotonicity might $p \models_{ut}$ might p, but might p, $\neg p \not\models_{ut}$ might p,

non-transitivity p, $\neg q \models_{0-ut} p \vee q$ and $p \vee q \models_{0-ut}$ might q, but p, $\neg q \not\models_{0-ut}$ might q

A PARADOXICAL IMPERATIVE INFERENCE

1 Slip the letter into the letter-box!
2 Slip the letter into the letter-box or burn it!
3 You may: slip the letter into the letter-box or burn it.
4 You may: burn the letter.
5 Therefore, if you ought to slip the letter into the letter box, then you may burn it.

	(Purportedly) holds in virtue of	*Intuitive acceptability*
1		
2	meaning of 'or'; from 1	ambivalent
3	relations between 'must' and 'may'; from 2	affirmative
4	distributivity of "free choice permission"; from 3	mainly affirmative
5		negative

- Unexpected behavior of 'or' in 2 and 4.

Paradoxical inference again: a deontic variant

0	$p \models p \vee q$	meaning of $\vee$
1	$\mathrm{O}p \models \mathrm{O}(p \vee q)$	Scott's principle
2	$\mathrm{O}(p \vee q) \models \mathrm{P}(p \vee q)$	D axiom
3	$\mathrm{O}p \models \mathrm{P}(p \vee q)$	by $\models$ transitivity; from 1, 2
4	$\mathrm{P}(p \vee q) \models \mathrm{P}q$	by free choice permission
5	$\mathrm{O}p \models \mathrm{P}q$	by $\models$ transitivity; from 3, 4

- The consequence relation 1, which is intuitively less plausible than 4, holds in normal deontic logic while 4 does not hold.
- Scott's principle

$$\{(p_1 \wedge \ldots \wedge p_{n-1}) \rightarrow q\} \vdash (\Box p_1 \wedge \ldots \wedge \Box p_{n-1}) \rightarrow \Box q$$

($n \geq 1$) characterizes normal propositional modal logic (e.g. it may replace K axiom and neccessitation rule). It may be read as stating that "meaning relations" of propositional logic, i.e. meaning relations holding in virtue of meaning of truth-functional connectives, are preserved in the modal context.

Avoiding the paradox

- The `tonk` example shows that syntactically defined "logical" terms have different properties given the diverse types of consequence.
- (Alf) Ross' paradox and free choice permission show that logical terms may "change their behavior" in the presence of other logical terms.
  - The odd result that if anything is obligatory than everything is permitted (i.e. $Op \Rightarrow Pq$) shows that one may have intuitions that confirm isolated consequence steps and still lack the intuition that confirms transitive closure of these steps.
- The pretheoretical understanding of logical relations may well be holistic in character: perhaps there is no unique understanding of logical terms that is *constitutive* for the understanding of consequence relations, and perhaps there is no unique understanding of admissible consequence relations that is *regulative* for the understanding of logical terms.
- In practical logic the phenomenon of unclear intuitions are noticeable. Both on the formal and on informal side the results and intuitions collide on the issues of existence of consequence relation for particular schemata and on the nature of consequence relation.

Geach's description

Peter Geach. Dr. Kenny on practical inference. *Analysis* (1966) **26**: 76–79

Definitions

Some years ago I read a letter in a political weekly to some such effect as this. 'I do not dispute Col. Bogey's premises, nor the logic of his inference. But even if a conclusion is validly drawn from acceptable premises, we are not obliged to accept it if those premises are incomplete; and unfortunately there is a vital premise missing from the Colonel's argument-the existence of Communist China.' I do not know what Col. Bogey's original argument had been; whether this criticism of it could be apt depends on whether it was a piece of indicative or of practical reasoning. Indicative reasoning from a set of premises, if valid, could of course not be invalidated because there is a premise "missing" from the set. But a piece of practical reasoning from a set of premises can be invalidated thus: your opponent produces a fiat you have to accept, and the addition of this to the fiats you have already accepted yields a combination with which your conclusion is inconsistent.

Defeasibility of conclusion and completeness of premises

The consequence relation described by Geach has two notable properties:

- (”locality”) conclusion holds in virtue of premises but it can be defeated by additional premises;
- (existence of the limit) if the premises are complete the conclusion cannot be defeated (where ’conclusion is defeated’ means ’premises are acceptable and conclusion is not acceptable’).
- By ’Geach’s problem’ I mean a problem of devising modeltheoretic notion of consequence relation that captures the pretheoretical notions of conclusion defeasibility and of ”completeness of premises”.

Tarskian consequence relation

Properties of Tarskian consequence relation

$$\text{(Reflexivity)} \quad \frac{}{\Gamma \models p, \text{ for all } p \in \Gamma}$$

$$\text{(Monotony)} \quad \frac{\Gamma \models p}{\Gamma, \Delta \models p}$$

$$\text{(Transitivity)} \quad \frac{\Gamma \models p, \text{ for all } p \in \Delta \qquad \Delta \models q}{\Gamma, \Delta \models q}$$

- Pretheoretical notion given in Geach's quote[2] is a notion of nonmonotonic consequence relation.
- The example shows that prethoretical notions concern the properties of a variety of consequence relation.

[2] But even if a conclusion is validly drawn from acceptable premises, we are not obliged to accept it if those premises are incomplete

IMPERATIVES

Geach's description of practical argument can be illustrated using a modified variant of Von Wright-Lemmon's syntax and semantics for change expressions. Imperatives are commanded changes and can be analyzed as two part sentences combining two kinds of direction of fit:

$$!(\underbrace{\overset{BEFORE}{initial\_situation}}_{word-to-world\ fit} / \underbrace{\overset{AFTER}{resulting\_situation}}_{world-to-word\ fit})$$

Basic semantics of imperatives:

- Imperatives are commanded actions.
  - Produce A: $!(\neg A / A)$; Suppress A: $!(\neg A / \neg A)$; Maintain A: $!(A / A)$; Destroy A: $!(A / \neg A)$; See to it that A: $!(\top / A)$
- Imperative $!(p/q)$ is true iff (i) in the initial situation p is the case, (ii) q is the case in the imperative future, (iii) q is possible in the future, (iv) q is avoidable in the future. [3]

[3] The problem of practical reasoning is to find out which one is the actual and which one is the ideal world (on the basis of available facts and commands).

A SIMPLE SYSTEM

Definicija

Syntax

Atom is a finite set of propositional letters. Language $\mathcal{L}_{PL}$

$$\begin{array}{rcl} a & \in & Atom \\ \mathcal{L}_{PL} & : & := a \mid \top \mid \varphi \mid \neg\varphi \mid \varphi \wedge \psi \end{array}$$

Language $\mathcal{L}_!$

$$\begin{array}{rcl} p, q & \in & \mathcal{L}_{PL} \\ \mathcal{L}_! & : & := \cdot(p/\top) \mid !(p/q) \mid \boxdot(\top/q) \mid \neg\varphi \mid \varphi \wedge \psi \end{array}$$

Language $\mathcal{L}_{!might}$

$$\begin{array}{rcl} p & \in & \mathcal{L}_! \\ \mathcal{L}_{!might} & : & := p \mid might\, p \mid \end{array}$$

SEMANTICS

Definicija

$\mathbf{W}_0 = \wp Atom$
Structures
$\Sigma = \{\langle W, R_!, R_F\rangle \mid W \subseteq \mathbf{W}_0, R_! \subseteq W \times W, R_F \subseteq W \times W\}$

Definicija

Ignorant structure: $0 = \langle \mathbf{W}_0, \mathbf{W}_0 \times \mathbf{W}_0, \mathbf{W}_0 \times \mathbf{W}_0\rangle$

SEMANTICS

Definicija

Valuation for $p, q \in \mathcal{L}_{PL}$
$w \models p$ iff $p \in w_i$ for propositional letters p
$w \models \neg p$ iff $w \not\models p$
$w \models p \wedge q$ iff $w \models p$ and $w \models q$

Definicija

Truth at w in σ
$\sigma, w \models \cdot (p/\top)$ iff $w \models p$ and $R_! (w, v)$ or $R_F (w, v)$ for some v
$\sigma, w \models ! (p/q)$ iff (i) $w \models p$ and (ii) $v \models q$ for all v such that $R_! (w, v)$, and (iii) $u \models q$ for some u such that $R_F (w, u)$, and (iv) $z \not\models q$ for some z such that $R_F (w, z)$
$\sigma, w \models \boxdot (\top/p)$ iff $v \models p$ for all v such that $R_! (w, v)$ or $R_F (w, v)$
$\sigma, w \models \neg\varphi$ iff $\sigma, w \not\models \varphi$
$\sigma, w \models \varphi \wedge \psi$ iff $\sigma, w \models \varphi$ and $\sigma, w \models \psi$
$\sigma, w \models \texttt{might}\ \varphi$ iff $\sigma, v \models \varphi$ for some v

Prima facie consequence

Definicija

Validity in $\sigma = \langle W \times R_! \times R_F \rangle$
$\sigma \models p$ iff $\sigma, w \models p$ for all $w \in W$ where $\sigma = \langle W, R_!, R_F \rangle$

Definicija (Substructure)

$\sigma \leq \sigma'$ iff $W \subseteq W'$ and $R_! \subseteq R'_!$ and $R_F \subseteq R'_F$ (where $\sigma = \langle W, R_!, R_F \rangle$ and $\sigma' = \left\langle W', R'_!, R'_F \right\rangle$).

Definicija (Minimal structure)

$(0 \mid p) = \sigma$ iff $\sigma \models p$ and if $\sigma' \models p$, then $\sigma' \leq \sigma$.

Prima facie CONSEQUENCE; COMPLETENESS OF PREMISES

Definicija

$$(0 \mid \Gamma) = \bigcap_{p \in \Gamma} (0 \mid p)$$

Definicija

$\Gamma \models_{\textit{prima facie}} p$ iff $(0 \mid \Gamma) \models p$

Definicija

Let $(0 \mid \Gamma) = \langle W, R_!, R_F \rangle$. Γ is a complete set iff $\left|mem_1(R_!)\right| = 1$ and $\left|mem_2(R_!)\right| = 1$.

Back to Ross' paradox

The letter is not burned.	(i)	$\cdot\left(\neg B / \top\right)$
It is not possible that the letter is in the letter box and that it is burned.	(ii)	$\boxdot\left(\top / \neg L \vee \neg B\right)$
Put the letter into the letter box!	(iii)	$!\left(\neg L / L\right)$
Put the letter into the letter box or burn it!	(iv)	$!\left(\neg L \wedge \neg B / L \vee B\right)$
It might be good to burn the letter!	(v)	$\texttt{might}\, !\left(\neg B / B\right)$

Creating the largest structure by eliminating relations

- For $\cdot(p/\top)$ remove all arrows starting at $\neg p$-worlds. For $!(p/q)$ test wheter there is an R_F arrow pointing to a q world and an R_F arrow pointing to a $\neg q$ world; if so, remove all $R_!$ arrows starting in a $\neg p$ world or ending in a $\neg q$ world; otherwise, remove all arrows. For $\boxdot(\top/p)$ remove all arrows ending in $\neg p$-worlds.
- Disjunction introduction partially vindicated

$$\left\{\underset{(i)}{\cdot(\neg B/\top)}, \underset{(ii)}{\boxdot(\top/\neg L \vee \neg B)}, \underset{(iii)}{!(\neg L/L)}\right\} \models_{\textit{prima facie}} \underset{(iv)}{!(\neg B \wedge \neg L/B \vee L)} \qquad (1)$$

Initial situation			Imperative future			Possible future		
w_1	$\{B, L\}$	$\times$ by (i)	w_1	$\{B, L\}$	$\times$ by (ii)	w_1	$\{B, L\}$	$\times$ by (ii)
w_2	$\{B\}$	$\times$ by (i)	w_2	$\{B\}$	$\times$ by (iii)	w_2	$\{B\}$	
w_3	$\{L\}$	$\times$ by (iii)	w_3	$\{L\}$		w_3	$\{L\}$	
w_4	$\emptyset$		w_4	$\emptyset$	$\times$ by (iii)	w_4	$\emptyset$	

- Free choice permission partially vindicated (here modified to choice offering imperative and suggestion)

$$\left\{ \underset{(iv)}{!\left(\neg B \wedge \neg L / B \vee L\right)} \right\} \models_{\textit{prima facie}} \texttt{might}\, \underset{(v)}{!\left(\neg B / B\right)} \tag{2}$$

Initial situation			Imperative future			Possible future	
w_1	$\{B, L\}$	$\times$ by (iv)	w_1	$\{B, L\}$		w_1	$\{B, L\}$
w_2	$\{B\}$	$\times$ by (iv)	w_2	$\{B\}$		w_2	$\{B\}$
w_3	$\{L\}$	$\times$ by (iv)	w_3	$\{L\}$		w_3	$\{L\}$
w_4	$\emptyset$		w_4	$\emptyset$	$\times$ by (iv)	w_4	$\emptyset$

- Avoiding the paradox ($\mathrm{O}p \Rightarrow \mathrm{P}q$)

The relation $\models_{\textit{prima facie}}$ is not transitive and in this case the unwanted conclusion does not follow;

$$\left\{ \underset{(i)}{\cdot\left(\neg B/\top\right)}, \underset{(ii)}{\boxdot\left(\top/\neg L \vee \neg B\right)}, \underset{(iii)}{!\left(\neg L/L\right)} \right\} \not\models_{\textit{prima facie}} \underset{(v)}{\texttt{might}\,!\left(\neg B/B\right)} \tag{3}$$

since $\{\cdot\left(\neg B/\top\right), \boxdot\left(\top/\neg L \vee \neg B\right), !\left(\neg L/L\right), !\left(\neg B/B\right)\}$ is not satisfiable.

Initial situation			Imperative future			Possible future		
w_1	$\{B, L\}$	$\times$ by (i)	w_1	$\{B, L\}$	$\times$ by (ii)	w_1	$\{B, L\}$	$\times$ by (ii)
w_2	$\{B\}$	$\times$ by (i)	w_2	$\{B\}$	$\times$ by (iii)	w_2	$\{B\}$	$\times$ by (v)
w_3	$\{L\}$	$\times$ by (iii)	w_3	$\{L\}$	$\times$ by (v)	w_3	$\{L\}$	$\times$ by (v)
w_4	$\emptyset$	$\times$ by (v)	w_4	$\emptyset$	$\times$ by (iii)	w_4	$\emptyset$	$\times$ by (v)

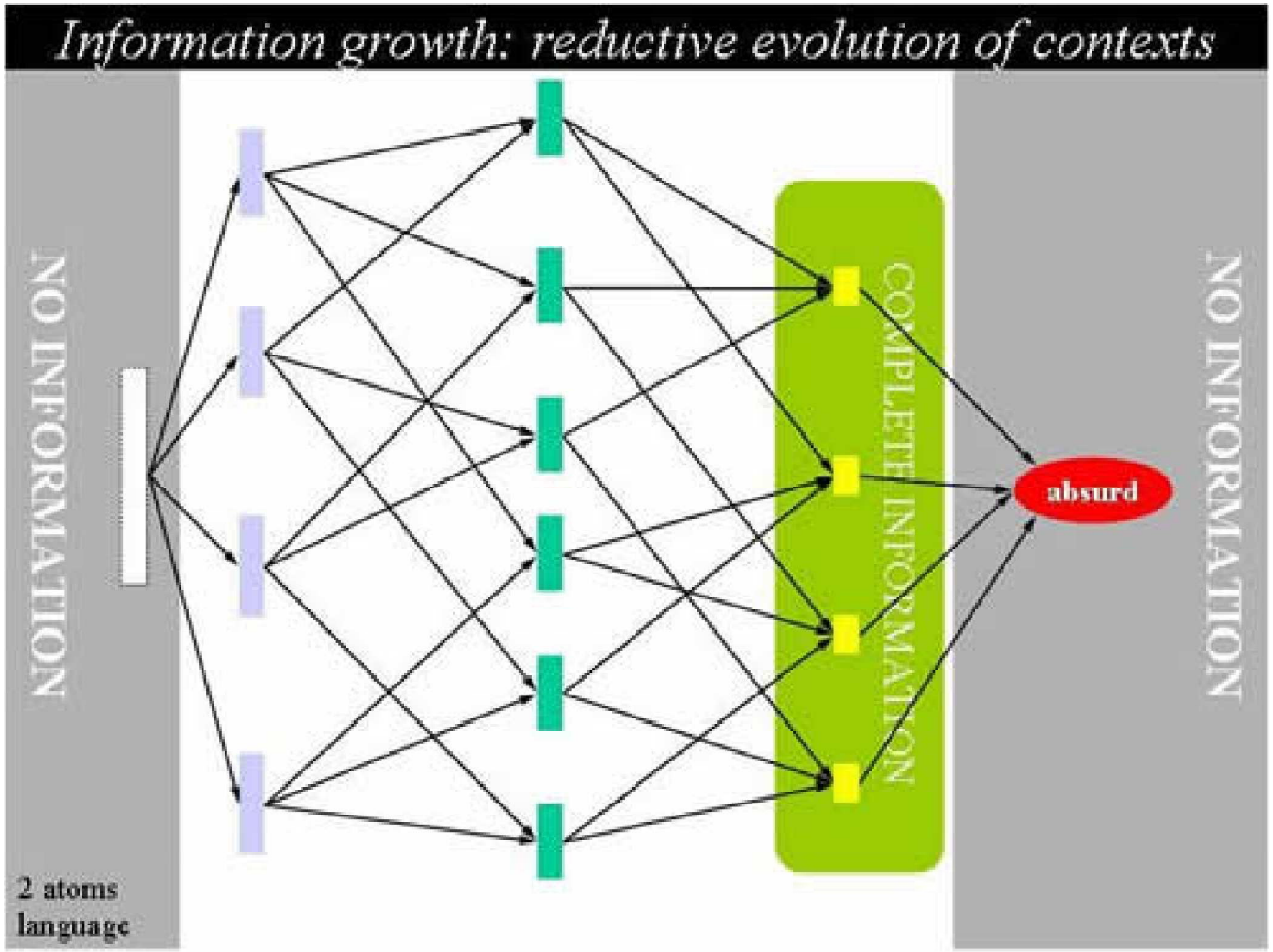
Conjectures

- The language practices do not support the hypothesis that understanding of meanings of logical terms is constitutive for the understanding of consequence relations.
- The language practices do not support the hypothesis that understanding of consequence relations is regulative for the understanding of meaning of logical terms.
- My conjecture is that understandings of logical terms and logical relations come to us bundled together as a collection of open notions.

Updates and downdates

- In our communication we frequently withdraw, cancel, unsay that what we said before.
- The sentences we use for unsaying must have their meaning, don't they?
- This retractive move in the language game can be modeled within dynamic semantics.
- In communicative update, the receiver undergoes a transition where his mental state becomes more precise or at least as precise as before. (We may think of precision as a number of answers to questions 'what is the case' and 'what am i to do.') Update is uncertainty reduction.
- In downdate, triggered by the sender's withdrawal of that which he said or implied before, the transition goes backwards, towards 'uncertainty escalation.'

Towards certainty



Permission as withdrawal of imperative

A number of authors has in the last decades drawn a distinction between two types of negation. E.g. the usual understanding of negation of assertion is that it is assertion too, but with negative content; on the other hand, some authors discuss "denial in a non-derivative sense," denial as a speech act sui generis (indicating a failure to obtain a reason for a certain assertion). The same goes for imperatives.

Primjer

In Searle's speech act theory, where illocutionary force indicator has the role similar to the role of modal element, while propositional indicator corresponds to sentence radical. Searle used the term 'illocutionary negation' for external negation, and he classified permissions as directives alongside other speech acts typically performed by uttering an imperative.

Good idea, but logical form got wrong

"Permit" also has the syntax of directives, though giving permission is not strictly speaking trying to get someone to do something, rather it consists in removing antecedently existing restrictions on his doing it, and is therefore the illocutionary negation of a directive with a negative propositional content, its logical form is $\sim !(\sim p)$.

Searle, John. 1999. *Expression and Meaning: Studies in the Theory of Speech Acts*.
Cambridge: Cambridge University Press.

Permitting by withdrawing

- If permissions are conceived as "removal of antecedently existing restrictions," then the idea of downdate comes as a natural solution.
- Still, the solution is not simple. To remove the motivational and obligation imposing effects of an imperative that had been either explicitly uttered or implied, it is not enough to "move backwards" to a state where removed imperative is not accepted. The downdated state must be such as to enable update with imperative with content opposite to the one being withdrawn. So, in this case to get the semantics of withdrawn imperative, i.e. permission giving sentence, the opposition between 'act' and 'let it happen' imperatives must be correctly established.
- Here we encounter a phenomenon of logical dynamics similar to theory revision, which has been elaborated by Alchourrón, Gärdenfors, and Makinson. In imperative case, to withdrawal of imperative there corresponds a specific "logical act" of contraction.

Logic of politeness

My research has shown that AGM theory of contraction together with downdate semantics entails the fact that external denial, instead od reducing, increases the degree of uncertainty. After a sentence has been withdrawn (canceled, externally negated, unsaid,. . .), Hearer's mental state not only becomes less determinate but also the path of change itself is under-determinate. It may turn out that the requirement of maximal preservation springs from the normative source of cooperative communication, but it might be just one among other admissible types of contraction. The negated speech acts do not make natural language more expressive, as it can be proved. Unsaying increases "communicative entropy" and is avoidable. Therefore, we should apologize if we negate a speech act. And not for the sake of cultural convention, but for the sake of logic.

Logic as ethics of communication

"One is a lonely number," and logic needs not be a study of loneliness, or, even worse, of universe of meaning of language expressions where there is no one for whom the words mean something. Now one can observe the process of "shifting the logical perspective from valid argumentation to cooperative communication," and in that respect logic should restitute its core position in the *trivium* part in humanistic education, and reestablish itself not only as "ethics of reasoning," but also as "ethics of communication," thus helping us to preserve a human world for the children of tomorrow.
Logic of imperatives plays an important theoretical role in that process.