Deontic logic as a study of conditions of rationality in norm-related activities

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Abstract

[a] The program put forward in von Wright's last works defines deontic logic as "a study of conditions which must be satisfied in rational norm-giving activity" and thus introduces the viewpoint of logical pragmatics. In this paper a formal explication for von Wright's program is proposed within the framework of set-theoretic approach and extended to a two-sets model which allows for the separate treatment of obligation-norms and permission norms. The three translation functions connecting the language of deontic logic with the language of the extended set-theoretical approach are introduced, and used in proving the correspondence between the deontic theorems, on one side, and the perfection properties of the norm-set and the "counter-set", on the other side. In this way the possibility of reinterpretation of standard deontic logic as the theory of perfection properties that ought to be achieved in norm-giving activity has been formally proved. The extended set-theoretic approach is applied to the problem of rationality of principles of completion of normative systems. The paper concludes with a plaidoyer for logical pragmatics turn envisaged in the late phase of Von Wright's work in deontic logic.

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Overview

1. Von Wright's reinterpretation of deontic logic

2. A formal explication of Von Wright's reinterpretation
   - The norm-set for obligation-norms and the counter-set for permission norms
   - Translations from SDL language to set-theoretic language

3. Classical deontic logic as the theory of ideal normative systems

4. Philosophy of deontic logic from the viewpoint of logical pragmatics

5. Appendix: The application of the two-sets model to the problem of completeness
   - Is it rational to complete a normative system using the meta-principle *everything not permitted is forbidden*?

Reinterpretation of SDL

Von Wright's late work on deontic logic

Von Wright's reinterpretation of deontic logic developed gradually and has introduced important conceptual distinctions and theses, among which the following stand out:

- the distinction between prescriptive and descriptive use of deontic sentences;
- the thesis that relation between permission and absence of prohibition is not conceptual but normative in character;
- this normative relation is one among other "perfection properties" of the normative system,
- the norm-giver (in the norm-giving activity by which the normative system is produced) ought to achieve perfection properties of the system.

Georg Henrik von Wright (1916–2003), one of the founders of deontic logic. His reinterpretation of deontic logic has been presented in his invited talk at the *Fourth International Workshop on Deontic Logic in Computer Science (DEON '98)*, Bologna, Italy, 8-10 January, 1998.

The two uses

The ambiguity of deontic sentences

The two uses can be distinguished by their effects and written down in an *ad hoc* dynamic logic type of notation [act]effect:

- In the prescriptive use a directive type of speech-act is performed; the effect takes place in the "social world", i.e., a change in the normative reality possibly occurs;
  after i says/writes: "It is obligatory for j that φ" , it is the case that it is obligatory for j that φ, $[i : \underline{O_j\varphi}]\ O_j\varphi$.
- In the descriptive use an assertive type of speech-act is performed; the effect takes place in the "subjective world", i.e., a change of the recipient's beliefs possibly occurs;
  after i says/writes: "It is obligatory for j that φ" , it is the case that j believes that it is obligatory for j that φ, $[i : \underline{O_j\varphi}]B_jO_j\varphi$.
- It should be noted that the "prescriptive use of a deontic sentence" has double direction of fit (the world to fit the words and the words to fit the world); "it makes it so by saying it is so"; thus deontic sentences in the prescriptive use are a kind of "performatives" (or "declarations").
- The interpretational hypothesis that will be followed in this talk is that standard deontic logic deals with the second-order deontic operator O that applies to acts of prescriptive use of deontic sentences, e.g., the second-order norm of internal consistency,

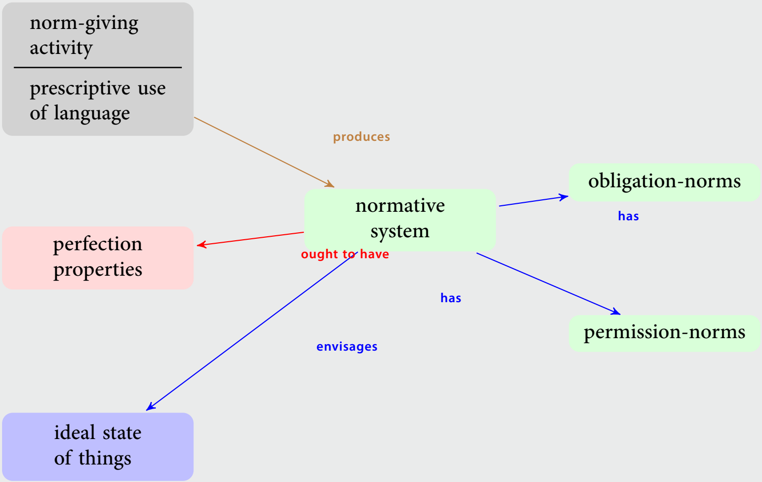
$$[i : \underline{O_j\varphi}]F_i i : \underline{F_j\varphi}$$

Citation

"...deontic sentences in ordinary usage exhibit a characteristic ambiguity. Sometimes they are used as norm-formulations. We shall call this their prescriptive use. Sometimes they are used for making what we called normative statements. We call this their descriptive use. When used descriptively, deontic sentences express what we called norm-propositions. If the norms are prescriptions, norm-propositions are to the effect that such and such prescriptions 'exist', i.e. have been given and are in force."

von Wright, G.H. (1963). *Norm and Action : A Logical Enquiry*. London: Routledge and Kegan Paul.

The conceptual map of Von Wright's late work



Deontic logic as the logic of prescriptive use of language is a study of its rationality conditions

"Deontic logic, one could also say, is neither a logic of norms nor a logic of norm-propositions but a study of conditions which must be satisfied in rational norm-giving activity. It is strict *logic* because the conditions which it lays down are derived from *logical* relations between states in the ideal worlds which normative codes envisage."

Georg Henrik von Wright (1993).

A Pilgrim's Progress.

In von Wright, G.H. *The Tree of Knowledge and Other Essays*, 103–113.

Leiden: Brill.

[p.111]

- This programmatic statement on the directions for the development of deontic logic together with the outline of the path of its realization puts once again Von Wright in the role of the "midwife" (to use his own words) of the new viewpoint in deontic logic.

The basic idea can be summarized as follows: Thanks to the prescriptive use of language normative systems come into existence. The logical properties of normative systems are described using the language of the "logic of norm-propositions". Some logical properties are "perfection-properties". The absence of a certain perfection-property does not deprive a normative system of its normative force. In the prescriptive use of language the norm-giver ought to achieve some perfection properties of the normative system thereby produced. Deontic logic is a study of logical perfection properties; properties which act as the normative source of requirements to which the norm-giver and the norm-recipient are subordinated.

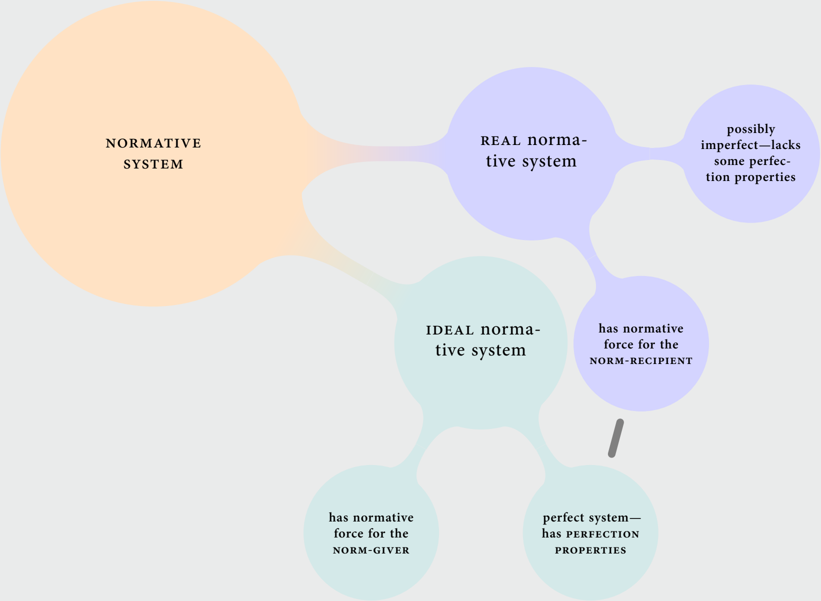
Von Wright's turn towards logical pragmatics

- The use and users of language and of language-constructions are taken into the picture.
- A normative systems can come into existence thanks to the prescriptive *use* of language. The logical properties of real normative systems can be described using the language of the "logic of norm-propositions". Some logical properties are "perfection-properties" of a normative-system. The absence of a certain perfection-property does not deprive a normative system of its normative force. In the *prescriptive use* of language the norm-giver *ought to achieve* some perfection properties of the normative system is subordinated. There are two types of *oughts*: the *ought* resulting from the norm-giving activity, and the *ought* to which the norm-giving activity is subordinated.
- According to Von Wright, deontic logic is a study of logical perfection properties; properties the achievement of which fulfils *rationality conditions of norm-giving activity*.
- The approach can be generalized so to include other roles, such as the role of norm-recipient, and other norm-related activities, such as normative reasoning.

Two-sets model

Set-theoretic approach in deontic logic

- Von Wright's reinterpretation of deontic logic can be formally explicated within the set-theoretic approach. The set-theoretic approach has been introduced into the logic of normative systems by Alchourrón and Bulygin in 1981. Within this framework deontic sentences are treated as claims on membership in the set of consequences $Cn(\mathcal{N})$ of "explicitly commanded propositions" $\mathcal{N}$. Thus, $\mathrm{O}\varphi$ in their approach means $\varphi \in Cn(\mathcal{N})$, while $\mathrm{P}\varphi$ is explicated as $\neg\varphi \notin Cn(\mathcal{N})$. More recently a refinement and generalization of the set-theoretic approach has been developed by Broome (e.g. *Rationality through Reasoning*, 2013) where the set of requirements is a set closed under equivalence and equated with the value of a code function, which takes as its arguments a normative source, an actor and a world.
- The point of divergence within the set-theoretic approaches lies in the properties that are assigned to sets of norms or requirements. It is in accord with the approach proposed by Von Wright to treat real norm-sets, the one corresponding to obligation-norms and the other to permission-norms, as simple sets consisting just of affirmed and negated propositional contents of explicitly promulgated norms, not presupposing any a priori given properties. Rather, it is the question of compliance with second-order normativity whether a real normative system posses desirable logical properties and approximates an ideal system. In the approach of this paper it is neither assumed that a norm-set is deductively closed nor that it is closed under equivalence. The *real normative system* is just a set of "explicitly promulgated norms".


NORMATIVE SYSTEM
REAL normative system
possibly imperfect—lacks some perfection properties
has normative force for the NORM-RECIPIENT
IDEAL normative system
has normative force for the NORM-GIVER
perfect system—has PERFECTION PROPERTIES

Norm-related activities: between the Real and the Ideal

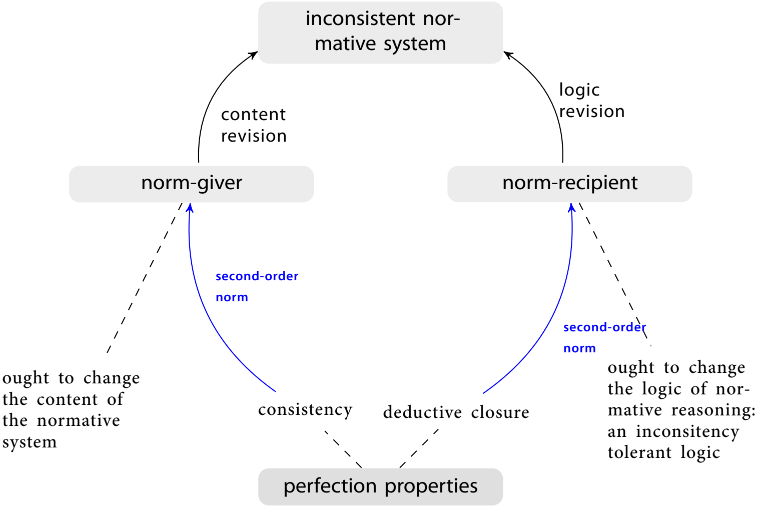


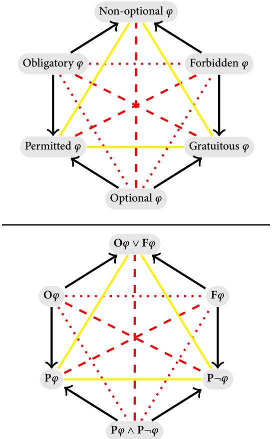
Figure : Corrective obligations with respect to an inconsitent normative-system.

Normative relation between normative concepts

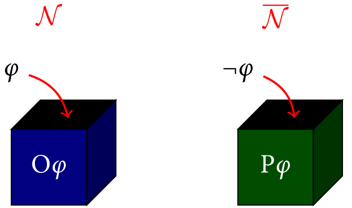
"Just as possibility is the negation of the necessity of the contradictory of a proposition, permission is the negation of the obligatoriness of the contradictory. P*p* ↔ ¬O¬*p* is a theorem of "classical" deontic logic.
I think that this opinion is mistaken. The relation between permission and absence of prohibition is not a *conceptual* but a *normative* relation. One may be able to give good reasons why such things which are not prohibited by the norms of a certain code should be regarded as permitted by the code in question. But to declare the non-prohibited permitted is a normative act. One could have a meta-norm to the effect that the not-prohibited is permitted. The well-known principles *Nulla poena sine lege* and *Nullum crimen sine lege* may be thought of as versions of this meta-norm. Or at least as closely related to it."

von Wright, G.H. (1991).
Is There a Logic of Norms? *Ratio Juris* 4:265–283.

The hexagon of logical relations holding in standard KD deontic logic. The red dotted line represents the contrariety relation, the red dashed line represents contradiction, the full line represents subcontrariety, and the arrows represent subalternation (implication). In a REAL normative system these relations need not obtain. The hexagon depicts normative relations between normative concepts, not conceptual relations. So, only in an IDEAL system deontic hexagon holds.



The metaphor: putting in boxes having different logical structure (I)



- The facts about a normative system are represented using the language of set-theory:
  - $O\varphi$ is represented by $\ulcorner\varphi\urcorner \in \mathcal{N}$,
  - $P\varphi$ is represented by $\ulcorner\neg\varphi\urcorner \in \overline{\mathcal{N}}$.

Although counter-intuitive at the first glance, the adequate metaphor for permitting is that of putting the negation of the content into the permission box. This corresponds to the standard definition "permitted that φ = not obligatory that $\neg\varphi$": $\neg\varphi$ cannot go into the blue box, so, it must be placed in the green box.

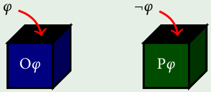
The perfection properties are different for different "boxes" since "ideal concepts" of obligation and permission have different logical structure. For example, having a contradictory pair is an imperfection property of the obligation box, but for the permission box this is neither a perfection nor an imperfection property. Similarly, completeness is a perfection property for permissions but not for obligations: it is indifferent whether $\ulcorner\varphi\urcorner \in \mathcal{N} \vee \ulcorner\neg\varphi\urcorner \in \mathcal{N}$ holds, while $\ulcorner\varphi\urcorner \in \overline{\mathcal{N}} \vee \ulcorner\neg\varphi\urcorner \in \overline{\mathcal{N}}$ ought to hold.

This model, as will be shown, can account for the fact that perfection properties come in pairs, one for obligations, another for permissions, both of which are characterized by the same theorems of standard deontic logic.

The metaphor: putting in boxes but with diverse logical structure (II)

Example

The difference in logical structure of the two "boxes" is also visible from the following facts:



- A perfect counter-set can have a contradictory pair of (negated) permission-norm contents, which means that a certain state of affairs is optional. This fact does not cause an "explosion" since in this box the principle *ex contradictione quodlibet* does not hold.
- Presence of a disjunct for each disjunction is a perfection property only for the "counter-set", i.e., permission-norm set.
  - In a perfect system $(\varphi \vee \psi) \in \mathcal{N}$ does not require $\varphi \in \mathcal{N} \vee \psi \in \mathcal{N}$.
  - In a perfect system $(\varphi \vee \psi) \in \overline{\mathcal{N}}$ does require $\varphi \in \overline{\mathcal{N}} \vee \psi \in \overline{\mathcal{N}}$.

According to the extended Von Wright's reinterpretation of deontic logic the norm-giver and the norm-recipient relate to the ideal concepts of obligation and permission.

Definition

Let $\mathcal{L}_{pl}$ be the language of propositional logic. A set $\mathcal{N} \subseteq \mathcal{L}_{pl}$ is called norm-set and contains contents of obligation-norms. A set $\overline{\mathcal{N}} \subseteq \mathcal{L}_{pl}$ is called counter-set and contains negated contents of permission-norms. A normative system is the pair $\langle \mathcal{N}, \overline{\mathcal{N}} \rangle$.

A technical remark

The ideal concepts of obligation and permission can be explicated by pointing out the "perfection properties" of their corresponding sets, namely, of the norm-set and the counter-set. Since the filter structure and the weak-ideal structure of the respective sets will be later recognized as responsible for their perfection properties these terms must be introduced. The first one is a well-know concept while the second will be introduced here.

Definition

A set X of truth-sets is a weak ideal iff (i) $[\![\varphi]\!] \subseteq [\![\psi]\!]$ and $[\![\psi]\!] \in X$, then $[\![\varphi]\!] \in X$, and (ii) if $[\![\varphi]\!] \cap [\![\psi]\!] \in X$, then $[\![\varphi]\!] \in X$ or $[\![\psi]\!] \in X$.

ideal theoretical type	set $X \subseteq \wp S$	its relative complement $Y = \wp S - X$
descriptive type: consistent and complete X	$[\![\varphi]\!] = \{v \mid v(\varphi) = t\}$ is the truth-set of φ; $\{[\![\varphi]\!] \mid \varphi \in Cn(T)\}$ is a maximal-filter (not containing $\varnothing = [\![\bot]\!]$; containing $\{v \mid v \text{ is a valuation}\} = [\![\top]\!]$; being closed under intersection and superset relation, having a member for each pair of a set and its complement)	$\{[\![\varphi]\!] \mid \varphi \notin Cn(T)\}$ is an (algebraic) ideal (containing $[\![\bot]\!]$, not containing $[\![\top]\!]$, being closed under union and subset relation)
normative type: consistent X	$\{[\![\varphi]\!] \mid \varphi \in Cn(T)\}$ is a filter	$\{[\![\varphi]\!] \mid \varphi \notin Cn(T)\}$ is a "weak ideal"

Von Wright's Conjecture

"…classic deontic logic, on the descriptive interpretation of its formulas, pictures a gapless and contradiction-free system of norms. A factual normative order may have these properties, and it may be thought desirable that it should have them. But can it be a truth of logic that a normative order has ('must have') these 'perfection'-properties?"

Georg Henrik von Wright (1999).
Deontic Logic: A Personal View.
Ratio Juris 12:26–38.

A formal explication of Von Wright's reinterpretation of deontic logic asks for the establishment of a connection between the theorems of standard deontic logic and properties of an ideal normative systems.

For this purpose the translation function has been introduced, connecting theorems standard deontic logic with the properties of the norm-set, in my paper (2010) and extended to include properties of the counter-set and relations between the two sets, in my DEON 2016 Proceedings paper.

Berislav Žarnić (2010).
A logical typology of normative systems.
Journal of Applied Ethics and Philosophy 2:30–40.

The three translation functions

$\langle \mathcal{N}, \overline{\mathcal{N}} \rangle$ is a normative system.
$\mathcal{L}_{pl}$ is the language of propositional logic.
Language $\mathcal{L}_{SDL}$: $\varphi ::= p \mid \mathrm{O}p \mid \mathrm{P}p \mid \neg\varphi \mid (\varphi_1 \wedge \varphi_2)$ where $p \in \mathcal{L}_{pl}$.
Language $\mathcal{L}_{\langle \mathcal{N}, \overline{\mathcal{N}} \rangle}$: $\varphi ::= p \mid \ulcorner p \urcorner \in \mathcal{N} \mid \ulcorner p \urcorner \in \overline{\mathcal{N}} \mid \neg\varphi \mid (\varphi_1 \wedge \varphi_2)$ where $p \in \mathcal{L}_{pl}$.
"Quine quotes", $\ulcorner \ldots \urcorner$, will be omitted at most places in the subsequent text for the ease of reading and writing.

$$\tau^+, \tau-, \tau* : \mathcal{L}_{SDL} \mapsto \mathcal{L}_{\langle \mathcal{N}, \overline{\mathcal{N}} \rangle}$$

To norm-set properties

$$\tau^+(\mathrm{O}\varphi) = \ulcorner \varphi \urcorner \in \mathcal{N}$$
$$\tau^+(\mathrm{P}\varphi) = \ulcorner \neg\varphi \urcorner \notin \mathcal{N}$$
$$\tau^+(\mathrm{P}\neg\varphi) = \ulcorner \varphi \urcorner \notin \mathcal{N}$$

To counter-set properties

$$\tau^-(\mathrm{O}\varphi) = \ulcorner \varphi \urcorner \notin \overline{\mathcal{N}}$$
$$\tau^-(\mathrm{P}\varphi) = \ulcorner \neg\varphi \urcorner \in \overline{\mathcal{N}}$$
$$\tau^-(\mathrm{P}\neg\varphi) = \ulcorner \varphi \urcorner \in \overline{\mathcal{N}}$$

To system properties

$$\tau^*(\mathrm{O}\varphi) = \tau^+(\mathrm{O}\varphi)$$
$$\tau^*(\mathrm{P}\varphi) = \tau^-(\mathrm{P}\varphi)$$

$\tau^\star(\varphi) = \varphi$ if $\varphi \in \mathcal{L}_{pl}$;
$\tau^\star(\neg\varphi) = \neg\tau^\star(\varphi)$;
$\tau^\star(\varphi \wedge \psi) = \tau^\star(\varphi) \wedge \tau^\star(\psi)$, where $\star = +, -, *$.

Perfection properties: Ideal NS

What SDL axioms say about normative systems?

POSTULATES AND RULES OF STANDARD DEONTIC LOGIC	OBLIGATION NORM-SET PROPERTIES	PERMISSION COUNTER-SET PROPERTIES
(D) $O\varphi \to P\varphi$	internal consistency (consistency of the obligation-norm set) $\tau^{+}(D) = \ulcorner\varphi\urcorner \in \mathcal{N} \to \ulcorner\neg\varphi\urcorner \notin \mathcal{N}$	completeness of the permission-norm set $\tau^{-}(D) = \ulcorner\varphi\urcorner \in \overline{\mathcal{N}} \vee \ulcorner\neg\varphi\urcorner \in \overline{\mathcal{N}}$
(2*) $(O\varphi \wedge O\psi) \to O(\varphi \wedge \psi)$	closure under conjunction $\tau^{+}(2^{*}) = (\varphi \in \mathcal{N} \wedge \psi \in \mathcal{N}) \to (\varphi \wedge \psi) \in \mathcal{N}$	having at least one conjunct for each conjunction contained $\tau^{-}(2^{*}) = (\varphi \wedge \psi) \in \overline{\mathcal{N}} \to (\varphi \in \overline{\mathcal{N}} \vee \psi \in \overline{\mathcal{N}})$
(Rc) $\dfrac{\vdash_{pl} \varphi \to \psi}{O\varphi \to O\psi}$	deductive closure $\tau^{+}(O\varphi \to O\psi) = \varphi \in \mathcal{N} \to \psi \in \mathcal{N}$ if $\vdash_{pl} \varphi \to \psi$	"closure under implicants" $\tau^{-}(O\varphi \to O\psi) = \psi \in \overline{\mathcal{N}} \to \varphi \in \overline{\mathcal{N}}$ if $\vdash_{pl} \varphi \to \psi$
	RELATIONAL PROPERTIES (SYSTEM PROPERTIES)	
(D*) $O\varphi \to \neg P\neg\varphi$	external consistency $\tau^{*}(D^{*}) = \varphi \in \mathcal{N} \to \varphi \notin \overline{\mathcal{N}}$	
(Com) $O\varphi \vee P\neg\varphi$	"gaplessness" $\tau^{*}(Com) = \varphi \in \mathcal{N} \vee \varphi \in \overline{\mathcal{N}}$	

An example

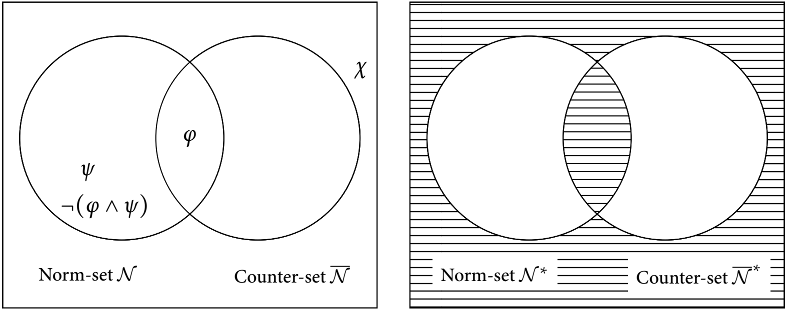


Figure : A comparison between imperfect system $\langle \mathcal{N}, \overline{\mathcal{N}} \rangle$ and system $\langle \mathcal{N}^*, \overline{\mathcal{N}}^* \rangle$, which is endowed with some perfection properties. System $\langle \mathcal{N}, \overline{\mathcal{N}} \rangle$ is internally inconsistent under deductive closure; it is externally inconsistent as the presence of φ at the intersection shows and makes both $\mathrm{O}\varphi$ and $\mathrm{P}\neg\varphi$ true; it is also incomplete since $\chi \notin \mathcal{N} \cup \overline{\mathcal{N}}$. System $\langle \mathcal{N}^*, \overline{\mathcal{N}}^* \rangle$ is externally consistent and complete.

Ideal normative system

An ideal normative system (*ins*) is internally (IntC) and externally consistent (ExtC), its obligation norm-set is closed under conjunction (2*) and entailment (Rc), and it is complete (Comp). An ideal normative system is characterized by the following axiom schemata and rules:

$$(1)\quad \vdash_{ins} \top \in \mathcal{N}$$

$$(2^*)\quad \vdash_{ins} (\varphi \in \mathcal{N} \wedge \psi \in \mathcal{N}) \rightarrow \varphi \wedge \psi \in \mathcal{N}$$

$$(\text{IntC})\quad \vdash_{ins} \varphi \in \mathcal{N} \rightarrow \neg\varphi \notin \mathcal{N}$$

$$(\text{ExtC})\quad \vdash_{ins} \varphi \in \mathcal{N} \rightarrow \varphi \notin \overline{\mathcal{N}}$$

$$(\text{Comp})\quad \vdash_{ins} \varphi \in \mathcal{N} \vee \varphi \in \overline{\mathcal{N}}$$

$$(\text{RcO})\quad \frac{\vdash_{pl} \varphi \rightarrow \psi}{\vdash_{ins} \varphi \in \mathcal{N} \rightarrow \psi \in \mathcal{N}}$$

$$(\text{RcP})\quad \frac{\vdash_{pl} \varphi \rightarrow \psi}{\vdash_{ins} \psi \in \overline{\mathcal{N}} \rightarrow \varphi \in \overline{\mathcal{N}}}$$

Other properties of an ideal normative system are consequences of these axioms and rules. In particular, perfection properties of the counter-set, closure under implicant and inclusion of at least one conjunct for each conjunction contained, can be derived in $\vdash_{ins}$. Warning: There is an error in the proceedings paper: the *Rc* rule from the paper should be replaced by the rules *RcO* and *RcP* given here. I apologize.

Classical deontic logic as a theory of ideal normative systems

The three translation functions when applied to theorems of standard deontic logic (*sdl*) yield the following descriptions:

- if $\vdash_{sdl} \varphi$, then $\tau^{+}(\varphi)$ describes a perfection property of the (obligation) norm-set;
- if $\vdash_{sdl} \varphi$, then $\tau^{-}(\varphi)$ describes a perfection property of the (permission) counter-set;
- if $\vdash_{sdl} \varphi$ and both O and P occur in φ, then $\tau^{*}(\varphi)$ describes a perfection relation between the norm-set and the counter-set.

Since translations of axioms and rules of standard deontic logic yield truths about an ideal normative system, they can be understood as the theory of ideal normative system thus confirming Von Wright's conjecture.

Theorem

If $\vdash_{sdl} \varphi$*, then* $\vdash_{ins} \tau^{+}(\varphi)$, $\vdash_{ins} \tau^{-}(\varphi)$, *and* $\vdash_{ins} \tau^{*}(\varphi)$.

Proof.

All axioms and rules of standard deontic logic can be derived in $\vdash_{ins}$. Therefore, any step of a proof in $\vdash_{sdl}$ can be reproduced within $\vdash_{ins}$. □

Proposition

$\vdash_{ins} \tau^+((\mathrm{O}\varphi \wedge \mathrm{P}\psi) \rightarrow \mathrm{P}(\varphi \wedge \psi))$

Proof.

$\tau^+((\mathrm{O}\varphi \wedge \mathrm{P}\psi) \rightarrow \mathrm{P}(\varphi \wedge \psi)) = (\varphi \in \mathcal{N} \wedge \neg\psi \notin \mathcal{N}) \rightarrow \neg(\varphi \wedge \psi) \notin \mathcal{N}$

1	$\varphi \in \mathcal{N} \wedge \neg\psi \notin \mathcal{N}$	
2	$\ulcorner\neg(\varphi \wedge \psi)\urcorner \in \mathcal{N}$	
3	$\varphi \in \mathcal{N}$	1/ Elim∧
4	$(\varphi \wedge \neg(\varphi \wedge \psi)) \in \mathcal{N}$	2, 3/ 2*
5	$(\varphi \wedge \neg(\varphi \wedge \psi)) \rightarrow \neg\psi$	$\vdash_{pl}$
6	$\neg\psi \in \mathcal{N}$	4, 5/ RcO
7	$\neg\psi \notin \mathcal{N}$	1/ Elim∧
8	$\ulcorner\neg(\varphi \wedge \psi)\urcorner \notin \mathcal{N}$	2–7/ Intro¬
9	$(\varphi \in \mathcal{N} \wedge \neg\psi \notin \mathcal{N}) \rightarrow \neg(\varphi \wedge \psi) \notin \mathcal{N}$	1–8/ Intro→

□

Proposition

$\vdash_{ins} \tau^{-}(O(\varphi \rightarrow \psi) \rightarrow (O\varphi \rightarrow O\psi))$

Proof.

$\tau^{-}(O(\varphi \rightarrow \psi) \rightarrow (O\varphi \rightarrow O\psi)) = (\varphi \rightarrow \psi) \notin \overline{\mathcal{N}} \rightarrow (\varphi \notin \overline{\mathcal{N}} \rightarrow \psi \notin \overline{\mathcal{N}})$

1	$(\varphi \rightarrow \psi) \notin \overline{\mathcal{N}}$	
2	$\varphi \notin \overline{\mathcal{N}}$	
3	$\varphi \in \mathcal{N}$	2/ Comp
4	$(\varphi \rightarrow \psi) \in \mathcal{N}$	1/ Comp
5	$(\varphi \land (\varphi \rightarrow \psi)) \in \mathcal{N}$	3, 4/ 2*
6	$(\varphi \land (\varphi \rightarrow \psi)) \rightarrow \psi$	$\vdash_{pl}$
7	$\psi \in \mathcal{N}$	5, 6/ RcO
8	$\psi \notin \overline{\mathcal{N}}$	7/ ExtC
9	$\varphi \notin \overline{\mathcal{N}} \rightarrow \psi \notin \overline{\mathcal{N}}$	2–8/ Intro→
10	$(\varphi \rightarrow \psi) \notin \overline{\mathcal{N}} \rightarrow (\varphi \notin \overline{\mathcal{N}} \rightarrow \psi \notin \overline{\mathcal{N}})$	1–9/ Intro→

□

Proposition

$\vdash_{ins} \tau^*(\mathrm{O}\varphi \to \mathrm{P}\varphi)$

Proof.

$\tau^*(\mathrm{O}\varphi \to \mathrm{P}\varphi) = \varphi \in \mathcal{N} \to \neg\varphi \in \overline{\mathcal{N}}$

1	$\varphi \in \mathcal{N}$	
2	$\ulcorner\neg\varphi\urcorner \notin \overline{\mathcal{N}}$	
3	$\neg\varphi \in \mathcal{N}$	2/ Comp
4	$\neg\varphi \notin \mathcal{N}$	1/ IntC
5	$\ulcorner\neg\varphi\urcorner \in \overline{\mathcal{N}}$	2–4/ Elim¬
6	$\varphi \in \mathcal{N} \to \neg\varphi \in \overline{\mathcal{N}}$	1–5/ Intro→

□

Logical pragmatics

What can be learnt on *deontic logic* form von Wright's reinterpretation?

- Deontic logic is not the logic of actual normative systems for they can fail in being perfect. It is neither the logic of normative reasoning since its basis can be imperfect.
- Deontic logic is the logic of perfect systems and their perfection properties, the ones that the norm-giver ought to achieve in normative system (re)construction, and the ones to which the norm-recipient relates in her/his reasoning in spite of the actual imperfections.
- Von Wright's late works on deontic logic (from 1980s onwards) show the need of supplementing the research from the viewpoint of logical pragmatics.

  Consider the example of second-order norm that forbids the norm-giver g to produce an externally inconsistent normative system for the norm-recipient r: $[g : \underline{O_r \varphi}]F_g g : \underline{P_r \neg\varphi}$. What is the source of the second-order norm? According to Von Wright it is an requirement of rationality. Nevertheless, there is another explanation: The second-order norms are the norms of language use. Thus, the language of normativity is subordinated to the normativity of language (in use).

Application

Normative gaps

"What is the difference 'in practice' between a state of affairs not being prohibited and its being permitted? Suppose there is a code of norms in which there is no norm P*p*. Now someone makes it so that *p*. What should be the law-giver's reaction to this, if any? Could he say: 'You were not permitted to do this and you must not do that which you are not permitted to do'? He could say this, making it a meta-norm that everything not-permitted is thereby forbidden. 'Logically' this would be just as possible, even though perhaps less reasonable, as to have a meta-norm permitting everything which is not forbidden. But one can also think of some 'middle way' between these two principles, a meta-norm to the effect that if something is not permitted by the existing norms of a code one must, as we say, 'ask permission' of the law-giver to do it."

Georg Henrik von Wright.
Is there a logic of norms?
Ratio Juris, 4:265–283, 1991.

According to von Wright there are three principles by use of which a normative system can be completed: 1. ($\neg F \rhd P$) everything not forbidden is permitted, 2. ($\neg P \rhd F$) everything not permitted is forbidden, 3. normative gaps are filled in communication between the norm-recipient and the norm-giver.
The third principle will be left aside because of its complexity. Using the two-sets model of normative system I will try to show why the first principle is to be preferred over the second one, i.e., why the first principle is "more reasonable". In addition to this I will show that the mere "logical possibility" of the second mode ($\neg P \rhd F$) of filling normative gaps is not a sufficient condition of its rationality, according to von Wright's own criterion of rationality of norm-giving activity.

The easy way to complete a normative system

Definition

A norm-system $\langle \mathcal{N}, \overline{\mathcal{N}} \rangle$ is gapless iff $\{\varphi, \neg\varphi\} \subseteq \mathcal{N} \cup \overline{\mathcal{N}}$ for all doable states of affairs φ and $\neg\varphi$, i.e., $\mathcal{L}_{\text{doable}} = \mathcal{N} \cup \overline{\mathcal{N}}$.

Remark

The notion of "doable state of affairs" is taken over from von Wright's works. The notion of doability introduces complex problems of logic of action. Here the set of sentences describing doable states of affairs will be simplified and identified with the set of contingent sentences, $\mathcal{L}_{\textbf{doable|relevant}} = \mathcal{L}_{pl} - \{\varphi \mid \vdash_{pl} \varphi \text{ or } \vdash_{pl} \neg\varphi\}$ *and restricted only to those which are relevant with respect to the normative system under investigation, i.e., those belonging to the "subject-matter" the normative system deals with. The notion of relevance does not play any decisive role in the argument that follows.*

The easy way of making a normative system complete is by applying the principle *everything which is not forbidden is permitted*. The way of filling in the gaps is straightforward, consisting in adding the missing sentences to the counter set and thus obtaining its extension $\overline{\mathcal{N}}^{\,*}$, as formula (1) shows.

$$\overline{\mathcal{N}}^{\,*} = \overline{\mathcal{N}} \cup \{\varphi \mid \varphi \notin Cn(\mathcal{N})\} \tag{1}$$

The long and branching road…

A completion of a consistent normative system under the principle *everything which is not permitted is forbidden* is not a functional relation. In this mode the process is under-determined and so does not result in a unique system. The completion proceeds in two steps, each of which includes a choice.

The first step

In the first step the counter-set must be completed in the view of perfection relations and properties. Also, the perfection-relation of external consistency between the obligation norm set and its counter-set ought to be preserved if present and so their intersection must remain empty. This means that it will be expanded to achieve perfection-properties of being closed under implicants and under the rule of having at least one conjunct for each member conjunction. Since the last condition has the disjunctive consequent (=at least one conjunct must be present) there may be different ways of performing the closure.

- Let us consider the case of completion of $\overline{\mathcal{N}}$ under the principle of "having at least one conjunct for each conjunction contained". Suppose that $\ulcorner \varphi \wedge \psi \urcorner \in \overline{N}$. There are two cases: (i) either $\varphi \in \mathcal{N}$ or $\psi \in \mathcal{N}$ (both sentences cannot be members thanks to consistency); (ii) $\varphi \notin \mathcal{N}$ and $\psi \notin \mathcal{N}$.
- Case (i) is deterministic: if one sentence is in $\mathcal{N}$, put the other in $\overline{\mathcal{N}}$.
- Case (ii) leaves three options: the two of putting only one sentence into the counter-set and the third of putting both of them there.

Therefore, the weak-ideal expansion of a counter-set results in a set of sets.

The second step

The second step in a completion of normative system is also under-determined and complex in itself. It consists of two phases.
In the first phase the obligation norm-set and its counter-set are closed under appropriate relations by taking into account "partially placed" sentences, i.e., those where only one sentence from a pair of contradictory sentences belongs to the closure of the system.
In the second phase "unplaced sentences" are being added in an iterative manner to the system. "Unplaced sentences" are those where no sentence from a pair of contradictory sentences belongs to the system, i.e., $\ulcorner \varphi \urcorner \notin \mathcal{N}_0 \cup \overline{\mathcal{N}}_0$ and $\ulcorner \neg\varphi \urcorner \notin \mathcal{N}_0 \cup \overline{\mathcal{N}}_0$. The choices abound here:

- Again the construction choice must be made: whether to put φ or $\neg\varphi$ in the expanded norm-set at some level of construction. Given a list of pairs of unplaced sentences the choice can be made at each level provided that the pair is independent from the norm-set constructed at that level.
- Furthermore, there is no preferred ordering of unplaced sentences. The outcome of the iterative process depends on the chosen ordering. In most cases the resulting systems will be radically different.

An example

The systems completed by the application of the principle *everything not permitted is forbidden* do not necessarily give a description of one and the same “ideal state of things”. Rather there is a multitude of “ideal states of things” described by a normative systems completed under this principle.

Example

Let $\mathcal{N} = \{p \vee q\}$ and $\overline{\mathcal{N}} = \varnothing$. Expansion of the counter-set by partially placed sentences and weak-ideal closure of the counter-set together yield the following set of sets: $\{\{\neg p \wedge \neg q, \neg p, \neg p \wedge q\}, \{\neg p \wedge \neg q, \neg q, \neg p \wedge \neg q\}, \{\neg p \wedge \neg q, \neg p, \neg q, \neg p \wedge q, p \wedge \neg q\}\}$. So, a choice must be made. Suppose that the norm-recipient r' chooses the first, and the norm-recipient r'' chooses the second. The expansion of the obligation-set with respect to the counter-set depends on the set being chosen, and thus it yields for r' the set $\mathcal{N}_0^{'} = \{p \vee q, p, p \vee \neg q\}$, while for r'' it is the set $\mathcal{N}_0^{''} = \{p \vee q, q, \neg p \vee q\}$. Finally, the expansion by unplaced sentences depends on the list used in the construction. Suppose that r' chooses List 1 given by: $[q], \ldots$, while r'' chooses List 2:$[\neg p], \ldots$. Then $\mathcal{N}_1^{'} = Cn(\mathcal{N}_0^{'} \cup \{q\})$, while $\mathcal{N}_1^{''} = Cn(\mathcal{N}_0^{''} \cup \{\neg p\})$. Therefore, $(p \wedge q) \in \mathcal{N}_1^{'}$, and $\neg(p \wedge q) \in \mathcal{N}_1^{''}$ The completion results in incompatible ideal states as translations show: For r' it holds that $O(p \wedge q)$ w.r.t. $\langle \mathcal{N}, \overline{\mathcal{N}} \rangle$ completed under the principle $\neg P \rhd F$. For r'' it holds that $F(p \wedge q)$ w.r.t. $\langle \mathcal{N}, \overline{\mathcal{N}} \rangle$ completed under the principle $\neg P \rhd F$.

A critique: how many ideal states?

> "Generally speaking: a legal order and, similarly, any coherent code or system of norms may be said to envisage what I propose to call an ideal state of things when no obligation is ever neglected and everything permitted is sometimes the case. If this ideal state is not logically possible, i.e., could not be factual, the totality of norms and the legislating activity which has generated it do not conform to the standards of rational willing. Deviations from these standards sometimes occur — and when they are discovered steps are usually taken to eliminate them by 'improved' legislation.
>
> Georg Henrik von Wright.
> Is and ought.
> In M. C. Doeser and J. N. Kraay, editors, *Facts and Values: Philosophical Reflections from Western and Non-Western Perspectives*, pages 31–48. Springer Netherlands, Dordrecht, 1986.

If a normative system is completed under the principle *everything not permitted is forbidden*, then, if consistent, it can "envisage more then one ideal state", each equally acceptable as any other. Thus, there will be no unique ideal state with respect to obligation-norms. If intending a unique ideal state is essential to rational willing on the side of the norm-giver, then the the principle *everything not permitted is forbidden* is not only "less reasonable", as von Wright claimed, but also not (instrumentally) rational.

The viewpoint of logical pragmatics

Definition

A consistent normative system $\langle \mathcal{N}, \overline{\mathcal{N}} \rangle$ supplemented with the completion meta-norm m is socially incoherent if it is possible for the two norm-recipients r and r' in their normative reasoning based on $\langle \mathcal{N}, \overline{\mathcal{N}} \rangle \cdot m$ to validly deduce incompatible conclusions.

- If viewpoints of other roles in norm-regulated activities are taken into account, then the completion principle $\neg\text{P} \rhd \text{F}$ cannot be a rational one.
- If norm-recipients use this principle in their normative reasoning, their normative conclusions might stand in exclusive opposition (either being contrary or contradictory). Thus, a normative system closed under the principle $\neg\text{P} \rhd \text{F}$ might be socially incoherent. Consistency (as Von Wright claimed) and social coherence (as is being claimed here) are conditions of rationality of norm-giving activity. So, the meta-norm $\neg\text{P} \rhd \text{F}$ is not only instrumentally non-rational, but non-rational with respect to the logic of normativity.

Deontic logic and rationality wider than individual

"Deontic logic, one could also say, is neither a logic of norms nor a logic of norm-propositions but a study of conditions which must be satisfied in rational norm-giving activity. It is strict *logic* because the conditions which it lays down are derived from *logical* relations between states in the ideal worlds which normative codes envisage."

Georg Henrik von Wright (1993). **A Pilgrim's Progress.** In von Wright, G.H. *The Tree of Knowledge and Other Essays*, 103–113. Leiden: Brill.

- The notion of social incoherence shows that rationality conditions of the norm-giving activity cannot be derived solely from consistency of a singular normative-system. If the task of completion of a normative-system is assigned to norm-recipients, the fulfilment of the task should not result in social incoherence.
- Thus, the meta-principle *everything not forbidden is permitted* is not just an option, but the second-order norm to which the norm-giver is subordinated.
- The norm-giver ought to produce normative systems endowed with perfection properties. Social coherence is a perfection property. Therefore, *everything not forbidden is permitted* is a truth of deontic logic as the theory of ideal normative systems.

Definition

The minimal weak-ideal closure $\mathrm{WI}(\overline{N})$ of a counter-set is the set of the smallest sets a satisfying the following conditions:

1. a includes $\overline{\mathcal{N}}$: $\overline{\mathcal{N}} \subseteq a$,
2. if $\psi \in \overline{\mathcal{N}}$ and φ entails ψ and $\varphi \in \mathcal{L}_{\mathrm{doable}}$, then $\varphi \in a$,
3. a satisfies one of the following conditions:
    1. if $\varphi \wedge \psi \in \overline{\mathcal{N}}$, $\psi \notin \overline{\mathcal{N}}$, $\varphi \notin Cn(\mathcal{N})$ and $\varphi \in \mathcal{L}_{\mathrm{doable}}$, then $\varphi \in a$,
    2. if $\varphi \wedge \psi \in \overline{\mathcal{N}}$, $\varphi \notin \overline{\mathcal{N}}$, $\psi \notin Cn(\mathcal{N})$ and $\psi \in \mathcal{L}_{\mathrm{doable}}$, then $\psi \in a$.

Definition

Function γ picks an arbitrary member of the set $\mathrm{WI}(\overline{\mathcal{N}})$ of weak-ideal sets: $\gamma(\overline{\mathcal{N}}) \in \mathrm{WI}(\overline{\mathcal{N}})$.

Example

Let $\mathcal{L}_{\mathrm{doable}} = \{p, q\}$. Let $\mathcal{N} = \varnothing$. Let the only norm be the norm-permission $\mathrm{P}(\neg p \vee \neg q)$. It follows that: $p \wedge q \in \overline{\mathcal{N}}$; $\mathrm{WI}(\overline{\mathcal{N}}) = \{\{p \wedge q, p, p \wedge \neg q\}, \{p \wedge q, q, \neg p \wedge q\}\}$

The second step in a completion of normative system is also under-determined and complex in itself. It consists of two phases. In each of the two phases lists of sentence are being used in the construction. Lists will be understood as lists of equivalence classes $[\varphi] = \{\psi \mid \vdash_{pl} \psi \leftrightarrow \varphi\}$ $[\varphi_1], \ldots, [\varphi_n], \ldots$.

1. In the first phase the obligation norm-set and its counter-set are closed under appropriate relations by taking into account "partially placed" sentences, i.e., those where only one sentence from a pair of contradictory sentences belongs to the closure of the system.

$$\overline{\mathcal{N}}_0 \in \mathrm{WI}(\gamma(\overline{\mathcal{N}}) \cup \{\varphi \mid \vdash_{pl} \varphi \leftrightarrow \neg\psi \text{ and } \psi \in Cn(\mathcal{N})\})$$

$$\mathcal{N}_0 = Cn(\mathcal{N} \cup \{\varphi \mid \vdash_{pl} \varphi \leftrightarrow \neg\psi \text{ and } \psi \in \overline{\mathcal{N}}_0\})$$

2. In the second phase "unplaced sentences" are being added in an iterative manner to the system. "Unplaced sentences" are those where no sentence from a pair of contradictory sentences belongs to the system.

$$\langle \mathcal{N}_{n+1}, \overline{\mathcal{N}_{n+1}} \rangle = $$

$$= \begin{cases} \langle Cn(\mathcal{N}_n \cup \{\varphi_n\}), \overline{\mathcal{N}}_n \cup \{\neg\varphi_n\} \rangle, & \text{if } \mathcal{N}_n \cup \{\varphi\} \text{ is consistent,} \\ \langle Cn(\mathcal{N}_n \cup \{\neg\varphi_n\}), \overline{\mathcal{N}}_n \cup \{\varphi_n\} \rangle, & \text{otherwise.} \end{cases}$$

$$\langle \mathcal{N}^*, \overline{\mathcal{N}}^* \rangle = \langle \bigcup_{0 \geq i} \mathcal{N}_i, \bigcup_{0 \geq i} \overline{\mathcal{N}}_i \rangle$$

There is no preferred ordering of unplaced sentences. The outcome of the iterative process depends on the chosen ordering. In most cases the resulting systems are radically different.